Same but not The Same: Describing and shaping meaning-making in engineering mathematics problems

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Abstract

Existing paradigms for mathematics education emphasise self-regulated learning, modelling competencies, and learning outcomes in all cognitive, psychomotor and affective domains of learning. However, there is a gap in the engineering education literature with regards to meaning making of mathematics concepts as applied to engineering problems. This paper is written as a configurative dialogue between two educators delivering and supporting teaching and learning of mathematics in two different European contexts and one supporting learning in a South African context. In sharing our practices, we realised that although an exhaustive list of frameworks for mathematics education exists, telling them apart in the context of engineering mathematics education can be difficult and often adds little actionable insights. This realisation motivates the title of this paper "Same but not The Same". We use the semantic plane of Legitimation Code Theory (LCT) to reflect on problem or scenario-based learning of abstract and/or complex mathematics topics such as linear algebra and differential equations through examples of their (engineering) applications. We argue that qualitative descriptions of meaning-making in open-ended, real-life mathematics problems, methods, and tasks are scarce, and that this can create significant challenges for educators to describe, share, and shape their practices.

Keywords: Engineering Mathematics, Applications, Abstraction, Real world problems, Semantics

1 Introduction

Engineering education change often envisions ambitious goals, such as rethinking what kind of engineer society needs (Guerra & Rodriguez-Mesa, 2021), what type of education best prepares students for dealing with open-ended, interdisciplinary and complex problems, and how curricula can be restructured to better reflect authentic engineering practice (Roach et al., 2018). Active learning approaches such as Problem-Based Learning (PBL) have become increasingly central to this vision, promising to develop students' academic achievement and transferrable skills such as social responsibility, communication, teamwork, and self-directed learning through engagement with real-world problems. However, there is a significant difference between envisioning educational change and operationalising and implementing it at the classroom level. Designing new curricula is one thing; designing actual activities, assessments, and learning environments that can operationalise any educational philosophy is another (Remillard & Heck, 2014).

In a review of the literature, Chen and coworkers (2020) identified a range of challenges faced by educators in the implementation of PBL, including the transformation in professional identity and acquisition of skills necessary for the transition from lecturers to facilitators, a lack of practical experience in multidisciplinary fields, the design and evaluation of materials and assessment, as well as limited institutional support or a lack practical resources in supporting the adoption of PBL. In mathematics-heavy disciplines such as engineering, these implementation challenges are particularly exacerbated.

Engineering and the natural sciences are fields with a "strong grammar" (Bernstein, 1999), where knowledge is built within a tightly regulated system of relational and methodological rules which might be out of scope for students to construct on their own. In adapting pedagogical approaches of such disciplines to contemporary educational paradigms, teachers must help students bridge abstract mathematical content via the naturalistic epistemologies of the applied sciences as well as the socio-constructive complexity and messiness of life in society (Sierpinska & Lerman, 1996). The practical implication is that educators seeking to transform their educational practices are not only required to rethink syllaba and classroom contents, but also how knowledge is presented, sequenced, transformed, and connected to other knowledges. In sum, there is a need to consider more carefully how mathematical meaning is made and how/whether meaning is construed in scenarios that are heavily bound by their context.

In this paper, we analyse meaning making in (engineering) mathematics by using the Semantics dimension of the Legitimation Code Theory (LCT) (Maton, 2014) to analyse educational practices within and across three different institutional contexts. In sharing our practices, we realised that although an exhaustive list of frameworks for mathematics education exists, telling frameworks apart in the context of engineering mathematics can be difficult, and the similarities in learning objectives are often far more important than differences. Therefore, our main research question is:

RQ: *What are the organising principles of meaning-making in engineering mathematics PBL education across different institutional implementations?*

To answer this question, the paper is structured as follows: First, a brief description of institutional environments and objectives in mathematics teaching is provided, then artefacts used in our subsequent analyses are introduced. We move onto a brief introduction to LCT Semantics as a tool to conceptualise and reveal how meaning-making is organised within educational artefacts which is followed by individual and comparative an analysis of the three artefacts. We conclude by identifying organising principles behind different educational practices, such as Mathematics PBL, Integrated Engineering Mathematics, and cross-faculty Mathematics teaching to engineering students.

2 Three Institutional Contexts and Related Artefacts

In this section, three institutional contexts shaping the experiences of the authors of this paper are outlined and discussed. For each context, an artefact around the teaching and learning of mathematics is presented which is then analysed through the theoretical framework presented in Section 3. Following initial discussions around the theoretical framework and research objectives, each author selected an existing teaching artefact representative of their context which has been used within their institution.

