Working in a problem-based manner with mathematics and crafts gives meaning to mathematics at Danish vocational schools

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Abstract

Students at Danish vocational schools often struggle with learning mathematics and finding it relevant. In response to this, the project KLUMP ("competence lifting of mathematics teachers at vocational schools") offered professional development of mathematics teachers and craft teachers through facilitating collaboration on creating teaching materials bridging mathematics and the craft. Five teams were established consisting of one mathematics and one craft teacher in the following areas: carpenter, blacksmith, mason, mechanics, and electrician. The teachers joined KLUMP voluntarily, and the school management supported KLUMP by coordinating time schedules and joining project seminars. The teachers selected relevant topics, where they developed, tested, evaluated, and refined their teaching material. The teachers received inspiration in workshops on, e.g. emergent modelling, inquiry- and problem-based learning, and concepts such as self-efficacy. A central theme is Jaworski's notion about learning communities. KLUMP found that the frameworks of a "bridge" between mathematics and crafts and emergent modelling were easy for the teachers to use. The students were happy when they experienced the bridge. Structures such as timetable planning, teacher allocation, and institutional support were essential for KLUMP to become successful.

Keywords: Mathematics, Vocational education, Teacher professional development, Upper secondary, Learning Community

1 The need for mathematics understanding and vocational education in Denmark

In Denmark, major technological developments have intensified the need for craftsmen to have more skills in mathematics. For instance, a mechanic connects electric cars to a computer to find the fault. It is no longer enough to open the bonnet or look at the mechanical parts. The soil and concrete worker has now replaced the shovel with large, expensive excavators. Therefore, today's craftsman is required to be both a theorist and a craftsman. This technological development alongside retirements from the workforce due to age means that by 2035, Denmark lacks around 150,000 people with a vocational degree, out of a population of 6 million (DA Analyse, 2023). Therefore, a high level of mathematics as well as a propensity for learning a STEM-subject, particularly in vocational fields is more important than before. Sadly, it is going the wrong way with the mathematics achievements in the Danish compulsory school system. The OECD PISA investigation from 2022 showed that mathematics achievement is falling, and there is now a larger proportion of low performers and smaller a proportion of high performers than before (Christensen et al., 2023). The written school leaving exam grades (9^{th} grade) has also dropped (Bjerril & Fahrendorff, 2024). The TIMSS study for 2023, in addition, found that the students' motivation for mathematics has declined since 2015 as well as their self-confidence in mathematics (Kjeldsen et al., 2023). Good mathematics grades (minimum '7', equivalent to grade 'C' on the European ECTS-scale) reduce the likelihood of dropout from a vocational education, while age and wellbeing are also important (EVA, 2023). Thus, the need for raising the level of mathematics understanding and the students' desire to study a vocational STEM subject is urgent.

Professional development of teachers is therefore essential. Only a few research and development projects in Denmark have explored these issues (Lindenskov, 2014; Lindhardt et al., 2025). Our project, KLUMP, focuses on professional development of teachers in various STEM vocational programmes, particularly for first-year students. The Danish education system consists of a 10-year compulsory unit school from grades 0 (aged 6 years) to grade 9. After this, youth can choose between a variety of general academic high schools (*gymnasium*) and a variety of vocational education and training (VET) programmes (Eurydice, 2024). The four main vocational fields are: (1) Care, health, and pedagogy, (2) Administration, commerce, and business service, (3) Food, agriculture, and hospitality, and (4) Technology, construction, and transportation. This paper focuses on programmes within the latter.

2 The diverseness of vocational teachers in Denmark

Teachers at vocational schools have very different backgrounds. The craftsmen teachers who teach in the workshops where the craft is being learnt prior to and parallel with apprenticeships, usually have a craft or vocational background, but little didactical training. The subject teachers, like mathematics, have the 4-year Professional Bachelor's Degree aimed for teaching in the compulsory school, but with typically no craftsman experience. Craftsmen teachers must take a full Professional Diploma (PD) in teaching, which corresponds to two years within a 6-year time limit. Hence, the teachers have a very diverse background.

