

The DKup24 uplift model for Denmark

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Abstract

A regional land uplift model for Denmark has been derived from geodetic data from the past century. The model reflects deformations in the national height systems and may be used to dynamically extend the quality of the Danish Vertical Reference DVR90 into the future. Also, the uplift model may play an important role in assessing impacts of climate scenarios when planning future facilities such as coastal protection.

The model is determined using an empirical approach and data from the three national precision levelling campaigns, tide gauges, and continuously operating reference stations acquiring data from Global Navigation Satellite Systems. The NKG2016GIA model by Steffen et al. (2016) is used as reference in the uplift model computations. Changes in coastal mean sea level are not included in the model. The DKup24 uplift model forms a so-called long-term release in the modelling of the land uplift rates in Denmark where the classical geodetic data such as levelling and tide gauge data, are used. Since the release of the previous model in 2016, the data has been reassessed to form a consistent data set.

Keywords: Geodetic vertical reference, crustal deformations, GIA, sea-level, GNSS reference stations, tide gauge data.

1. Background

An accurate and reliable height reference system has become an increasingly important part of the geodetic infrastructure. Various types of construction work need levelling and sea level variations impact on land assessed. To fulfil the requirements, it is important that the height reference system is stable and accessible for the users. In Denmark there are two main challenges in maintaining a stable and accessible height reference system: One is due to vertical deformation of the subsurface; another is due to degradation of benchmarks.

The vertical deformations have been observed and studied since the 18th century where Swedish Anders Celsius observed that sea level was decreasing relative to fix points along the coast (Ekman, 2009). In the 19th century it was acknowledged that the “vanishing water” was caused by land uplift and that this uplift might be associated with the last ice age. De Geer (1888) draw a map of the total uplift in metres since the end of the latest ice age (Figure 1). The magnitude of the total uplift agrees quite well with the about 1 cm/year uplift that are observed with tide gauges and Global Navigation Satellite Systems (GNSS) stations in the Baltic area. In Northern Jutland, Denmark, evidence in the form of sea floor from the stone age (5000 b.c.) has been elevated by

about 13 metres above current sea level (Mertz, E.L., 1924, Jackson et al. 2024), corresponding to a rate of 2-3 mm/year.

Degradation of the network of benchmarks defining the height system is also caused by local subsidence. Most structures are built on sediments and may experience subsidence due to compaction of the sediments which eventually results in inaccurate heights. Finally, loss of the physical markers, e.g., during renovation of houses etc., adds to the degradation and especially the access to the height reference system.

Due to the reasons described above three national precision levelling campaigns have been completed over the last century to redefine the height system.

The first campaign was carried out in 1885-1905 leading to the system DNN GM1891, the second in 1940-1953 leading to the system DNN GI1944, and the third in 1986-1992 leading to current height system DVR90 (Schmidt, 2000). All systems refer to mean sea level at the epoch, so tide gauge information was included as well. Since the late 1990'ies the national reference frame and height system have been monitored by permanent GNSS reference stations.

The purpose of the investigations as described in this manuscript has been to develop an uplift model for Denmark that may extend the use of DVR90 by introducing a dynamic reference framework for the height system. A similar corresponding effort has been conducting studies of the Fennoscandian uplift in a joint Nordic collaboration within the Nordic Commission of Geodesy (NKG). The effort and its results are described in Vestøl et al. (2019). As in this analysis, Vestøl et al. (2019) have derived an empirical uplift model, NKG2016LU, based on the GIA model NKG2016GIA (Steffen et al., 2016) and national levelling and GNSS data. Though the approaches are similar, more data sources are considered in this investigation.

2. Methodology

As the national height systems have been defined through the associated precision levelling campaigns it was decided that the uplift model should describe height changes relative to the geoid. Hence, the uplift rate at a location (ϕ, λ) may be described as the difference between the uplift relative to the ellipsoid minus the vertical change in the geoid:

$$u(\phi, \lambda) = u^{\text{ellipsoid}}(\phi, \lambda) - u^{\text{geoid}}(\phi, \lambda) \quad (1)$$

where ϕ and λ are latitude and longitude, respectively. In contrast to levelling, GNSS measure heights relative to the ellipsoid, so information about the geoid change is needed to link the two types of measurements. In this context the change in the geoid is caused by the changes in the Earth's gravity field due to mass transports inside the Earth associated with the viscoelastic processes of the glacial isostatic adjustment (GIA) (Lambeck et al., 1998).