2.1 Mathematics PBL at Aalborg University

Aalborg University (AAU) in Denmark has since its establishment in 1974 run a problem and project-based learning (PBL) curriculum at all faculties. Half of the students' time is typically used in traditional courses and the other half in PBL groups of up to eight students. Behind the curricula are shared PBL principles of, e.g., problem orientation, project organisation, students being responsible for their own learning as well as project planning, and feedback supervisors who act like facilitators.

The problems addressed in each study programme vary, but there is a tradition for contextualising disciplinary knowledge. Usually that students find a problem in society or within an application of mathematics in professional life fitting the thematic frames of a semester, then they analyse the problem in its context, formulate a more specific problem statement, and finally embark on a problem solution. In earlier semesters, students are often being given suggestions to problems in project catalogues as their level of subject knowledge is often not high enough to be able to formulate relevant problems from scratch by themselves (Kolmos et al. 2004, de Graaff and Kolmos 2007, Askehave et al. 2015).

AAU also has a study programme in Mathematics. Dahl (2018) argues that mathematics fits PBL as (1) The methods of working with mathematics are quite similar in PBL and in research, (2) The concept of society includes the society of researchers and professionals to whom (pure) mathematics is highly relevant, thus suitable for both the pure and applied part of mathematics. Dahl et al. (2023) further discuss different types of PBL-projects in mathematics ranging from projects that are solely driven by an external case, over projects where the concrete case more functions as an opportunity to learn the theory, to finally purely theoretical projects where the problem in internal to mathematics theory.

AAU Artefact

The artefact of analysis from AAU is taken from a 2nd semester group project with five students at Mathematics in 2018 (Simonsen, 2018). The project amounted to 15 ECTS, thus intending to take up have the semester (NB: In most of European countries, 60 ECTS is considered the workload of a full semester). A 5 ECTS course in Linear Algebra was provided at the semester as well as two other mathematics courses of each 5 ECTS. The overall thematic framework for this semester's project is linear programming, particularly

LPP (linear programming problems). With this framework, students chose their own specific angle. Author 2 acted as a co-supervisor of this project with the main supervisor being from the Department of Mathematical Sciences.

The group focused on the diet problem in linear programming. Here, the objective is to maximize or minimize for instance cost while finding the optimal amounts of foods satisfying various conditions of, e.g., nutrition, weight, etc. The specific problem the group worked on was to find the best foods to pack for a fictional one-week hike in the Swedish wilderness. This was a problem solely chosen by students themselves. The students took outset in discussing how heavy a backpack ideally can be considering the weight of a person, discussed limitations such as just putting an estimated weight on sleeping bags, tent, etc. They focus on the weight of the food, its nutrition in terms of proteins, carbohydrates, and fat and no other factors, to simplify the problem. The problem is solved using the simplex method.

2.2 Integrated Engineering Mathematics at UCL

UCL is a research-intensive university with a multi-faculty structure and a student cohort of over 6000 students in the Faculty of Engineering Science. In 2014, following a major curriculum review, the faculty introduced the Integrated Engineering Programme (IEP) as a shared programme framework across eight engineering departments (Mitchell et al., 2019). The establishment of the IEP resulted in a restructuring of mathematics education at UCL Engineering, where mathematics teaching was centralised under a new faculty-wide curriculum and delivered by engineering academics with expertise in modelling, simulation, and design. This was a move away from mathematics as a discipline-in-itself towards mathematics as a language and a tool, which was reflected in the name Mathematical Modelling and Analysis (MMA) for the main mathematics course undertaken by engineering students over their two first years of study.

In 2020, as a response to the migration to emergency teaching during the COVID-19 pandemic, MMA was redesigned to further align the teaching contents to disciplinary engineering practice. In each of the ten weeks of MMA1, over 900 students at UCL Engineering are introduced to a case-study that activates mathematical concepts and techniques and orients them toward a real-world problem or engineering application (Graham, 2022). For example, students learn about derivatives through modelling optimisation in family-run agriculture settings and integrals by designing non-invasive surgery protocols. The problems explored in the course are rooted in themes of social or environmental relevance (de Andrade & Makramalla, 2023), shaped by principles of critical mathematics literacy (Frankenstein, 1990) and critical mathematics education (Skovsmose, 2023).