Based on Jaworski's (2005) concept of "Learning Communities" (LC), we see a potential strength in the diverse backgrounds of the vocational teachers. Both types of backgrounds are valuable, and the teachers can learn from each other – and from educational research. A distinctive feature of LC is that all members are considered equal and learners in professional discourse. Mutual respect is essential. Another distinctive feature is a recognition that teachers do not only "receive" information from experts (e.g. researchers) and then simply implement these ideas. Teachers are seen as translators and interpreters of the knowledge presented to them by researchers into the practice they teach. Teachers have central knowledge of their practice, both in terms of students, the subject matter, and the subject area, and they are therefore also inventors of teaching methods and materials. This means that VET teachers in KLUMP are considered as experts in their own practice, and with unique insight into their subject area. Similarly, researchers and consultants participate on an equal footing with teachers as sparring partners in the process - not as judges.

3 The KLUMP project

The purpose of KLUMP ("competence lifting of mathematics teachers at vocational schools", an acronym in Danish) is to find and communicate answers to how mathematics and craft teachers can achieve professional teacher development through collaborating on teaching, particularly on developing joint teaching material.

3.1 Organisation of the KLUMP project

KLUMP consisted of a consortium of Aalborg University, NCUM (National Centre for the Development of Mathematics Teaching) at Aarhus University, and Tradium Vocational School in Randers. KLUMP took place with teachers and students from Tradium. Teacher teams (pairs) consisted of a craft teacher in a VET programme with a considerable mathematics content, and the same programme's mathematics teacher. Five teams completed the tasks. The carpenters' mathematics teacher was on the KLUMP steering board representing Tradium (fifth author). Each team selected relevant areas in their teaching and analysed, developed, tested, evaluated, and refined the lessons. This included co-teaching, hence being in each other's room (workshop versus classroom), for some for the first time. The new lessons were intended to make mathematics more accessible and relevant for vocational students by creating a bridge between mathematics and crafts. The teams got feedback for further reflection from consultants, where one was a university researcher (second author), the other a VET teacher from another vocational school (third author). The consultants also documented the project through interviews and observation of the students' reception of the new teaching, as well as the teachers' experience with participation in the LCs. The evaluation primarily consisted of a combination of qualitative interviews and observations of students and teachers (Kvale & Brinkmann, 2014; Robson, 2002). The analysis uses Bishop's (1988, 2017) framework on the interactions of mathematics education with culture, where culture is understood broadly to include occupational cultures. A central idea here is that mathematics is practiced differently in different cultures.

KLUMP ran August 2023 – May 2025 and was funded by the Novo Nordisk Foundation with DKK 1,423,948 (ca. 190,000 EUR). The autumn of 2023 consisted of establishing the teams, providing the teachers with an Inspirational Workshop, beginning the work with developing the lessons, and interviews with teachers and students. The interviews were done by the consultants and focused on how the students viewed the relevance of mathematics and finding out what challenges the teachers experienced. During the spring of

2024, the last lessons were developed, tried out, evaluated, and revised. To do this, KLUMP organised an Inspiration and Knowledge Sharing Seminar with KLUMPs Advisory Board. Unfortunately, some of the developments were delayed due to, e.g., teachers leaving the school for other jobs, and therefore, some of the final development and trying out of the lesson were postponed to the autumn of 2024. In November 2024, KLUMP held its Final Conference sharing its results with vocational teachers, developers of teaching materials, and researchers in VET from the whole country. About 35 people participated. The spring of 2025 focused on the documentation and dissemination of the results and lessons.

