The Beauty of Mathematical Order
A Study of the Role of Mathematics in Greek Philosophy and the Modern Art Works of Piet Hein and Inger Christensen

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Abstract: *Since antiquity, mathematics has been identified as a science that allows humans to comprehend the inner workings of harmony. In Greek philosophy, mathematical form is celebrated as an eminent source of beauty. The Greek understanding of cosmology is the principal reason for the strong connection between mathematics and beauty in Greek philosophy. A harmonious and definite cosmos is described by the geometrical figure of the circle. The mathematical images of the cosmos are an important starting point for seeing beauty in forms that can be described mathematically. The question, however, concerns whether the connection between beauty and mathematics survives in modern conceptions of beauty. Of course, even in ancient Greece, the concept of beauty was not exhausted by objects that could be grasped mathematically. In the modern context, it would be even more peculiar to argue for a necessary connection between beauty and mathematics. My focus is much humbler: I intend to illuminate how mathematics continues to play a role in certain conceptions of beauty. I also argue that the way in which mathematics is connected to beauty illuminates a specific mode of embodying intellectual insights. The two examples I consider are taken from design and poetry: the invention of the super-ellipse by Piet Hein, the Danish designer, poet, mathematician, and artist; and the Danish poet Inger Christensen's collection of poems,* Alphabet.

Keywords: *Mathematical beauty, Plato's Timaeus, Eudoxus, Piet Hein, Inger Christensen, embodiment of mathematical beauty, mimesis and modern art.*

My aim is two-fold. On one hand, I argue that mathematics is a special and, in many respects, an extraordinary theory of art, which is rooted in the Greek philosophical tradition of viewing mathematical forms as carriers of beauty. On the other hand, I explore how this inherited Greek philosophical tradition has undergone far-reaching changes in our modern world.[1] The Greeks

1 In *Aesthetic Measure* (1933), G. E. Birkhoff attempted to create a general mathematical formula to describe aesthetic beauty. In contrast, I do not mean to argue that *all* beauty can be described mathematically but that mathematics historically has played a great role in shaping notions of beauty and that some modern artists and designers continue to take inspiration from mathematics. Their interest in mathematics reveals a relationship in the philosophical tradition stemming from Plato and Aristotle. The modern aesthetic inspiration by mathematics rests

would have experienced the world as a finite, eternal, and ordered cosmos, whereas we have come to know ourselves as placed in an infinite universe in which our solar system is just one of many such systems in the cosmos. The developments and achievements of modern science have overthrown the culturally inherited belief in an ordered cosmos. Thus, modern humans are confident that we are able to create new technologies and designs to meet the challenges of human life. At the same time, we are aware that the ability to recreate nature by way of science and technology also poses a self-inflicted threat because we destroy natural resources and create technological means of mass destruction. The ambiguity of our place and role in the universe is clearly reflected in the poetry of Inger Christensen. Moreover, it is apparent that Piet Hein's design is a new way of imitating nature. Modern humans are no longer at rest in a harmonic cosmos that is fitted for human life. Instead, we must ourselves to create harmony while we also realize our destructive influence on life.

The article has two parts. In part one, I provide a rudimentary summary of mathematics' role in Greek philosophy in order to delineate the deep relationship between mathematical form, beauty, and art, which we have inherited by way of history. In the discussion of Greek philosophy, I will focus on how geometry, as beautiful form, mediates intellectual insight and sensible perception. I argue that we can find a link from the Greek conception of beauty to modern art and design in the aesthetic theory of Kant. In part two, I apply the theoretical ideas described in part one to two short interpretations of works by Piet Hein and Inger Christensen. I start by discussing how, as a designer, Hein brought mathematical insights into play in his work. From the example of Hein's very elegant and simple creation of a modern form of mathematical beauty, I move to discuss the poetry of Inger Christensen. In the collection of poems called *Alphabet*, Inger Christensen uses mathematics as an artistic means to convey poetic meaning. The beauty of harmonious mathematical forms becomes its opposite in the poems. Inger Christensen was inspired by the Fibonacci sequence of numbers in the construction of her poems. I will explain the alphabet's eerie and alarming connection to the Fibonacci sequence. In the conclusion, I return to the interconnections between beauty, mathematics, and art to consider how Hein and Christensen employ mathematics in the search for beauty and briefly outline some differences between the role of mathematics for the creation of beauty in antique Greece and our modern world. A very evident difference is the significance accorded to the human body.

Part One: Mathematical Beauty in the Greek Tradition

Plato's *Timaeus* is a prime source to consult to apprehend the immense importance of mathematics in Greek philosophy and its legacy in Western thought. The dialogue is atypical, as Plato gives the lead to the Pythagorean philosopher Timaeus from Locri, which is present-day Calabria, Italy. Traditionally, we know Plato as a composer of philosophical dialogues in which Socrates debates with and questions the people of Athens, including philosophers and sophists. *Timaeus* has only a very short prologue that consists of a dialogue among Socrates, Timaeus, Hermocrates, and Critias (17a–27d), during which they decide to listen to Timaeus' speculative account of how the cosmos, human beings, animals, and plants were created. I will not follow Timaeus' deliberations step by step but instead focus on the most significant assertions indicating the strong tie between mathematics and beauty in Greek thought.

Mathematical forms, such as the conic sections, were described by Euclid around 300 BC. The

on a very different understanding of humankind and our place in the universe. Thus, employing mathematics in modern design and art is also an interplay with interesting layers of cultural and philosophical meaning.

intellectual conception of conic figures, such as the circle, ellipse, parabola, and hyperbola, gave rise to geometrical descriptions and rules of transformation and construction. However, Euclid's neat axiomatic comprehension of geometrical forms was preceded by the long development of mathematical thinking. A major difficulty was understanding the relation between geometry and arithmetic. Greek arithmetic was based upon rational numbers, and its explanatory power was challenged by the discovery of irrational numbers, such as √2, which do not have a solution in rational numbers. These difficulties were solvable in the advanced understanding of geometry by Eudoxus, who was a close colleague of Plato, around 330 BC.