UCL Artefact

The artefact of analysis from UCL is taken from MMA1's teaching resources introducing students to ordinary differential equations (ODEs) via energy balance on planet Earth through a zero-dimensional model (Marshall et al., 2007). The artefact is framed as a case-study explained via a series of videos recorded by a UCL academic. In the videos, the lecturer models Earth's average temperature in terms of incoming energy from solar radiation, outgoing energy due to light reflection, and energy trapped in the atmosphere caused by the presence of greenhouse gases. The videos outlining the case study are accompanied by content videos explaining the connections between ODEs and differential calculus, ODE terminology, notation and properties, as well as analytical and numerical solution techniques for ODEs which are connected to integration concepts and techniques. Videos explaining mathematical content are followed by automatically graded exercises implemented in the institution's Virtual Learning Environment (VLE) Moodle via the STACK plug-in which allows for personalised feedback and step-by-step solution guidance.

Later in the same week, students practice the contents learned through the asynchronous material in departmentally offered face-to-face workshops led by UCL Academics and facilitated by Teaching Assistants (TAs). The workshops are designed to provide students with opportunities to solve complex problems individually and in groups, where activities are scaffolded and facilitated by staff. In the workshops, mathematical problems range from specific examples that extend or simplify the case-study as well as exercises in applying underlying modelling principles to different situations that are amenable to similar mathematical treatment.

2.3 Stellenbosch University's Mathematics in your Engine(ering)

South Africa (SA) faces significant challenges in the Higher Education (HE) sector, with consistently low throughput and retention, particularly in Science, Technology, Engineering and Mathematics (STEM) fields (CHE, 2022). Given the importance of these fields in addressing national and international Sustainable Development Goals (SDGs), the Department of Higher Education and Training (DHET) have invested significant funding to build educator capacity. All HE institutions have professional and academic support staff to assist lecturers in better understanding and implementing educational strategies across their curriculum, teaching, learning and assessment activities. The SA case study is based on an interdisciplinary group of engineering education researchers, along with science and engineering faculty academics at Stellenbosch University (SU), a research-intensive university in South Africa. Collaboratively, the group – around 10 key members - has been developing explanatory and foundational resources to better equip themselves and their students to integrate physics, chemistry and mathematics in engineering contexts.

SA has grappled with poor mathematics preparedness for HE for several years. Only 6% of the approximately one million school leavers have school mathematics eligible for application in STEM and Medical degrees (Nxasana, S., 2025). This means the available pool of students qualifying for Engineering is very small. The second challenge is that most SA HE institutions which offer engineering qualifications are reliant on science faculties to 'service' the required science and mathematics courses, particularly at first year level. 1st year engineering students are not, therefore, taught mathematics that explicitly links to their chosen degrees. Having conducted a 10-year cohort analysis on student performance and context in the engineering faculty, along with observational data from various academics and surveys, the collaborative research group targeted the first-year mathematics content. It became clear that students were not aware of why or how the mathematics they were learning was relevant to their chosen engineering degrees. The artefact analysed in this paper is a collaborative, multi-modal video demonstrating mathematical concepts such as vectors, differentiation and integration in the context of an internal combustion engine.

3 Theoretical Framework

We use Legitimation Code Theory (Maton, 2014) to analyse the organising principles within and across the different practices outlined in Section 2 in the teaching and learning of mathematics. LCT is a sociological toolkit based on Basil Bernstein's code theory. As a social realist framework, LCT is "theory agnostic" and can be used to analyse practice within and across diverse social ontologies. The basic premise of LCT is that actors in fields of practice simultaneously cooperate and struggle to align the field's language of legitimation closer to their own practice. In this sense, a language of legitimation is the compound system of values that define and measure achievement in a field, for example successfully applying abstract mathematical concepts in engineering practice.

3.1 LCT Semantics

Semantics is one of the dimensions of LCT that is employed to analyse meaning making in practices. LCT Semantics offers two analytical dimensions upon which meaning making is organised. The first dimension is semantic gravity (SG) which refers to the degree in which meanings depend on a specific context and is activated to analyse a continuum of possibilities between "abstract" and "pure" mathematics in engineering education. The second dimension is semantic density (SD), which refers to the degree in which meanings are condensed within a set of practices. LCT has been previously applied in the teaching of chemistry (Blackie, 2015), engineering (Wolff, Blaine & Lewis, 2021), and in defining semantic discontinuities in mathematics teaching (de Andrade et al., 2025). The two dimensions SG and SD exist theoretically within a continuum ranging from strong (+) to weak (-) and can be topologically organised into a Semantic Plane, as shown in Figure 1.