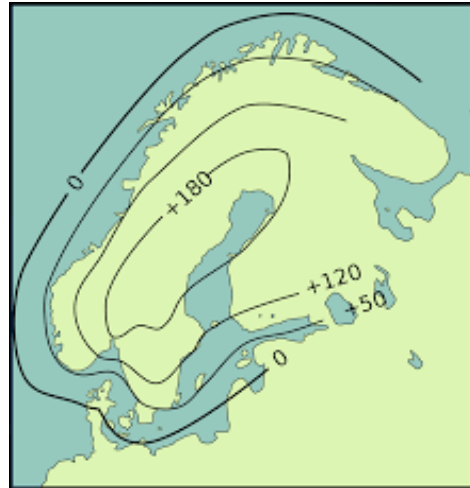


Figure 1. Scandinavia land uplift according to De Geer (1888) (total uplift in metres) (from: https://da.m.wikipedia.org/wiki/File:Scandinavia_land_uplift_according_De_Geer_1888.svg).

Note that temporal changes in the uplift rates are ignored. The uplift rate is assumed to be constant in time. This is a fair assumption considering the time scales of the GIA. Present day changes such as melting of glaciers and icesheets, are considered in the data processing but not included in the modelling of the uplift.

The levelling, however, measures relative height changes and does not provide any reference height by itself. For each of the three national height systems, mean sea level had been used as reference. Mean sea level may change from one epoch to another, though. Hence, an unknown height reference, u_0 , is needed to relate the height changes derived from levelling, Δu , to the uplift. That is

$$u(\phi, \lambda) = \Delta u(\phi, \lambda) + u_0 \quad (2)$$

Observations of sea level at tide gauges depend on the vertical uplift as well. Hence, sea level recordings may be used as observations of the vertical uplift relative to sea level. That is

$$u(\phi, \lambda) = u_{MSL} - r(\phi, \lambda) \quad (3)$$

where u_{MSL} is a constant unknown value representing a rise in sea level relative to the ellipsoid and $r(\phi, \lambda)$ is the observed relative sea level height.

A regional or global GIA model may be used as reference model. Such GIA models integrate models for the glacial history of the ice coverage and observations of sea level in an estimation of the Earth's viscoelastic response. Subsequently, present day uplift rates may be derived from the models. The associated change in the gravity field may be derived from the estimated mass transport to model the corresponding change in the geoid. Alternatively, data from the satellite mission GRACE (Tapley et al. 2019) may be used to estimate the geoid change. In total, the reference uplift model may be obtained as

$$u^{Ref}(\phi, \lambda) = u^{GIA}(\phi, \lambda) - u^{Geoid}(\phi, \lambda) \quad (4)$$

where the reference model expresses uplift rates relative to the geoid. Hence, the residual uplift rates are obtained as

$$du(\phi, \lambda) = u(\phi, \lambda) - u^{Ref}(\phi, \lambda) \quad (5)$$

The residual uplift rates may be estimated from observations from where the reference model has been subtracted as well. The unknown parameters u_0 and u_{MSL} need to be estimated jointly. This may be carried out using least squares collocation as described by Vestøl (2006). However, other representations of the surface, such as polynomials, may be used as well. In this analysis, the following expression, inspired by the gravity potential from a set of point masses, was used

$$du(\phi, \lambda) = \sum_i^N \frac{a_i}{\sqrt{\Delta x_i^2 + \Delta y_i^2 + z_0^2}}; \quad \Delta x_i = \frac{\lambda - \lambda_i}{\Delta \lambda} \cos(\phi_i), \Delta y_i = \frac{\phi - \phi_i}{\Delta \phi} \quad (6)$$