The great invention of Eudoxus was to reinterpret irrational numbers such as √2, Φ, and π as non-quantified mathematical relations of magnitude to explain and describe continuous geometrical entities, such as lines, angles, areas, and volumes as well as geometrical proportions, such as the golden ratio, Φ. For example, in school, we learn that the area of a circle is calculated by multiplying the square of the radius by π: Area = $r^2 * \pi$. An illustrative approximation of the proof can be shown by cutting the circle up into smaller parts, of all which have the same length as the ratio (see figure 1). This geometrical invention is thought to be the basis of Plato's mature treatment of mathematics (see Fossa & Ericson 2005 for a historical and mathematical argumentation for the relationship between Eudoxus and Plato), which laid the foundation of Euclid's axiomatic proofs in geometry. Thus, geometry became the most important mathematical discipline, and the proofs of geometrical figures, such as the circle, the ellipse, the triangle, and the square, became the epitome of human intellectual achievement.

The Divine Craftsman

For Plato, geometry was a fitting model of intellectual knowledge because the drawn geometrical figures were images (*eikon*) of the intelligible model figure (*paradeigma*). By means of a sensible hand-drawn image, the intellect is able to understand the model's figures. We may draw an approximation to a perfect circle to comprehend that the round line of the circle is exactly the same at any point, and the radius stays constant. The intellectual grasp of the intelligible geometrical figures entails drawing those that function as images of the properties and relational proportions of the corresponding geometrical forms. There is an intriguing relationship between the intelligible geometrical forms and the physically and sensuously created images of circles, triangles, and squares we use to comprehend geometry. There is also a stimulating relationship between the embodied drawings of geometrical figures and the intelligible grasping of their mathematical significance, which is the only possible means of drawing the sensible images. For Plato, the interrelation between sensible images and the intellectual grasping of their mathematical nature functioned as an image of the intricate interconnections between the sensible and the intelligible. Using an analogy, Plato argued that our beliefs (*doxa*) concerning the sensuous world relate to the eternal knowledge (*episteme*) of being in the same way that the drawn geometrical figure relates to the geometrical form. Moreover, these two kinds of knowing were proportionally related. The ability to draw the physical image (*eikon*) of a geometrical figure is furthermore a craft (*technê*) that we acquire in practice and education. Hence, there is a strong tie between knowledge and craft.

In *Timaeus*, the analogy is taken even further, as Plato argues that the knowledge of the cosmos that Timaeus presents is only an image "that is no less likely (*eikôs*) than any anyone else's" (29c). Hence, it is a likely account of an image of the model. Humans strive to comprehend the created cosmos, but as mortal and finite beings endowed with a spark of intellect (*nous*), we are only able to give a likely account of how the intellect of the demiurge created the cosmos.

The cosmos was created; thus, its coming into being was a product of the craft (*technê*) of the demiurge. Any human intellectual comprehension of this process will always fall short. Consequently, the best account of the cosmos is the one that is not less likely than anyone else's. Plato thereby builds fallibility into his cosmology, not as something that makes his account less reasonable but that demarcates the relationship between the fallible human intellect and craft and the eternal, beautiful, and good cosmos created by the demiurge or divine craftsman. However, just as the divine craftsman created the cosmos, humans can create images that assist in our knowledge of the cosmos. Hence, knowledge is intimately connected to an embodied and sensual relationship to the world. The creative practice of *technê* by the demiurge as well as the human consists of producing something that the material with which both are working does not contain from the outset. In Plato's understanding of this process, mathematical proportions enable humans to envision this creative act (see Gleen 2011), which is what Timeaus employs as he gives his likely account of how the cosmos was created.

By following the lead of geometry, Timaeus gives an account of the creation of cosmos by the demiurge. The first premise is that the cosmos is unitary, created, and beautiful; moreover, the demiurge is good and has crafted the most perfect cosmos. The cosmos was crafted by the *technê* of the demiurge to represent the eternal model. Timaeus conveys his understanding of the beauty of the cosmos and the goodness of the demiurge by giving a likely account of its mathematical structure. Thus, the beauty of mathematical proportions and simple geometrical figures is represented in the images to which Timaeus, as a human being, clings in order to give a likely account. Therefore, the route to insight commences in the sensible embodied drawing of geometrical figures so that intelligible knowledge emerges from this practice. Timaeus argues that cosmos was formed as a perfect sphere, which is unitary and self-sufficient. Hence, the cosmos is "a happy god" (34b) and contains all beings within it. The inner cosmos contains two concentric circles that continually revolve in the same spot (36c). The outer circle is interpreted as the sphere of the fixed stars. It has the same motion, whereas the inner circle is interpreted as the space designated for the planets, which has a different motion. Thus, the retrograde movements of the planets are explainable as instances of difference compared with the motion of the fixed stars. Only after the perfect and eternal movements of the heavenly bodies is in place did the demiurge create time and appoint the planets to be the keepers of time (see 37c–39e).

The space between the fixed stars and planets is occupied by the four elements: fire, air, water, and earth. These are described as standing in a harmonious relation to each other by means of fine mathematical proportions. Furthermore, the elements are imagined in the form of a platonic solid, which according to Plato, are "the most beautiful bodies, dissimilar to one another, but capable in part of being produced out of one another by means of dissolution" (53e). The five platonic solids constitute the only perfectly symmetrical three-dimensional forms arranged in the use of only one figure. In *Timaeus*, Plato argues that the four elements, as the smallest parts of the cosmos, are describable as unique figures that change internally. Because three of the solids are composed of triangles, simpler forms can be made into the most complex forms. Fire, which is the most heavenly and unstable element, is given the form of a tetrahedron—a pyramid-like form of four triangles. Air is identified with the octahedron, which consists of eight triangles that form a spherical double pyramid. The elementary form of water is an icosahedron that consists of 20 triangles. Thus, two fire elements and one element of air become one element of water. Earth, the most stable and corporeal element, is given the form of a cube that consists of six squares. The fifth platonic solid is the dodecahedron, which consists of 12 regular pentagons and resembles a sphere. According to Plato, "god used it up for the decoration of the universe" (55c). This strange passage is often interpreted as Plato's concept of the dodecahedron as the canvas on which the constellations of the stars are depicted (see Gregory 2001: 43).