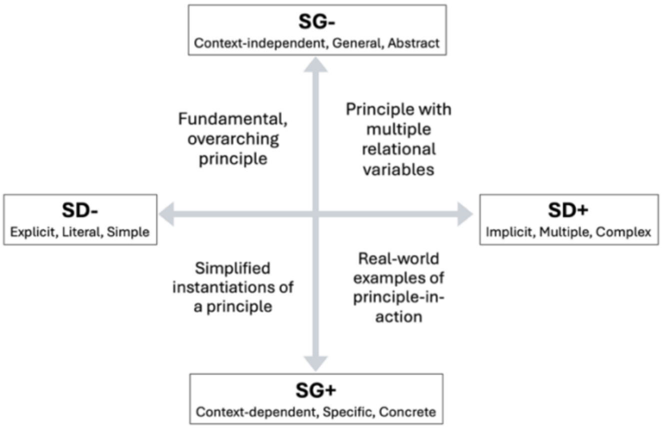

Figure 1: Semantic Plane, adapted from (Wolff et al., 2023).

4 Analysis and Discussion

Each author in this study individually coded the artefact related to their institution, and then all authors reconvened around the first author to discuss and compare coding.

4.1 AAU: Linear Programming Problems

This project is an Aalborg University project, where PBL is the overall educational framework. In PBL, the outset of work is usually an ill-structured problem, which through a problem analysis is then developed into a more narrow and concrete problem statement, which the project group then works on by applying elements from the general theory to find specific solutions. Hence, the *envisioned* semantic tour to be undertaken by students starts as concrete and complex problem (SG+, SD+), to be simplified as students choose boundary conditions and make choices and simplifying assumptions (SG+, SD-). Mathematics is activated via abstract and complex symbolic systems (SG-, SD+) which are used to solve break down the problem into simpler and general parts (SG-, SD-).

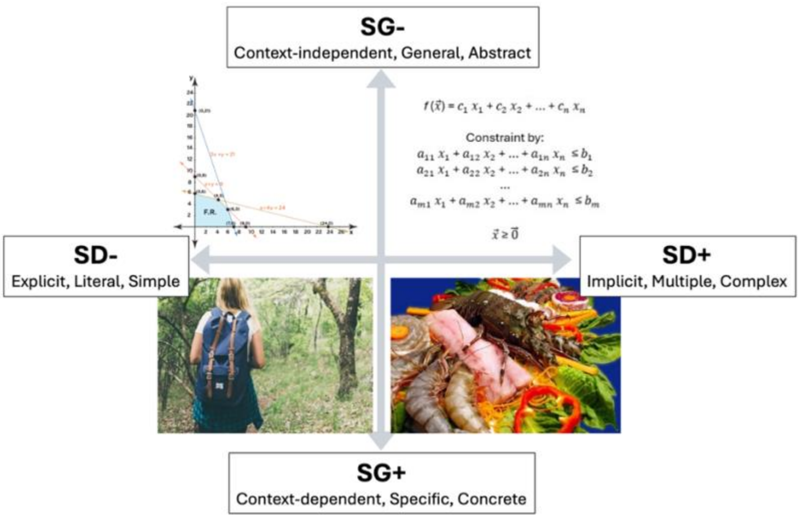

Figure 2: AAU Artefact on the Semantic Plane.

The final conclusions could, and should ideally, move the students back to the abstract world, both the concrete and the abstract to provide some insight into these, based on the group's work. However, often in practice, projects are what is termed Type 2 in (Dahl et al., 2023), where the case from practice often functions as an opportunity to learn a piece of theory. Therefore, the actual journey of the students might omit or skip the complex and concrete step (SG+, SD+), which in this case could be that humans in very different contexts need healthy food for reasonable prices, which naturally is subjective and context-dependent. This is only mentioned it in passing, as the real purpose of the project is to learn the theory with its multiple relational variables (SG-, SG+). In this case, the actual simple and concrete case (SG+, SD-) to some extent does drive the curiosity and motivation, but the case is not overly complex, as most students would instinctively know what to pack for a hike in the wilderness; or in practice they would never have asked mathematics for help.