Then du is expressed as a sum of a series of N base functions located in the nodes of a regular grid covering the area and having a grid spacing of $\Delta \phi$ and $\Delta \lambda$ in latitude and longitude, respectively. The z_0 value may be adjusted to obtain a proper width of the base functions. The

unknown parameters a_i are estimated together with the parameters u_0 and u_{MSL} from a set of observations using

$$\begin{pmatrix} u_0 \\ u_{MSL} \\ \mathbf{a} \end{pmatrix} = \mathbf{x} = (A^T C_e^{-1} A + C_x^{-1})^{-1} A^T C_e^{-1} \mathbf{y} \quad (7)$$

where \mathbf{a} and \mathbf{y} are vectors of the coefficients a_i and the residual observations, respectively. C_e is the error-covariance matrix. C_x is the covariance matrix associates with the unknown parameters and is substituted by empirically determined constraints to ensure smoothness. This expression was used to reduce the effects of the highly inhomogeneous distribution of the observations, the levelling data in particular.

3. The data

3.1 The reference model

The NKG2016GIA model by Steffen et al. (2016) is used as the GIA model. It was derived in a comprehensive collaboration in the framework of the Nordic Geodetic Commission (NKG). A series of Earth models were used in determining uplift models that were assessed using time series from GNSS and other geologic data (Vestøl et al., 2019). The final model (NKG2016GIA_preI0306) is shown in Figure 2. The uplift rates reach values up to 10-12 mm/year in the Northern parts of Sweden. In Denmark values of 0-3 mm/year are found. The associated geoid trend model was derived using a series of spherical harmonic coefficients up to degree and order 60 of GRACE (Tapley et al. 2019) and GRACE Follow-On data (Landerer et al., 2020) from April 2002 to July 2024 (GRACE Release 6.0 and GRACE Follow-On Release 6.3 from Jet Propulsion Laboratory, California Institute of Technology). The degree one-degree terms and C_{20} were modified as recommended by Landerer et al. (2020). In addition to a weak filtering of harmonic coefficients of degree and order 14, 28, 32, and 46 no smoothing was applied. The geoid trends were obtained using linear regression and reach values of 0.6-0.7 mm/year in Scandinavia (also Figure 2). In Denmark values of 0.0-0.3 mm/year are found.

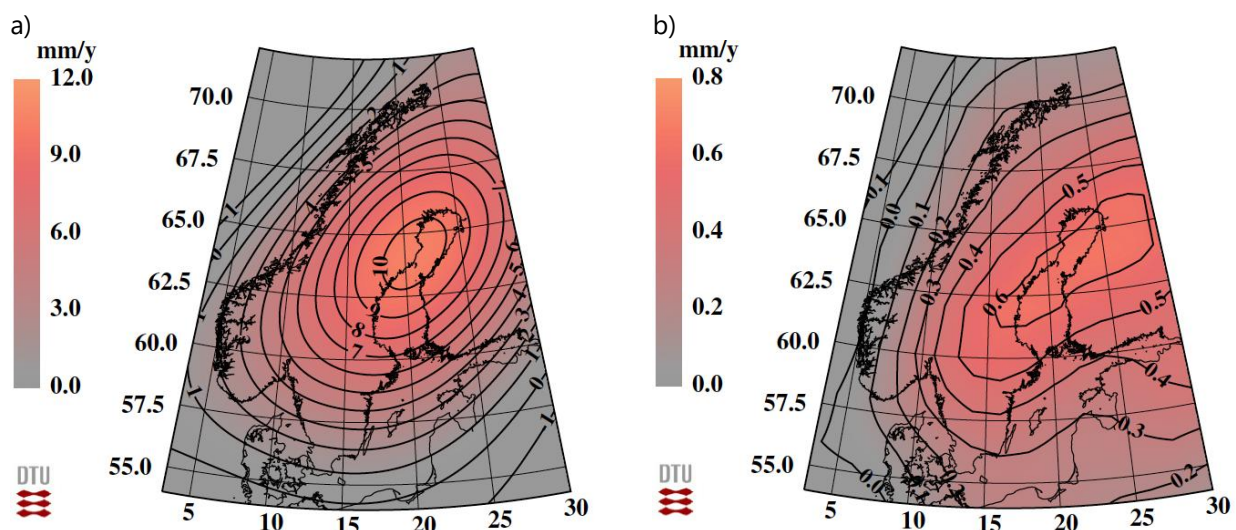


Figure 2. Vertical uplift rates in mm/year from the NKG2016GIA model (a) and geoid rates in mm/year from GRACE (b).