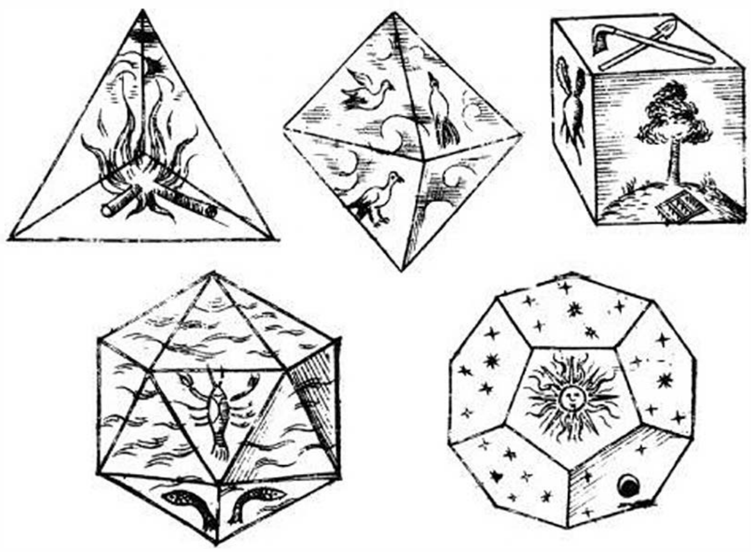

Figure 1: Illustration by Johannes Kepler of the five Platonic solids and the 4 elements as well as the cosmic sphere from the book *Harmonices Mundi*, 1619

Plato shows the human ability to grasp and gives a likely account of the creation of the cosmos that is dependent on mathematical insight. As mortal humans, to appreciate the beauty of the cosmos, we need mathematics, which not only gives us insights into the workings of the cosmos but helps us gain its equanimity in our minds. By contemplating the cosmos, which is the demiurge's creation, we imitate the "revolutions of reason in the heaven and use them for the revolving of the reasoning that is within us, these being akin to those, the perturbable to the imperturbable" so that "through learning and sharing in calculations which are correct by their nature, by imitation of the absolutely unvarying revolutions of the god [cosmos] we might stabilize the variable revolutions within ourselves" (47b–c). The detection of harmonious forms in the cosmos is closely affiliated with mathematical forms and with the individual's peace of mind. Therefore, the pursuit of mathematical knowledge as well as the pursuit of beauty advances serenity. By this means, the ascription of beauty to mathematical forms and the cosmos is by analogy the ascription of beauty to the perpetual forms of nature and our imitation of them in art.

The demiurge created the cosmos as "a living creature endowed with soul and reason" (30c) and imbued it with the heavenly bodies, the four elements, and the gods. After these creations, the demiurge requested the gods to populate this most beautiful and harmonious world with living beings. As beings created by the demiurge, the gods can only be torn apart by the demiurge itself. However, their creations are finite and mortal beings (see 41c–b). Thus, the frailty and fallibility of humans, animals, and plants are due to the gods rather than the demiurge as a divine craftsman.

Human and Divine Technê and the Creative Process

There is a sense in which the job of the divine craftsman does not differ from that of any other craftsman. The demiurge has to act on a material, and the material must be receptive to the model in order to be made into an image (*eikon*) of the model (*paradeigma*). The idea that the world is a product of design has enormous consequences for the relationships among mathematics, design, and art. It implies that human craftmanship can attain perfection in forming matter analogous to the divinely created cosmos. Even though it is obvious that humans are less perfect in all their doings, they may strive to become good by imitating the beautiful creation of the demiurge. We come closest to intellectually understanding the perfect forms of the cosmos by way of mathematics, especially geometry. The human understanding of geometry commences with the bodily practice of drawing and constructing geometrical figures and images. Intellectual knowledge thus emerges from this embodied practice. If we can combine mathematical comprehension with craftmanship, we can create sensuous forms, such as buildings, as imitations of the divine craftsman. We can by analogy become craftsmen who create harmonious forms that will be deemed beautiful if they are akin to the divine bodies created by the demiurge. In the Greek understanding of the human activity of creating objects, human craftsmanship is an imitation of the created cosmos, and thus its beauty is measured by mathematical and natural forms.

According to Aristotle, craftsmanship, not the craftsman, matters. Therefore, he directs attention to the *technê* of craftsmanship and to the process of creation rather than the individual creator. One reason for Aristotle's shift in focus from the agent to the process is his overall critique of Plato's cosmology. According to Aristotle (see *Physics* 251b), the ideas of the beginning of the cosmos as the demiurge's creation and the later creation of time are nonsensical. In Aristotle's view, the cosmos was eternal, and he envisioned it in its totality as an unmoved mover that was perfectly beautiful and indivisible and that contemplated nothing but perfect contemplation (see *Metaphysics* XII, 1072a). Thus, human craftsmen and their creations cannot be understood as analogous to the divine craftsman. Aristotle understood the process of creation as one in which the art of building (the *technê*) was the moving or efficient cause of a house, which resides in the builder (see *Physics* III. 3, 195b). The person who creates new objects as a craftsman is also a practitioner of a certain craftsmanship or *technê*. Hence, form resides in *technai*, such as architecture, painting, carpeting, and so on, which is responsible for its own realization; the medium is the person. Moreover, to become a medium for the realization of form, the craftsman must practice his craft. Thus, Aristotle focuses on practice, which is bodily training.

Plato and Aristotle agree that humans create objects of beauty by imitating the inherent structure of the intelligible cosmos. In Plato's view, we imitate the demiurge; in Aristotle's view, we become media for the process of forming the cosmos. Thus, in the Greek conception of creation—processes of designing and creating art works—individual humans participate in the act of forming by bringing the right *technê* into play. In human creation, it is vital that it takes place in a sensuous environment and involves forming sensuous images that allow intelligible forms to emerge. The sensible and intelligible are deeply intertwined, and there is no access to the intelligible realm of forms without the embodied creation of the sensuous.

As an underlying and defining structure, mathematics enables us to envision how the design product or the art work participates in the much larger world of mathematics and thereby carries connotations to other fields of significance. Mathematics is essential in understanding the close connection between beauty and intellectual experience. The divine heavenly bodies are archetypes of beauty, and any human comprehension of the workings of the cosmos is through mathematics. Thus, mathematics can be viewed as the science that explains beautiful forms in a language that humans can learn. Understanding the mathematical explanation of the conic sections entails both intellectual pleasure and the possibility of being able to create objects that are approximations of these forms. The interdependence between mathematical knowledge and craftsmanship in designing and creating objects emerges. However, according to Aristotle, not only the intellectual endeavors of the mathematician or the mathematically trained craftsman foster intellectual pleasure, but also the connection between art and intellectual comprehension concerns all types of knowledge. Aristotle noted in his *Poetics*, "to be learning something is the greatest of pleasures not only to the philosopher but also to the rest of mankind" (1448b: 13–15).