3.2 Theoretical underpinning

At the Inspirational Workshop in the autumn of 2023, researchers (first and fourth authors) and consultants in KLUMP presented some theoretical frameworks for inspiration. One was “emergent modelling” (EM) by Gravemeijer and Stephan (2002). In EM, the focus is on four stages of the students’ learning. At the beginning, the situatedness is in focus, i.e. the students’ own informal strategies, later comes a specific reference to a subject, after which comes the general, formal, and generic. The model creates a connection from practice to the formal mathematics. We, however, extended the model to create a connection both ways, namely also creating a connection from formal mathematics to practice, i.e. a bridge between practical knowledge and formal mathematical knowledge. Teachers were also introduced to principles and examples of inquiry-based learning (IBL) and problem-based learning (PBL). These teaching methods have in common that the teaching is based on a specific problem in context, where students acquire mathematics through problem solving and modelling (Artigue & Blomhøj, 2013; Kolmos et al., 2004). Finally, teachers were also made acquainted with the concept of “self-efficacy” (Bandura, 1997) which means that if students believe that they can handle a challenge, they will be motivated to get started and to overcome any difficulties along the way.

4 The case of blacksmiths

Due to page limitations, we can only provide details from one of the teams. However, in Section 5, we summarise some of the overall findings across teams. The blacksmith team moved the mathematics teaching to the workshop. The classes were co-taught, and the teachers had several meetings developing the material. The example in Figure 1 is an explanation of what they did with the students when teaching them some mathematics calculations prior to plate rolling.

Figure 1: Blacksmith-Mathematics in the workshop. Here the mathematics teacher (white sweater) illustrates to participants at the KLUMP conference the plate rolling calculations he did with the students. The craft teacher stands next to the mathematics teacher. Conference participants are around the table. The first author took the picture.

The mathematics teacher explained that he normally does the standard generic mathematics formula with the students in the classroom, but that the collaboration with the craft teacher taught him that since material expands during plate rolling, his calculation would be wrong. He should not use the diameter when calculating, but the mean diameter. Then all would be correct. The mathematics teacher taught this in the

workshop, with the craft teacher telling the students, that they should now listen to the mathematics teachers, as “what he is saying is very important”. By doing that the craft teacher also acknowledges mathematics and the mathematics teacher as important and valuable in the education as blacksmith, which can increase students’ motivation for mathematics.

From the interviews with the blacksmith-students and teachers, we learnt that overall, the students are positive towards bridge-building between mathematics and the craft. However, it was not without practical problems moving the mathematics teaching to the workshop. One problem was that using a black/white board is not always easy in a workshop. This was solved by putting up chairs in a half-circle. Another problem was disturbances such as noise from other people working in the large workshops. This clearly presents a dilemma: By moving mathematics into the crafts, a multitude of distractions appear, which may hamper the students’ learning. Both the mathematics and the craft teacher learnt a lot from each other’s field.

Two concrete observations: One blacksmith student who according to his teacher usually did not like mathematics, discovers that the mathematical calculation with the mean diameter makes his plate rolling fit perfectly. It became very motivating for him to get useful mathematics competencies. This also contributed to the self-efficacy of the student. It was also observed that it was literally “hands-on” when some of the students put their hands around the cylinder to get an approximate measurement of the diameter, and when they used a string for the circumference and then suddenly grasped what diameter means. Concretely, the students said (all quotes are translated by the authors to English): “Mathematics in the classroom is boring – much better in the workshop where one can see the meaning of it”; “Like the mathematics that is useful as a craftsman”, “Would like more mathematics in the workshop”. However, we also learnt that some of the students hated being filmed, which meant that on the days when we were filming, which the students knew in advance, less students were present.

5 Overall results for all teachers and students

From the interviews and observations, it was evident that collaboration between a mathematics teacher and a craft teacher is efficient and enjoyable for both. Mathematics aimed at the craft is central for the students who like that the mathematics teacher also teaches in the workshop and the craft teacher also teaches in the mathematics classroom. Mathematics gives meaning for the students when it is relevant and interesting to them, and when the mathematics language is understandable. This was regardless of mathematical level being targeted in class. The students remember mathematics better and they did not get so tired. These findings are in line with the theoretical frameworks of problem- and inquiry-based learning in which relevant problems are motivating to students.

The table below gives an overview of some general experiences across the five teams.