3.2 The levelling data

As described above three national precision levelling campaigns have been completed over the last century. Though the height systems were referenced to mean sea level at the relevant epochs, a benchmark in Aarhus was kept fixed when computing rates between them for this analysis. See also Mäkinen et al. (2005) for details. The rates $\Delta u(\phi, \lambda)$ are shown in Figure 3. Note that the Southernmost part of Jutland was a part of Germany from 1864 to 1920 and therefore not covered by the first levelling campaign. Each set of rates show the general pattern of uplift very consistently.

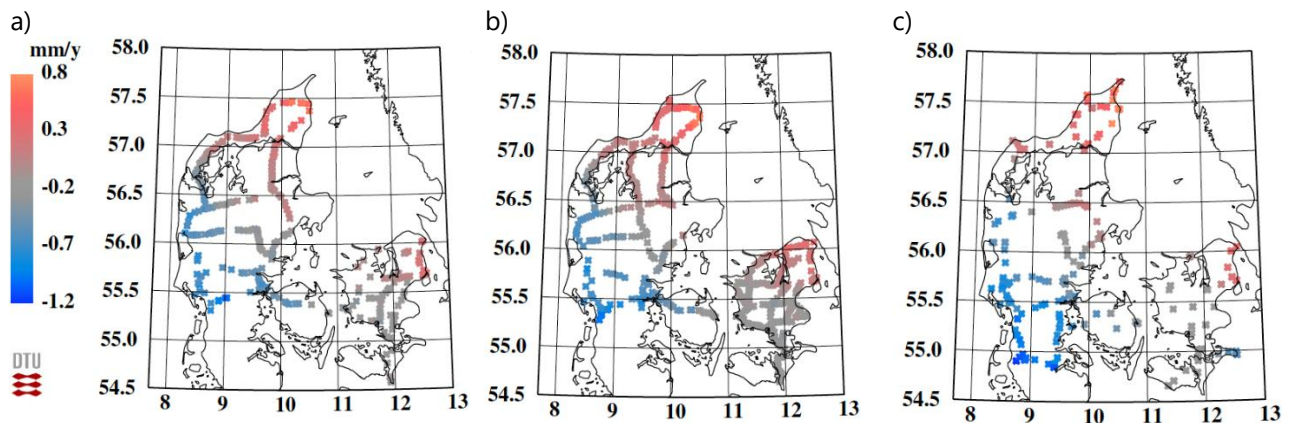


Figure 3. Uplift rates in mm/year from levelling referenced to a benchmark in Aarhus from precision levelling campaign #1 to #2 (a), #1 to #3 (b), and #2 to #3 (c).

3.3 The tide gauge data

The tide gauge data were obtained from PSMSL (Holgate et al., 2013, PSMSL, 2017). The Danish Meteorological Institute has been operating tide gauges since about 1880. Ten stations have time series covering a period of more than 100 years. More stations were established in the early part of the 20th century. It was decided to use the 100 years period 1900-2000 as reference epoch for the computation of sea-level trends. The sea level records are shown in Figure 4. The following stations have almost complete time series in that period of time: Gedser, København, Hornbæk, Korsør, Slipshavn, Fredericia, Aarhus, Hirtshals, and Esbjerg. In addition, data from the Swedish station Klagshamn located close to København was included. Before the linear trends were computed, a seasonal cycle was removed to reduce possible effects of data gaps. The trends are shown in Table 1. Time series from stations at Rødbyhavn, Fynshav, Frederikshavn, and Hanstholm start later. Also, data from Frederikshavn before 1950 had to be ignored to avoid effects of subsidence due to gas extraction during 1930-1950 (Trap Danmarks redaktion, 2025). To use those time series, trends for the time period 1950-2006 were computed for all station. By comparing trends for that period of time, values representing the 1900-2000 trends were derived empirically considering an apparent acceleration at nearby stations. Those sea level trends are shown in Table 1 as well. The uplift rates relative to sea level, $u_{MSL} - r(\phi, \lambda)$, obtained from the tide gauge data records are shown in Figure 5.