According to Aristotle, if a product of design or an artwork helps us learn something about the world, it will give rise to intellectual pleasure. If a crafted object alludes to a mathematical structure, the viewer has direct access to the intellectual pleasure of grasping this structure. Aristotle viewed "imitation, … sense of harmony and rhythm" (1448b: 20) as natural to humans and thereby also something in which we instinctively find delight. From antique Greece throughout the Middle Ages and the Renaissance, cosmology implied a harmonious relationship between the cosmos and humans. This conception enabled humans to grasp cosmic harmony and create beauty in human designs using imitation, proportion, or rhythm in a manner that was analogous to the order of cosmos. Its metaphysical and epistemic foundations were the geocentric worldview. Thus, in the early modern period, with the appearance of the heliocentric world view and its mechanical description of the infinite universe, the role of beauty and creation in art and design shifted, as the analogous relationship between the human creator and the divine craftsman lost its metaphysical foundation.

A Modern Reformulation of Beauty: Immanuel Kant

The idea of a certain intellectual pleasure connected to the experience of beauty has a long history, but it was given its modern formulation by Immanuel Kant in the *Critique of Judgement* (1790). In aesthetic judgements, Kant argues that we experience intellectual pleasure as the subjective counterpart to the experience of an object that we deem beautiful. For example, the sensuous experience of *this rose* gives rise to a harmonious play between the power of judgement and the understanding (see §8 of *Critique of Judgement*, AA V: 215). The understanding suggests different concepts to capture what makes the rose beautiful, while the power of judgement insists that these concepts are inadequate: there is something more in the beauty of the rose than can be grasped by concepts and understanding.

Less often recognized in aesthetic discussions of the *Critique of Judgement* is that in the *Introduction* Kant claims that this feeling of intellectual pleasure also accompanies our discovery of order in nature. Thereby, within the framework of his own critical philosophy, Kant reformulates a modern version of the relationship between beauty and nature formulated by Plato. According to Timaeus, "all that is good is beautiful and the beautiful is not void of due measure" (*Timaeus* 887c). From the modern metaphysical and epistemic perspective, there can be no ontological relation between the good, the beautiful, and mathematical proportionality. However, Kant offers a perspective on reflective judgement, which enables us to consider the relationship between our epistemic functions and our search for patterns in the sensible phenomena. He speaks of a *technique of nature* (see *Critique of Judgement*, AA V: 216) that guides us in comprehending a subject to apprehend a unified conception of nature. The understanding of sensuous experience as guided by laws, which was the concern of his First Critique, Kant now argues, explained only how we acquire knowledge of singular experiences. The conceptualization of sensuous experience that arises when the understanding applies concepts to the empirical sensations of intuition cannot explain *why* and *how* we grasp the various different empirical laws of nature as a united and ordered whole. It is, declares Kant in the Introduction to the Third Critique, only through the workings of the reflective power of judgement that we impose a unity on nature, as a feeling of pleasure is attached to the concept of purposiveness in nature (see *Critique of Judgement*, AA V: 186-188). The reflective power of judgement proposes the unification of the singular laws into a purposeful whole, which exceeds the determining judgements that guide the understanding. This understanding can help us apprehend individual natural laws. However, these empirically heterogenous laws can be united under a principle that is not empirical, which is a result of reflective judgement and hence gives rise to the intellectual feeling of pleasure. Kant asserts that unification under a non-empirical principle is incidental; there is nothing in the understanding or in nature that necessitates such ordering. When we judge that nature is an ordered whole, we are solely justified in arguing that we experience nature *as if* it was ordered. We do not have access to any metaphysical or ontological resources in our attempts to understand nature, which enables us to claim that nature is ordered according to a harmonic structure that we can grasp. Nevertheless, we do comprehend nature as an ordered whole.

However, we no longer feel any special pleasure in the comprehensibility of nature even though humanity must have felt such a pleasure in the distant past. This leads Kant to put forward the astonishing claim that if reflective judgement was not used to search for connections between clusters of empirical knowledge, "even the most ordinary experience would not be possible" (*Critique of Judgement*, AA V: 187). Because reflective judgement has an important but undetected role in the comprehension of nature, the feeling of pleasure that accompanies the success of proposing principles of order and unity in our knowledge of nature has become so interwoven with knowledge that it is no longer deliberately noted.

Within the framework of his critical philosophy, Kant develops a modern version of the Aristotelian idea that humans acquire an intellectual feeling of pleasure from comprehending what is empirically given as a system of connections, which mirrors the Greek idea that the closest we can come to grasping the cosmos is through our mathematical descriptions of the movements of the heavenly bodies. However, Kant's understanding of nature is no longer the finite cosmos of the Greeks but an infinite universe. The mathematization of nature in the natural sciences, as founded by Kepler and Galileo in opposition to the cosmology of Greek philosophy and medieval Christianity, were developed from specific empirical laws to unite them under general principles. The movement from the concepts of the understanding to the reflective principles of the power of judgement held the promise of attaining the conception of a unified natural world, but we do not know how far our endeavor reaches. Kant seems to think that if we devise with new principles that unite empirical laws into a whole, they will be accompanied by an intellectual feeling of pleasure that is similar in kind to the feeling of pleasure that accompanies the experience of beauty. Hence, we acquire from Kant's critical philosophy the path for a modern interpretation of why the application of mathematical insights to design processes and artworks gives rise to a distinctive experience of beauty.

In parallel reflective judgements, nature is observed *as if* it is ordered according to a *technique of nature* and view objects *as if* they are beautiful, which introduces within a modern framework a new interpretation of the interrelationship between nature and beauty. Although aesthetic judgements do not single out ontological traits of beauty, and even though the technique of nature is only a perspective of reflective judgement, these judgements give rise to a feeling of pleasure, which encourages us to view nature as an ordered whole and to judge our own creations as analogous. Furthermore, just as geometrical insight has to start with the sensuous embodied practice of drawing and constructing geometrical images and figures, the content of the reflective power of judgement is the singular sensuous experience. We cannot develop general claims about what is beautiful or about which experiences should give rise to a feeling of intellectual pleasure. Reflective judgement arises only from our bodily interactions with the sensuous environment.

Based on this modern formulation of a purely functional relationship between intellectual insights, beauty, and the modern mathematical natural sciences, we now focus on two examples of the application of mathematics in design and art.