4.2 UCL: Earth's Energy Balance

The MMA course has two main objectives: the first is to foster mathematical modelling competencies in students, and the second is to provide a solid knowledge foundation to aid students in carrying out complex algebraic manipulations and in implementing engineering problems in MATLAB (Nyamapfene et al., 2022). The case study of Earth's energy balance starts out with the presenter anchoring the concepts into real-world phenomena, such as incoming solar radiation, energy reflected from the Earth's surface, and energy trapped due to greenhouse gases (SG+, SD-). These components are then *conceptualised* as sources or sinks within an energy conservation system, where equilibrium is given by no change in energy.

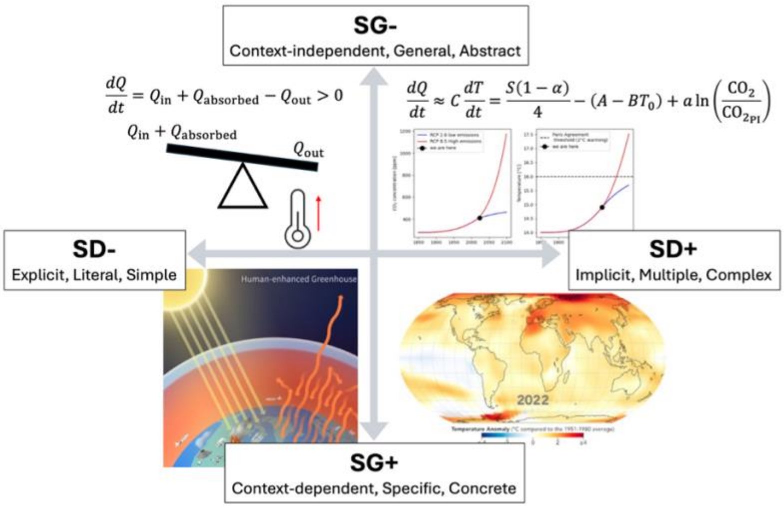

Figure 3: UCL Artefact on the Semantic Plane.

In natural language, this model is given as $\text{Change in Energy} = \text{Energy in - Energy out}$, which is then mathematised in canonical conservation law form (SD-, SG-). This movement towards abstraction serves to bridge concrete meaning into the abstract equation, which is then packed with more meanings via an exposition of the relational variables that compose it. For example, "Energy in" is given by solar radiation, but this radiation only reaches a part of Earth's surface at a time; or "Energy out" would be given by full reflection from the surface, but with changing areas of the polar cap this might need to be adjusted in the model on a yearly basis based on satellite data and must also deduce the energy that is trapped due to the presence of greenhouse gases in the atmosphere. This semantic heuristic that oscillates between complex and abstracts ideas and simple and concrete terms has been called a "semantic wave" elsewhere(SG-, SD+)⟷(SG+, SD-).

As a final step, the model denoting the principle of energy conservation and all its relational variables is linked back to the initial problem situation as a theoretical description of a real-world phenomenon. Later in the week, students have the opportunity to deal with this problem in a more concrete and complex form as they implement the model in MATLAB and retrospectively fit coefficients for energy trapped due to greenhouse gases from real-world average temperature on planet Earth (SG+, SD+).

4.3 SU: Mathematics in your Engine(ering)

The narrative in the SU artefact begins with "*Have you ever wondered where mathematics can be applied in engineering? Luckily, you don't have to look too far – If you've ever driven in a car (SG+, SD-), you've probably heard the noisy rumblings of the internal combustion engine (SG+, SD+)".* The narrator starts in the real world at a fairly simple level and quickly moves to the 'invisible' world of the internal combustion engine (ICE). Literally zooming into the ICE, she then isolates the slider crank mechanism, describes its sub-components (SG+, SD-). To understand how the mechanism works, she explains, we need "to understand some basic maths concepts", and she moves into the simple, abstract domain while providing an explanation of vectors (SG-, SD-). She keeps the explanation in the simple, abstract domain, including a graphic representation. As she unpacks vector basics, she adds layer upon layer of abstract explanations, such as the difference between 'dot' and 'cross' products, and then the role of differentiation and integration. These moves collaboratively increase the semantic density (SD ↑), but each time she adds a new element, she returns to the simple slider crank mechanism (SG+, SD-) to explain the relevance of the mathematics.