3.4 The GNSS data

GNSS data have been acquired from continuously operating reference stations. The Danish Agency for Climate Data (KDS) is responsible for the geodetic definition of the reference network in Denmark. The permanent GNSS stations play an important role in the monitoring and maintenance

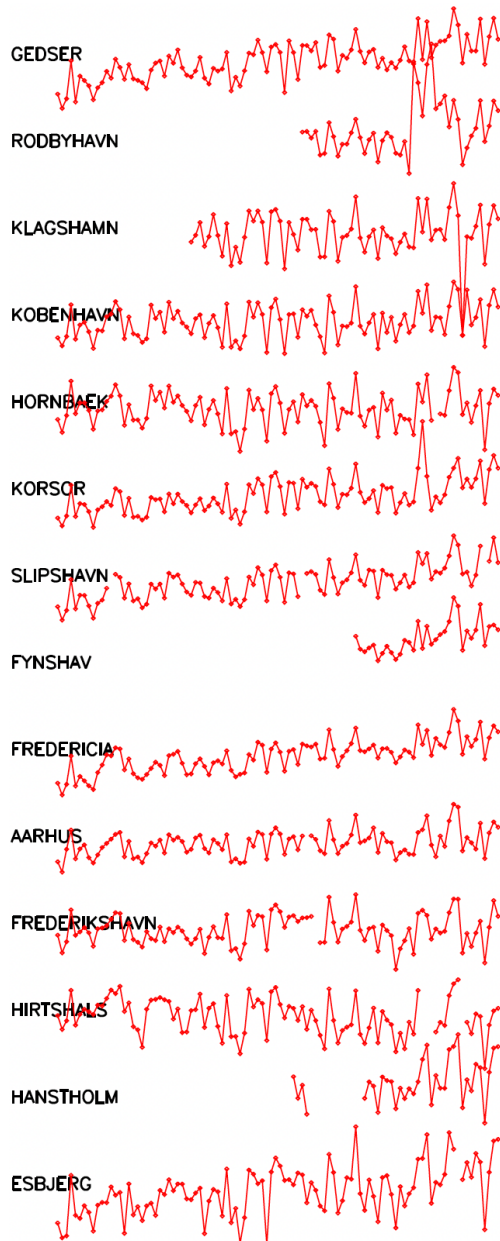


Figure 4. Sea level records of annual means for the Danish stations for the 1900-2000 period.

Station	1950-2006	1900-2000
GEDSER	1.24	1.03
RODBYHAVN	1.53	1.36
KLAGSHAMN	0.82	0.15
KOBENHAVN	1.74	0.39
HORNBAEK	1.09	0.10
KORSOR	1.01	0.74
SLIPSHAVN	1.30	0.85
FYNSHAV	1.58	1.18
FREDERICIA	1.25	0.93
AARHUS	0.96	0.43
FREDERIKSHAVN	-0.13	0.18
HIRTSHALS	0.28	-0.20
HANSTHOLM	1.67	1.21
ESBJERG	1.56	1.10

Table 1. Sea level trends in mm/year for the two time periods, 1950-2006 and 1900-2000, respectively. Red values are estimated.

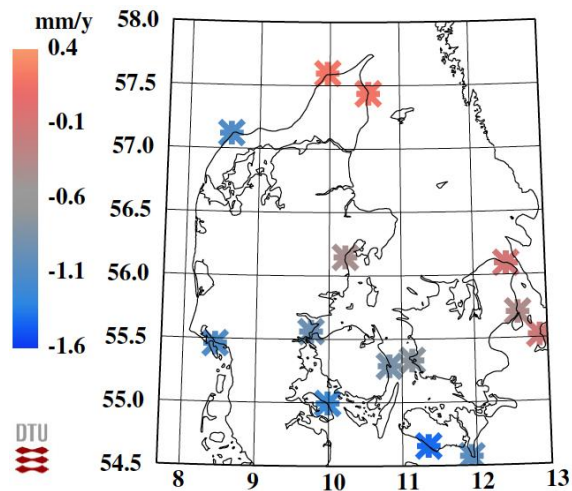


Figure 5. Uplift rates in mm/year relative to sea level from tide gauge data.

of the geodetic network. During 1998 and 1999 KDS established three permanent GNSS station in Denmark, SMID, SULD and BUDP, and later in 2004 ESBH, ESBC, GESR, and HIRS were

established. Using data up to 2016.5 time series of about 12-16 years of daily GNSS solutions are available to determine secular vertical and horizontal displacement rates.