Part Two: Mathematical Beauty in Modern Design and Art

In modernity, the analogous relationship between a created and intelligible cosmos and the objects created by humans has been reduced to a myth. However, as we have seen in Kant's foundation of modern aesthetics, the concept of beauty is connected to reflective judgements of our environment *as if* it is conducive to human comprehension. The functional relationship between our judgements of beauty, our comprehension of nature as ordered, and our feeling of intellectual pleasure enlightens our understanding of modern craftsmanship and its relationship to mathematics. In the following, I start by considering Piet Hein's practice of design by revealing that human creation can recreate order in environments that are shaped by humans. The poetry of Inger Christensen, in contrast, indicates that the human reordering of nature by means of scientific and technical developments confronts humans with our civilizational limits.

Creation of a New Form: Piet Hein and the Super-Ellipse

When the city center of Stockholm was being modernized in the late 1950s, the city planners encountered a problem that at first seemed unsolvable. They intended to remodel *Sergels torg*, one of the main squares of the city, so that it would both accommodate the increased car traffic and preserve it as a beautiful space and meeting place for pedestrians in the city. *Sergels torg* is rectangular, and the city planners envisioned a makeover that would allow for a plaza with fountains within a traffic circle. The form of the plaza would repeat the shape of the peripheral road with shops and restaurants on the outskirts. The problem was how to design the road and the interior plaza. A simple circle would waste too much space, the curves in an ellipse would be too narrow for the traffic to move smoothly, and a rectangle would not add novelty to the newly designed square. The city planners tried a modified ellipse with wider curves. However, although this shape functioned as road, it looked awkward when it was repeated in the interior plaza. The chief architect of the renovation project knew Piet Hein from their student days and decided to phone him to ask for help.

As the story goes, Hein immediately had a suggestion for the solution: "What we want," he told the architect, "is a curve that mediates between the circle and the square, between the ellipse and the rectangle. I think a curve with the same equation as an ellipse but with an exponent of two and a half would do it" (Hicks 1966: 56). With the aid of the calculation powers of computers in those days, Hein produced a drawing of the curve with the exponent of two and a half, which fit the specific rectangle of the square. This curve could be increased and decreased without losing its original shape, and thus the inner plaza could be fitted harmoniously into the traffic circle.

Figure 2: Sergels torg with super-elliptical round-about and interior plazza. Stockholm early 1960'ies.

By thinking about the problem as a geometrical challenge, Hein invented a new shape, the super-ellipse, which could be increased and decreased in size to fit the different areas of the rectangular square. Hein pointed out that its superiority lay in its "unity, like a piece of music" (Hicks 1966: 56). The city planners' freely constructed modified ellipse was doomed as a solution to the challenge because it "isn't fixed, isn't definite like a circle or square. You don't know what it is. It isn't esthetically satisfying" (Hicks 1966: 66). Hein stressed that a form drawn freehand would not be aesthetically appealing because it would lack definiteness. This argument shows that he adhered to the Greek ideal that the symmetry and definiteness of mathematical forms are more beautiful than the contingency of a form created using only the human eye. The proportions and the precise calculation of the super-ellipse are appealing not only to the human eye but also to the human intellect. Designed in the curves of the super-ellipse, the road enabled both people and cars to move more freely than they would have on a road in the shape of an ellipse or a rectangle. Thus, Hein's design was not merely an intellectual achievement but a practical invention highlighting that the practical use of the design object is central to good design processes. The obvious but necessary restriction in all design is that it must fulfill its practical purpose. At *Sergels torg*, Hein's super-ellipse enables humans to drive cars and enjoy café life in the same square. Good design always starts by considering the embodied human being and the humanly shaped environment.

The mathematical formula of the super-ellipse has been known at least since the French mathematician Gabriel Lamé described it in 1818 (see Solomon 1999: 48 for a concise description of the mathematical properties of Laméian ellipsoidal harmonics). The super-ellipse is a special instance of the theory of ellipsoidal harmonics, which Lamé developed to describe the equation by which the curve of an ellipse is transformed.[2] The curve of an ordinary ellipse becomes increasingly similar to the square if the value of n becomes greater than 2. It will become increasingly pointed and star-like when the values of n are less than 2. Hein knew about these mathematical descriptions of transformations of the ellipse, and, as a designer, he drew on his mathematical knowledge to create an aesthetically satisfying solution to the challenge posed by *Sergels torg*. The first to calculate the form *and* to bring it into being as a physical object, Hein

2 An ordinary ellipse is described by the equation $(x/a)^2 + (y/b)^2 = 1$. If a = b the ellipse becomes an ordinary circle. Ellipsoidal harmony, as an equation for describing all variations of an ellipsoidal form, is defined as: $|x/a|^n + |y/b|^n = 1$. The super-ellipse has the exponent of n = 2.5.

called his favorite exemplar of ellipsoidal harmonics the super-ellipse. Its curve is close to the rounding of an ordinary ellipse, but it is also squared when the value of n is 2.5.

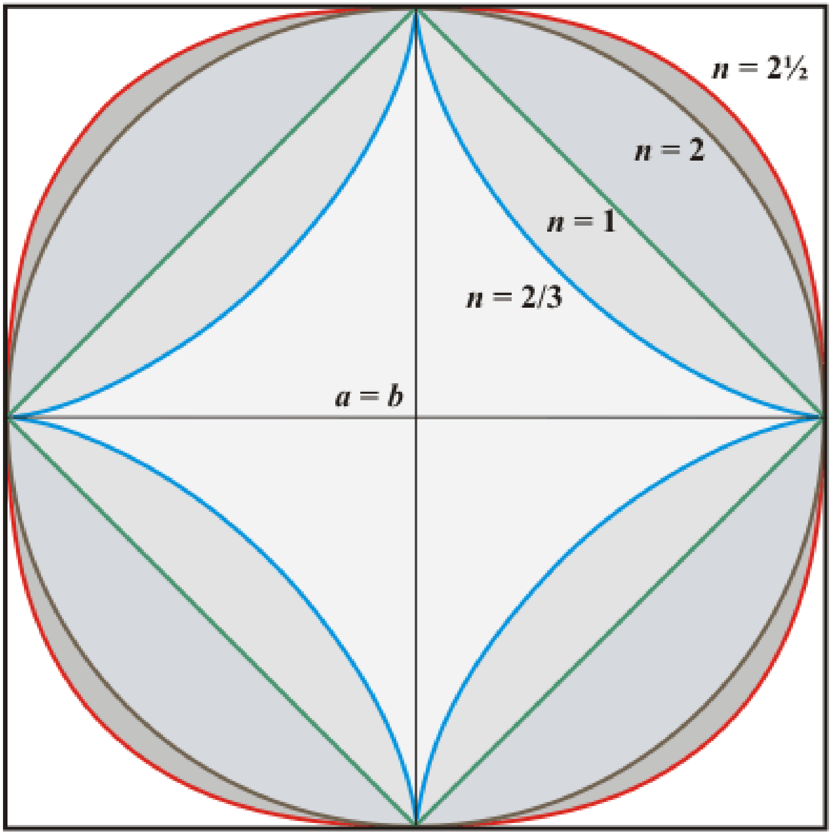

Figure 3: Different curves of an ellipse with one centrum. n = 2 is the ordinary circle. For any value of n smaller than 2 the figure becomes more and more pointed. n = 2,5 displays the rounding of Hein's super-ellipse. Values of n larger than 2 makes the figure moreand more square.