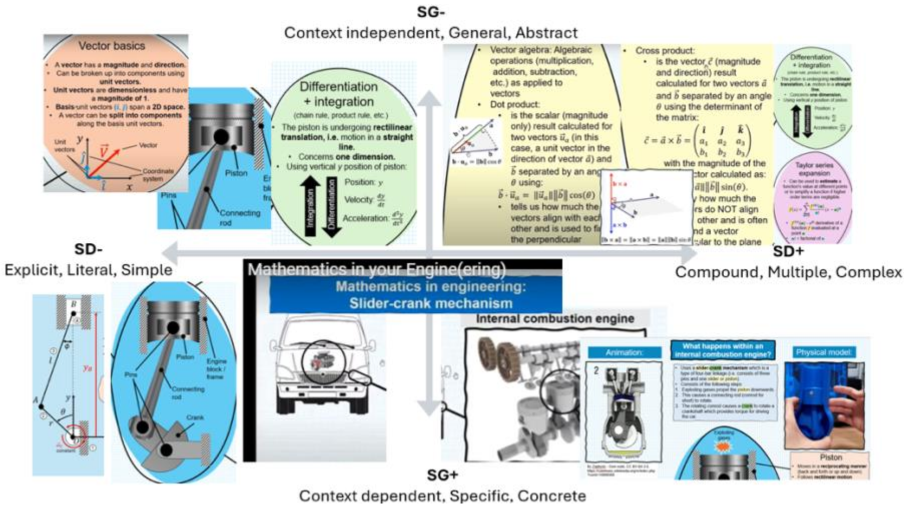

Figure 4: SU Artefact on the Semantic Plane.

The video resource demonstrates the continuous code-shifting along both axes as the narrator builds a picture of how mathematics is relevant in this particular engineering system. As a resource for students, it offers simple and increasingly complex guidance on each of the mathematical concepts using verbal, visual, symbolic and graphical modes of meaning-making. The team working on this particular resource report a significant development in understanding each other's disciplinary fields and how they can be combined to better aid students.

4.4 Revealing Organising Principles Within and Across Practices

In this paper, we have described three institutional practices aiming at bridging abstraction and concreteness in mathematics education. This description was followed by a configurative analysis through the lens of LCT Semantics, which analytically distinguishes between the degree of context dependence of meanings and the degree to which meanings are condensed within practices. By employing the Semantic Plane (Figure 1) we were able to outline a realist description of individual practices (Figures 2 – 4), revealing the principles that underpin meaning making in each of them.

The AAU artefact represented the semantic journey undertaken by students across a second semester project in Linear Algebra. Our analysis of this artefact revealed that student work in this project consisted of breaking down the complexity of a problem by setting boundary conditions and making simplifying assumptions (SD↓) as well as linking concrete simplified ideas to their abstract counterparts (SG↓). In this artefact, the (SG+, SD+) code is used as a motivator, but the life experiences of students supersede the intended socio-complexity of the problem and thus the (SG+, SD-) code serves as a relational anchor to access specialised mathematical objects and procedures (SD↑) that would have been otherwise counter-intuitive or difficult to construct individually.

The semantic journey in the AUU problem is thus based on a simple-concrete code (SG+, SD-) and focused on complex-abstract or simple-abstract codes (SG-, SD+) or (SG-, SD-). In discussing this artefact, we were able to identify that in Mathematics PBL there might be a distinction between *envisioned* and *enacted* semantic journeys. That is, ideally a Mathematics PBL project would start from a real-world problem (SG+, SD+), proceed to simplify it (SD↓) and then abstract and re-pack meaning into it (SG↓, SD↑). However, the *enacted* journey might mention the (SG+, SD+) code in passing such that students work mostly from the more relatable (SG+, SD-) code, defined as a Type 2 Project (Dahl et al., 2023).

One could discuss if this is a *weak* form of PBL practice since the main drive for student work is not based on the complex, ill-posed problem (SG+, SD+). However, using Skovsmose's (2001) framework of landscapes of investigation, we could argue that (PBL) investigation can also happen with reference to pure mathematics (as seen in SG-, SD+). Moreover, as discussed in Simonsen (2018) an investigation can occur in a *semi-reality* (SG+, SD-). Therefore, one could think of PBL as a way of working within the interplay of theory and practice, hence the actual "route" one takes is less important than the fact that one's work eventually embodies all semantic codes.