The computation of site coordinates followed the procedure of Khan et al. (2010). We use the GIPSY OASIS 6.4 software package developed at the Jet Propulsion Laboratory (JPL) and released in March 2016. We use JPL final orbit products which include satellite orbits, satellite clock parameters and Earth orientation parameters. The orbit products take the satellite antenna phase centre offsets into account. Receiver clock parameters are modelled, and the atmospheric delay parameters are modelled using the Vienna Mapping Function and corrections to remove the solid Earth tide and ocean tidal loading was applied. The reference frame is ITRF2008 as realized by IGS08. The results of the computations are described in Khan (2017).

In addition to the core stations mentioned above, data from newer stations established by KDS, i.e. HABY, FYHA, FERR, and KMS3, was used as well. Also, data from the DTU station SKAG and data from private stations (AALBK, AARH2, BRNS, BLAH, FEH3, NAK1, and OEHU) was used. More stations could have been used but their time series were considered to be too short. The vertical rates are shown in Table 2 and in Figure 6.

Station	Rates
SMID	0.74
SULD	1.43
BUDP	1.05
HIRS	2.43
ESBC	0.53
GESR	0.45
FERR	0.71
BLAH	0.14
KMS3	1.58
SKAG	2.77
FYHA	0.56
FER3	0.50
BRNS	0.16
HANST	1.13
HVSA	0.91
NAK1	0.66
OEHU	1.74
AALBK	2.31
AARH2	1.29

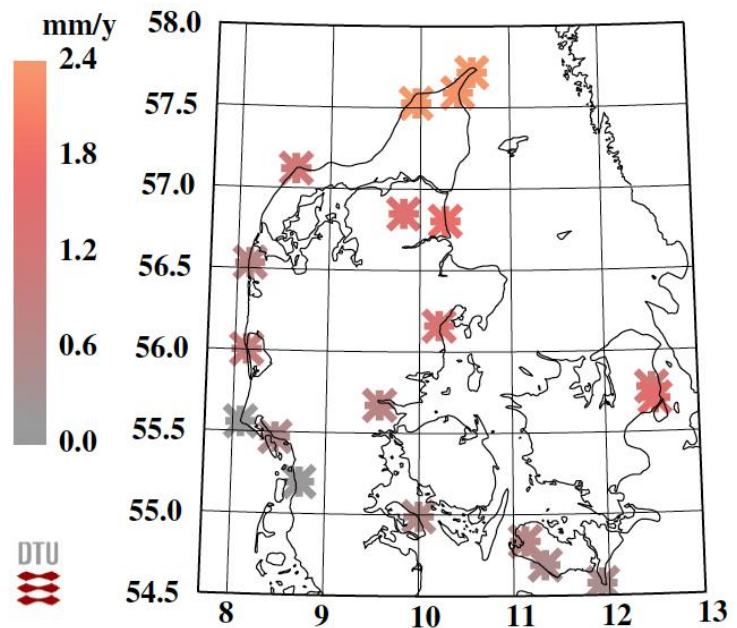


Figure 6. Vertical uplift rates in mm/year from GNSS

Table 2. Vertical uplift rates in from GNSS in mm/year.

4 The results

As part of the initial processing of the data, the contributions of the reference models are subtracted. The results are shown in Table 3 where mean values and standard deviations are shown. The rates from levelling were referenced to a benchmark in Aarhus and have a mean value about -0.2 mm/year. After the removal of the reference models the mean values drop to about -1.1 mm/year which agrees well with the actual uplift in Aarhus according to the reference model. The standard deviations increase slightly from about 0.3 mm/year to 0.4 mm/year.