Because the super-ellipse mediates between the square and the circle, it can be used to solve the human problem of directing traffic smoothly and harmoniously in a rectangular square. Hein's drawing of a new path for traffic at *Sergels torg* is, in my opinion, an example of how *technê* works through the individual. The combination of Hein's skills as a designer and mathematician enabled him to envision a new solution. Hein has described the creative process of design as mystical, which resembles Aristotle's insistence that *technê* works through the individual. He emphasizes that "you must know the field … but *thinking* you know too much can be a hindrance to creativity. I [Hein] work according to a principle of being unwise. It helps to be unwise. But I had to develop sheer stupidity into unwisdom" (Hicks 1966: 66). The insistence on unwisdom sounds strikingly familiar to Socrates' declaration that he knew nothing. For Hein as for Socrates, the important thing is to be open to new solutions and new knowledge rather than dogmatically adhere to human views. In the process of design, the move from "sheer stupidity" to "unwisdom" develops through continuous engagement with practice. It is by forming objects, drawing images of mathematical figures, and improving these practical skills that the designer can be open to yet unknown solutions.

According to Hein and the Greek tradition of *technê*, the creative process and the invention of beauty are nurtured when we let the knowledge of the field play in an open mind. As I have already pointed out, the mathematics of the super-ellipse was well-known. However, the application of this mathematical form as the most harmonious and elegant way to manage the traffic at the square was possible because Hein was able to think as both a designer and a mathematician. The Aristotelean idea that *technê* speaks through the designer, who in this case was Piet Hein, sheds light on how he thought of the super-ellipse. Because Hein understood ellipsoidal harmonics, he could invent the new physical form of the super-ellipse. Because the unity of a mathematical form makes it possible to enhance or decrease its size, the harmony of the road at *Sergels torg* could be reiterated in harmonious forms within the roundabout, thus generating harmony and beauty in the plan of the square.

The process of designing *Sergels torg* was based on the intellectual comprehension of form as a solution to a concrete and contingently shaped human urban space. According to Hein, the solution that he envisioned for *Sergels torg* addressed a problem that was specific to modernity, which mediated between "two tendencies" of civilization: "one toward straight lines and rectangular patterns and one toward circular lines" (*Life*, 66). These tendencies are humanly produced, and "there are reasons, mechanical and psychological, for both tendencies. Things made with straight lines fit well together and save space. And we can move easily—physically or mentally—around things made with round lines. But we are in a straitjacket, having to accept one or the other, when often some intermediate form would be better" (*Life*, 66). Thus, as a form mediating between the circle and the square, the super-ellipse is a human solution to the "lines [drawn by man] which he himself then stumbles over" (*Life*, 66).

After he designed *Sergels torg* in the form of the super-ellipse, Hein created other objects using the same form. The best known of his designs is the super-ellipse table and the super-egg, which is a three-dimensional, solid super-ellipse. One of the most beautiful features of the super-ellipse table is that it has no end. The rounding of the corners is not done simply by cutting the sharp edges. Hein's creation of tables in a super-ellipse form is an example of how a new form is created in the design process, one that is not found in nature, but has nature-like features because it is mathematically well-described. It is an improvement of the ellipse. The super-egg form is an improved shape of a bird's egg. The super-egg can be stood erect, and its shape is perfectly symmetrical and harmonious. Hence, Hein has imitated nature by employing mathematical figures to create forms that exceed natural forms.

In using the example of Piet Hein's use of the super-ellipse, I have demonstrated that the design process can be helped by applying geometrical forms that surpass the forms of nature. We do not encounter super-eggs in nature; the orbits of the planets are not super-elliptical. Instead, design objects are mediations between the two archetypical directions in human orientation: straight lines and circular lines. Hein's invention of the super-ellipse is a modern response to the needs that arise in the course of fitting human-designed objects to a more or less unified whole. We need cars and desire to drive smoothly in the city, and we have square rooms in most houses but wish to fit many people at a table. The super-ellipse is an efficient shape for our roundabouts and tables. Furthermore, as a definite mathematical form, the super-ellipse satisfies our intellect, thereby providing intellectual beauty in addition to its immediately pleasing form. The artificial environments in human culture require the excellent designer to possess a *technê* that will be able to imitate the human understanding of geometrical forms in nature in order to create design objects with new geometrical properties.

As a bridge to the next example, Inger Christensen, it is fitting to point out that the dimensions of all Piet Hein's super-ellipse tables are approximations of the irrational number $\Phi=$ 1.618034…, which is also known as the golden ratio. Thus, the ideal mathematical proportion is fundamental in Hein's designs. Throughout our history, Φ has given rise to various more or less mystical interpretations. Some have argued that Φ not only proves the mathematical structure of the universe but also that a universe with such unity and order under the surface must have been designed by God—the great demiurge. This is, however, not what should interest us here.

The Play Between the Finite and the Infinite: Inger Christensen

In turning to Inger Christensen, I direct the focus away from design and toward art, specifically poetry. I have already mentioned Aristotle's *Poetics*. In the treatise, Aristotle puts forward the rather questionable theses that all art is imitation or *mimesis*. Since then, critics have asked how can a poem be viewed as an imitation? Does the love poem imitate the feelings of the lover? If so, in what sense? Much of the criticism against imitation as the central and defining feature of art is reasonable. However, Inger Christensen's collection of poems, *Alphabet* (1981), is an example of how the artistic play with the theory of imitation could be used to create intelligent and evocative art. Of course, the collection could be interpreted in many ways. However, in accordance with my theme, which is the mathematical structures in art and nature, I interpret the poems as a playful interaction with the ideal of mimesis.