The UCL artefact represented a *scaffolded* journey presented to students in contextualising the importance and applications of ODEs, rooted in the urge to align engineering practices to Sustainable Development Goals (SDGs). In this practice, meaning is anchored in visual descriptions of planet Earth as a natural/engineered system so that modelling competencies of mathematising phenomena (SG↓) foreground formal mathematics. The semantic journey follows in waves towards "packing" real-world and algebraic meaning into the basic equation representing energy conservation principles, and each source or sink in the system receive a specific algebraic form that is rooted in close approximations to the real-life situation at hand (SD↑). The explanation ends back at the (SG+, SD-) code, where the initial visual representation of the problem is compared to the principle of energy balance with its relational variables.

The semantic journey in the UCL Artefact is then also based in a simple-concrete code (SG+, SD-), but focuses on preparing students to solve problems in a (SG+, SD+) code. That is, later in the week students implement the model of energy balance on planet Earth in MATLAB with the objective of calculating both Earth's equilibrium temperature (stationary approximation) and also parameterising coefficients the equation based on real-world data (SG+, SD+). Within this stage of learning, students face complex questions such as: "*what is the effect of a shrinking polar cap on the energy reflected from Earth's surface?*" or "*how could this model be improved toward modelling space-dependent reflections in areas of high concentration of greenhouse gases?*".

Similarly, the SU artefact is based on a (SG+, SD-) code which is an anchor to the complexity of an engine (SG+, SD+). Mathematical topics are introduced in the (SG-, SD-) code and the density of meanings is progressively strengthened in abstraction (SD↑), recurring to simple-concrete descriptions (SG+, SD-) in order to avoid losing practical relevance. Most interestingly, although this artefact cumulatively "packs" meaning into the ideas of vector and their products, as well as oscillations and their differential and integral forms, it avoids lingering in a (SG-, SD+) code. This absence points to the purpose to which the artefact has been designed which is to contextualise how mathematics is present in engineering design and practice, or in other words, to strengthen the semantic gravity of mathematics topics (SG↑). As mentioned in Section 2, the SU artefact is different from the AAU and the UCL counterparts in that was designed collaboratively between engineers, physicists, chemists and mathematicians as a support resource to enrichen students' understanding of mathematics in practical engineering design. The semantic journey in the SU Artefact is then based on (SG+, SD-) codes but does not focus on a particular code, but rather on foregrounding semantic gravity shifts (SG↑↓) which naturally build up semantic density (SD↑).

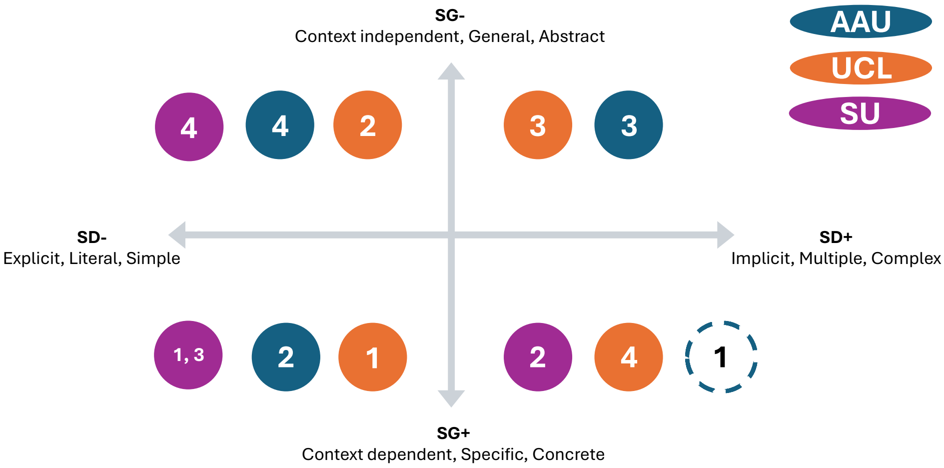

Figure 5: Comparison of semantic journeys across practices. Numbers within circles represent main semantic stages within practices. Blue circles are used to represent AAU, orange circles represent UCL, and purple circles represent SU. The dashed line in stage 1 for the AAU artefact accounts for a possible weakening of SD in this code in practice.

Figure 5 shows a two-dimensional visualisation of the three practices analysed in this paper mapped onto the semantic plane with respect to four of their main stages as coded individually by the authors of the study in their home institutions. The analysis shown in Figure 5 is then conceptualised as an analysis within, as opposed to across, practices to gain insight onto situated semantic journeys related to different ways of thinking about bridging the gap between mathematical abstraction and concrete applications.