For the rates obtained from the tide gauge data a mean value of -0.65 mm/year indicate that land on average is subsiding relative to sea level. After the removal of the reference models the mean drops to about -1.5 mm/year agreeing well with a general sea level rise of less than a few millimetres. The standard deviations are about 0.5 mm/year. For the rates from GNSS the mean value shows a general uplift in the region of about 1.1 mm/year. This indicates that the sea level rise in the region for the period 1900-2000 is about 1.7-1.8 mm/year. After removing the contribution from the GIA model, the mean value drops to about zero and the standard deviation dropped from about 0.8 mm/year to 0.3 mm/year.

In Vestøl et al. (2019) the GIA model was assessed to have an overall accuracy of 0.34 mm/year. Hence, the relatively good agreement between the GIA model and the GNSS rates may be caused by the fact that data from a subset of the Danish GNSS stations was included in the determination of the GIA model. The fit of the levelling data is affected by errors in the levelling too. The corresponding standard deviations of the differences are about 0.37-0.39 mm/year. Those numbers indicate that the overall errors of the levelling rates are about 0.15-0.19 mm/year. The fit of the tide gauge rates is 0.50 mm/year which indicate that the overall error of those data is about 0.37 mm/year. Regional differences in the observed sea-level rates may play a role explaining this relatively high error.

Data set	N	Observations		Residual observations	
		Mean	St.dev	Mean	St.dev
Lev 1 – 2	307	-0.18	0.32	-1.12	0.37
Lev 1 - 3	537	-0.13	0.31	-1.00	0.38
Lev 2 – 3	301	-0.25	0.45	-1.05	0.39
TG	14	-0.65	0.53	-1.46	0.50
GNSS	19	1.11	0.77	0.03	0.32

Table 3. Statistical parameters of uplift rates from observations and from residual observation where the reference models have been subtracted, from data sets of rates from levelling (between first and second campaign, first and third campaign, and between second and third campaign), rates from tide gauge data (TG), and rates from GNSS data in mm/year.

Prior to estimating the parameters describing the residual field, $du(\phi, \lambda)$, the error variances need to be assessed. This was done in an empirical way to assure that systematic errors would be considered and, not at least, to assure a proper weighting of the different data sources. Examples of systematic errors may be local subsidence at the stations, accumulated distance dependent errors in the levelling, and inter-annual sea-level variability. A proper weighting is required to avoid

that the large number of levelling data will dominate the estimation. In total the number of levelling data is 1145 which is about 60-80 times more than the number of tide gauge rates and GNSS rates, respectively. By empirically adjusting the errors of the levelling rates a more even contribution of those data is obtained.

Hence, the errors of the levelling rates were set to 1.0, 2.0, and 1.0 mm/year for the three data sets, respectively, to compensate for the large number of data and avoid that the levelling rates dominate the solution. The errors of the rates derived from the tide gauge data were set to 0.25 mm/year for stations having a complete coverage over the 1900-2000 period and to 0.4 mm/year for the other stations to include the uncertainty in the estimation of those rates. The errors of the rates from the GNSS were set to 0.25 mm/year for the stations having time series longer than 10 years and 0.35 mm/year for the remaining stations.

The base functions describing the residual field were placed in the nodes of a regular grid covering the area, of $54.1^\circ < \phi < 58.0^\circ$ and $7.7^\circ < \lambda < 13.1^\circ$ and having a grid spacing of $\Delta\phi = 0.1^\circ$ and $\Delta\lambda = 0.2^\circ$ in latitude and longitude, respectively. The z_0 values were kept constant at a value of 0.9. Hence the spatial resolution is about 12 km. A regularization was applied empirically through the diagonal elements of the covariance matrix, C_x , i.e. only constraining the variance of the a_i parameters and avoiding fitting the data beyond the respective noise levels.

Then a solution was computed using the methodology described above. The unknown parameters, U_0 and U_{MSL} , were estimated to be 1.05 mm/year and 1.62 mm/year, respectively. Hence, the sea level rise came out slightly smaller compared to the preliminary estimate described above. The estimated residual uplift rates are shown in Figure 7. The values range from about -0.6 mm/year in the Northwestern part of the area to about +0.6 mm/year in the Southeast. The estimated errors are lowest on land where the data are located and increase outside as expected (Fig. 8). The errors on land are about 0.1-0.2 mm/year, slightly higher at the coast.