Christensen's collection of poems is based on a stringent system, which is the imitation of the Fibonacci sequence correlated with the order of letters in the alphabet. The Fibonacci sequence is a sequence of numbers in which each successive number in the sequence is obtained by adding the two previous numbers: 1 + 1 = 2, 2 + 1 = 3, 2 + 3 = 5, 3 + 5 = 8, and so on. The ensuing sequence of numbers has the characteristic that the ratio between any two succeeding numbers is an approximation of Φ, which is defined as 1+ √5/2 = 1.618034. The Fibonacci sequence is an arithmetical expression of the golden ratio. For example, 2+3/3 = 1,666. The larger the Fibonacci numbers are, the closer the approximation to Φ; so, for example, the ratio between the 11^{th} and 12^{th} Fibonacci numbers is 144 + 233/233 = 1.618026.

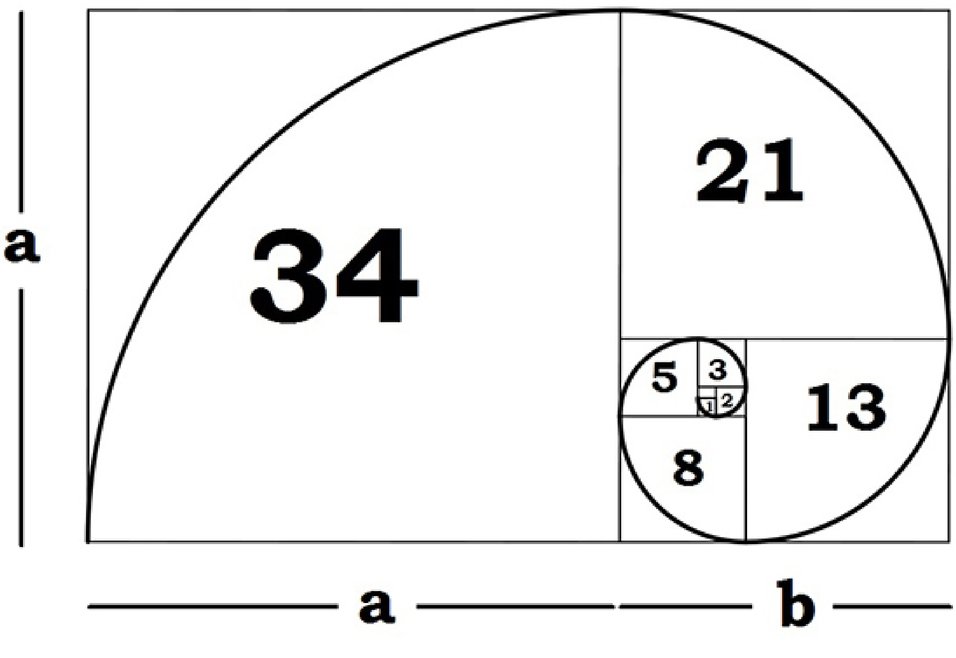

Figure 4: The illustration displays how the proportions of the golden rectangle consists of Fibonacci numbers. a and b being two succeding Fibonaccis numbers.

The golden ratio or golden section expresses the harmonious division of space. It was used by Plato in his famous divided line analogy in the fifth book of the Republic (509d-511e – see Fossa & Erickson 2005) as an analogy of the different kinds of knowledge and the route the individual has to undergo to develop the ability to recognize in mere meaning (*doxa*) the eternal and divine knowledge (*episteme* and *nous*). In Plato's use of the golden ratio, we see the ideal of emergent qualities arising from mathematical proportion as well as the acute interrelationship between sensible and intelligible understandings of this proportion. In the analogy of the divided line, Plato suggested that eternal knowledge reveals to the human intellect that a mathematical structure underlies the phenomena of the world. Plato thus read apparently non-mathematical contexts as though their inherent deep and hidden truth were written in mathematical language (Fossa & Ericson 2005: 76).

The golden ratio gives us an immediate view of a harmonious relationship within a form, whereas the exponential development of the Fibonacci sequence points to vast and relentless growth. Hence, the Fibonacci sequence does not depict a unified form but an eerie, unified formulation of infinite expansion. In *Alphabet*, Inger Christensen exploits the intricate relationship between the golden ratio, which fosters the beauty of a closed form by means of mathematical proportions, and the open and explosive development of the Fibonacci sequence of numbers, which expands exponentially and quickly develops into numbers that are too large to be comprehended in images.

Because the Fibonacci sequence develops exponentially, the 28^{th} number in the sequence is 832,040. In the Fibonacci sequence, it is impossible to write poems with succeeding numbers of lines following the rapidly increasing numbers and for the reader to comprehend a collection of 28 poems in which each succeeding poem has the length of the next number in the Fibonacci sequence. The large numbers are too large for us to view in sensible images, and a poem of 832,040 lines is immensely difficult to read as a unified whole. In *Alphabet*, Christensen does not follow exactly the Fibonacci sequence but plays with the effect of conjoining the closed and contingent system of the order of letters in the alphabet with the open system of the exponentially developing Fibonacci sequence, the form of which is endowed with mathematic necessity.

The Danish alphabet has 28 letters. Each poem in *Alphabet* is connected to a letter by the first sentence in each poem. The first sentence announces the existence of a natural object by correlating the starting letter with its place in the alphabet. For example, the third poem starts with the line, "The cicadas exist"; c is the third letter in the alphabet. The collection of poems is thus structured by two formally structuring principles: the order of letters in the alphabet and the exponential increase of the numbers in the Fibonacci sequence. The beginning letter of each poem follows the alphabet, and the length of the number of lines in each poem develops according to the Fibonacci sequence. Depending on which letter in the alphabet has been reached, the poem has as many lines as the correlated Fibonacci number has. The first poem is simply a reiteration of its first and only line: "Apricot trees exist, apricot trees exit." It is followed by the next poem, which has two lines, and thereby the Fibonacci sequence is begun:

Bracken exists; and blackberries, blackberries;
bromine exists; and hydrogen, hydrogen

The imitation of the Fibonacci sequence as an arithmetical expression of the golden ratio is inhibited because it develops so quickly into large numbers that the finite numbers of letters in the alphabet cannot be represented. Thus, there is a mismatch between the finite and the infinite, which is similar to the mismatch between the sum of letters represented by a Fibonaccian number and the infinity of meanings created by language. Thus, imitation and ability to supersede nature, which Hein's designs exemplify, fall short in Christensen's poems. The failure to represent all letters in the alphabet in this collection of poems conveys artistic meaning. If Christensen had followed the Fibonacci sequence stringently, the last poem would relate to the Danish letter *å* and consist of 832,040 lines. Christensen stops at the letter *n*, which ought to be represented in a poem of 610 lines, instead stopping at line 377. The letter *n* is used in modern mathematical annotation as a space holder for any number. Thus, because the poem stops in the middle of *n*, it suggests that we may comprehend that the entire inventory of letters and things exist as space holders for other things, which might just as well exist but have not been mentioned in Christensen's *Alphabet*.