The most important similarities between practices seemed to be an anchoring in (SG+, SD-) codes so that mathematical content is relatable, and the mathematical theory has an explicit equivalent in real-life that does not include complexity that is added from technical or socio-cultural aspects, which would result in (SD↑). Across practices, a (SG-, SD+) code was only present in instances where students were involved in solving a problem that involved a high degree of technicality and condensation of mathematical meaning, such as the Linear Programming Project or the Earth Energy Balance scenario.

In our analysis, a (SG+, SD+) code was strongly visible in two instances. First, when contextualising a problem that has both technical and socio-cultural aspects (AAU), and second when implementing complex models via computer programming (UCL). It is important to note that although within the SU artefact there were clear instances of (SG-, SD+), these were not highlighted by Author 3 as stages where the artefact "stayed" for too long. Rather, the objective of the artefact was not to guide students into a semantic context where complexity was required, but to contextualise the connections between mathematical objects and operations such as vectors and vector products and their real-life counterparts in an engine (SG↑↓). Another similarity between the AAU and the SU artefact was an emphasis in "packing" and "unpacking" meaning within (SG+), that is, both artefacts involved a shift from (SG+, SD+) to (SG+, SD-), where the SU artefact presented a "there and back again" tour with stages 1 through to 3 within (SG+).

5 Conclusions and Future Work

In this paper, we have used LCT Semantics to investigate the organising principles underpinning educational practice aimed at bridging the gap between mathematical abstraction and real-world engineering applications. We individually coded three separate artefacts from different institutional contexts in terms of semantic gravity (context dependence of meanings) and semantic density (condensation of meanings into a practice). These artefacts served different educational purposes, where one was undertaken as a semester-long project in a PBL course in linear algebra, the other was presented as a series of videos introducing the real-world significance of ODEs in engineering practice, and the last one aimed at connecting mathematical and engineering meanings in a combustion engine for engineering students and multidisciplinary faculty. Our analyses revealed three distinct semantic journeys shown in Figure 5, each related to the specific context of education and the learning objectives of the artefacts under analysis.

Across all three cases, LCT Semantics provided a configurative language for describing and comparing how mathematical meanings are introduced, transformed, and operationalised. This analysis makes visible the often-implicit pedagogical designs behind mathematics tasks and offers a conceptual toolkit for ensuring that design of mathematics education resources is intentional, inclusive, and connected to students' lived experiences. More importantly, LCT Semantics proved equally useful in mapping out practices across seemingly incompatible social ontologies such as socio-constructivism in PBL and the scientific realism of research-based educational case-studies.

Linking back to our motivating problem, which was the absence of clearer analyses of what characterises PBL teaching and learning in engineering mathematics, we could analytically distinguish three important components in contextualising mathematics in engineering mathematics which are useful for educators in designing problem-based learning of mathematics in engineering.

- Firstly, the presence of (SG+, SD-) codes in all artefacts pointed towards the use of simple everyday language as well as visual aids in grounding and anchoring the understanding of students in studying mathematics in engineering. This was a seemingly invariant code in all artefacts where all practices effectively started (note that the AAU artefact was classed as a Type 2 problem).
- Secondly, two main shifts in meaning-making were identified. The first comprised weaking semantic density within strong semantic gravity (simplifying problems), which involves breaking down a complex (SD+) and concrete (SG+) situation into constituent parts in everyday language (SD↓). The second included weakening semantic gravity (mathematising phenomena and their relationships), which consisted of using equations and symbolic variables to represent a concrete problem-situation via a mathematical model (SG↓). This second practice took place across SD+ and SD- instances.
- Finally, we have identified the possibility that semantic journeys in education might present both envisioned and enacted configurations, much similar to issues of alignment between intended and enacted curricula (Remillard & Heck, 2014).

As limitations to our study, we would like to highlight that the three artefacts coded were not one-to-one equivalent in that they were designed for different applications in different institutional settings. Furthermore, our LCT coding of the artefacts involved coding within artefacts, such that no comparisons between SG and SD across artefacts was made.

Future work in this area might involve research problems such as:

- Exploring intra- and inter-coder reliability when using LCT Semantics. Relevant research questions could be "To what degree different actors and their orientations change their coding of artefacts on the Semantic Plane?" as well as "To what degree the codes assigned by an actor change temporally or situationally?".

- Exploring student’s perspectives on LCT Semantics. A relevant research question would be “To what extent do students perceive and conceptualise abstraction and complexity when solving real-world engineering problems and projects using mathematics?”.

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