Table 1: The experience of KLUMP on several factors.

Teaching	Collaboration	Frames	Students and classes
We experienced:	Teachers said:	We experienced:	We experienced:
- Craft-related math is the only right thing - Using the hands makes math easier - More motivating than traditional teaching - Bridge-building can happen everywhere - Positive to see the two teachers together, ping-pong between them	- Collaboration is the only right thing with crafts math - Requires joint meetings - Requires management and schedule planning on board - Would be useful with a common teacher room - Know each other’s schedule - Develop a mutual language - Respect for each other’s fields	- The workshop gives more differentiated teaching - The classroom gives a shared presentation for all students at the same time - Difficult to get access to the workshop as a math teacher without a craft	- The math content should give meaning and be relevant to the students - Prefer classes with only one craft, not mixed - Prefer teaching the same formal level of math in a class - Good if math classes are located in the crafts

Building bridges worked both when mathematics was taught in the workshop, and when crafts were taught in the classroom. The two teachers in each team met more frequently about the development of the teaching material compared to before. A collaboration like this, however, demands time, overview, and resources. It is not enough to encourage teachers to collaborate if teachers in practice are busy, located in different buildings where it takes time to move to and from. In a busy daily life, collaboration does not happen. Operation trumps development, which is seen if, e.g., a teacher is sick, and another teacher needs to take over, and this will always have priority. A part of the KLUMP budget gave release to the participating teachers, so they had less teaching than normal to be able to develop the teaching materials for KLUMP.

6 Conclusion – and some how-to suggestions

The five teams chose different approaches for collaboration which all seemed to work. This means that one does not have to choose among (1) mathematics entering the workshop, or (2) the craft entering the mathematics classroom. KLUMP shows that teachers are motivated by spending professional time with other teachers and sharing experiences, and teachers are motivated by seeing more mathematics in the profession.

Students supported what happened in KLUMP as it created meaning with mathematics. Motivation is promoted when students discover that they use mathematics skills to find solutions and streamline their craft. It is essential to create situations in the teaching where the students experience that they are successful. This creates learning and helps the students' self-efficacy. Wellbeing and feeling safe are also essential in order for the teachers to enter into such a collaboration. Teachers open their classrooms/workshops to someone else, and they go into unknown territory.

Teachers supported KLUMP but they were challenged by the difficulty of finding meeting times in a very full teaching schedule. Better scheduling, allowing for teacher meeting times, stable employees so the "investment" in the development can be applied many years ahead, and shared teacher rooms are desired. It is therefore necessary to have a system perspective on teacher development. Without the practical support from the administration in terms of actual planning of when the teachers could meet, including release from teaching/time to do the development, it is not possible. Also, there needs to be facilitators to inspire and push the project to give inspiration and feedback to the teachers on how the implementation goes.

KLUMP found that the frameworks of (1) a "bridge" between mathematics and the crafts and (2) emergent modelling were easy and intuitive for the teachers to use. But the bridge-metaphor can also be discussed. It was an advantage that the metaphor focused on the fact that mathematics and vocational subjects can and should meet, as well as the fact that they can meet many places on the bridge, including at the ends. It may be a disadvantage of the metaphor that it leaves the two places 'mathematics' and 'craft' untouched, and that the bridge itself remains at the same level. Something happens at both places when you collaborate with each other and when you have 'raised your skills'.

Learning mathematics is much more than cognitive and learning to manipulate symbols, as described by Bishop (1988, 2017). Learning mathematics is to become 'enculturated', i.e., through broad learning processes becoming part of a mathematics culture with specific norms and responsibilities. Bishop (1988) recognised many different mathematics cultures from around the world, with these having common grounds based on six fundamental mathematical activities: counting, locating, measuring, designing, playing, and explaining. Bishop saw modern symbolic mathematics as a tool to communicate experiences from such fundamental activities. In KLUMP we find that Bishop's ideas suit vocational education well as the crafts are filled with fundamental mathematical activities and as mathematics is about modern symbolic representations and communication.

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