The poems begin by naming a natural object based on the first letter of the alphabet. One way in which Christensen plays with the idea of mimesis is by imitating the human endeavor to name and list all things in the world. Christensen has called this list a "sloppy dictionary" (Christensen in Holm 2016: 138). By naming things, we move from the sensuous practical engagement with things to an intellectual and theoretical comprehension of them, which is potentially detached from any bodily interaction with the world. The failure to develop 28 poems following the Fibonacci sequence indirectly hints that the endeavor to name and list all things is in vain. Furthermore, Christensen not only lists ordinary and friendly natural objects that exist but also merges this image of the natural world with negative phenomena such as death, pollution, and killers. The relation between words indicates the human use of nature to create weapons, such as the atom bomb. Beneath the surface lurks catastrophe that is our own creation. The alarming possibility that humanity creates disasters by the scientific manipulation of nature gives new meaning to the sudden ending of the poem. Perhaps the catastrophe has already taken place. The final lines of poem 14, which commences with the statement, "nights exist," ends with a vision of children seeking shelter in a cave from what might be an exploding atom bomb:

but they are not children
nobody carries them anymore

In the uncanny and frightening ending of Christensen's collection of poems, children lose their childhood because they are deserted or dead. The collection simply breaks off in a manner that is analogous to death cruelly ending a life. The significance of the application of the Fibonacci sequence as the structure of *Alphabet* can be seen as an imitation of the growth as well as the sudden termination of life. Furthermore, it can be read as an imitation of the fission caused by an atom bomb. Because the letters in the alphabet are a closed and contingent unity, humans can produce names and theories about anything in the world. Similarly, we have produced the means of our own nihilation through modern science and technology. Thus, in Christensen's poems, the usual positive connotations related to the symmetry of the golden ratio become an alarming and uncomfortable mirror of the human demiurge. That is, our quest not only to comprehend but also to transform the natural order seems on one hand to lead to failure because of the infinity of possibilities that Christensen represents in the Fibonacci sequence as a structure representing the finite alphabet. On the other hand, we are also threatened by failure because we have produced artificial objects that can destroy us as well as the planet.

The Beauty of Mathematical Forms in Modern Art: An Outline of a Conclusion

In designing objects, Piet Hein employed mathematics to augment the beauty of our world, and he invented a form to address the challenges posed by modern urban landscapes. In *Alphabet*, Inger Christensen applied the classical trope of mathematical beauty—the golden ratio—and the Fibonacci sequence to represent the potentially uncanny and alarming effects of human endeavors to design and redesign our natural world. Hein's use of mathematics is paradigmatic of the concept of beauty as a finite form, and his manner of creating designs for the modern world adheres to the ideal of reordering the potential chaotic modernity in a neatly finite human-created cosmos. Christensen's play with mathematics has a critical connotation, the subtext of which questions the sanity and serenity of our intellectual quest for understanding and controlling nature. The ability to theoretically model the natural world entails the possibility of transforming the natural order, thus enabling humanity to become a powerful and potentially destructive demiurge. In *Alphabet*, Christensen displays this new powerful human position, which threatens to obliterate our physical existence.

The loss of belief in a finite and eternal cosmos has led to difficult questions about how humans operate in potential infinity. The beauty of the finite cosmos of the Greeks has vanished, but the language of mathematical beauty continues to suggest itself as a reference that can be used to construct novel forms of harmony from the chaotic modern world or as a mimetic approach to articulating the predicaments of our intellectual achievements. Hein's designs exemplify that the practical engagement with mathematics and specific problems of design can reshape the natural world into a pleasing artificial environment that fits both the human body and technological invention. On one hand, Hein augments nature and the space of human movement. Christensen, on the other hand, reveals that human-augmented nature and the human desire to control it by means of science and technology pose potential threats to humankind. She thereby exposes the fragility of the human body and the natural environment as well as the ambiguity of the persistent human desire to control nature.

References

Aristotle (1984) "Metaphysics", *The Complete Works of Aristotle*, vol. I, transl. W.D Ross, Princeton N J: Princeton University Press

Aristotle (1984) "Physics", *The Complete Works of Aristotle*, vol. II, transl. W.D Ross & Hardie & Gaye, Princeton N J: Princeton University Press

Aristotle (1984) "Poetics", *The Complete Works of Aristotle*, vol. I, transl. W.D Ross & Bywater, Princeton N J: Princeton University Press

Birkhoff, George David (1933) *Aesthetic Measure*, Cambridge M. A.: Harvard University Press

Christensen, Inger (2001) *Alphabet*, transl. Nied, New York: New Directions

Fossa, John & Ericson, Glenn (2005) "The divided line and the golden mean." *Revista Brasileira de Historia da Mathematica*, 5(9), 59–77)

Gleen, S. (2011) "Proportions and mathematics in Plato's Timaeus", *Hermathena*, 190, (11–27)

Gregory Andrew (2001) *Plato's philosophy of science*. London UK: Bloomsbury

Hicks, Jim (1966) "A poet with a slide rule. Close-up: Piet Hein bestride art and science." *Life Magazine, 14*.10.1966, (55–66)

Holm, Isaak Winkel (2016): "Et hvidkalket, gudsforladt lys: Inger Christensen's alfabet og katastrofen", *European Journal of Scandinavian Studies*, vol. 46:1 (137-156)

Kant, Immanuel (1790) *Critique of Judgement / Kritik der Urteilskraft*, Akademie Ausgabe V

Plato (2003) *The Republic*, transl. Lee, Pereia, Jowett, London UK: Penguin Classics

Plato (1925) *Timaeus*, transl. W.R.M. Lamb, Cambridge, MA: Harvard University Press

David Salomon (1999) *Computer graphics and geometric modelling*, Berlin: Springer