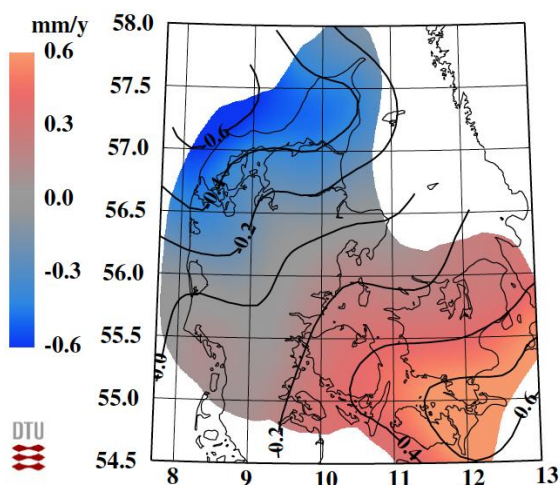


Figure 7. Estimated residual uplift rates in mm/year.

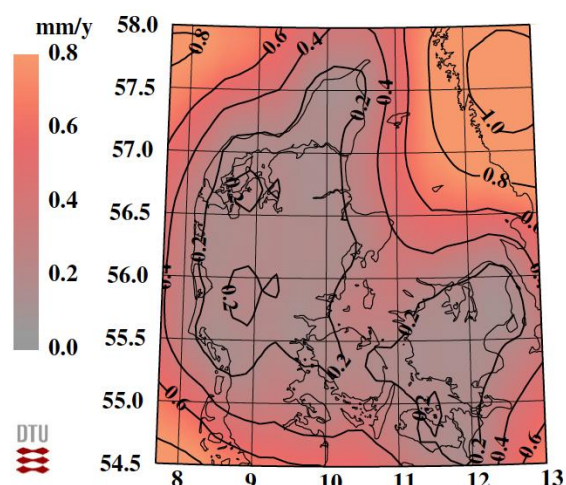


Figure 8. Errors of estimated rates in mm/year.

To assess the solution post-fit residuals were computed as the difference between the data used in the estimation, i.e. the residual uplift rates, and the corresponding quantities modelled by the

solution. The statistics are shown in Table 4. The mean values of the post-fit residuals are close to zero for each data set which indicates that the solution fits the level of the observations and that there are no internal biases between the data sets. The standard deviations have been reduced from about 0.3-0.5 mm/year to about 0.1-0.2 mm/year. Hence, most of the uplift has been adopted by the estimated model. Furthermore, the standard deviations of the post-fit residuals agree very well with the estimated errors shown in Figure 8.

Data set	N	Residual observations		Post-fit residuals	
		Mean	St.dev	Mean	St.dev
Lev 1 – 2	307	-1.00	0.38	0.00	0.10
Lev 1 - 3	537	-1.05	0.39	-0.05	0.09
Lev 2 – 3	301	-1.12	0.37	0.01	0.16
TG	14	-1.46	0.50	-0.02	0.16
GNSS	19	0.03	0.32	0.01	0.18

Table 4. Statistical parameters of residual uplift rates and post-fit residuals from levelling (between first and second campaign, first and third campaign, and between second and third campaign), from tide gauge data (TG), and from GNSS data in mm/year.

A more detailed assessment of the solution and its fitting of the data may be carried out visually by inspecting plots of post-fit residuals. In general, the plots of the post-fit residuals (Fig. 9-11) show that small discrepancies are found in the central parts of the region. However, larger discrepancies are found in the outskirts of the region where the data from different sources do not agree. Examples of large discrepancies are found near Hanstholm where the rates from levelling do not agree with the rate derived from the tide gauge data. At Hirtshals the rates from levelling and from GNSS do not agree. Also, at Gedser the levelling does not agree with the GNSS while at Rødbyhavn the levelling does not agree with the tide gauge data. It is important to consider, though, that the stations and benchmarks are established in sedimentary materials and may experience local subsidence. Furthermore, the actual sea level trends may not be constant in the different seas.

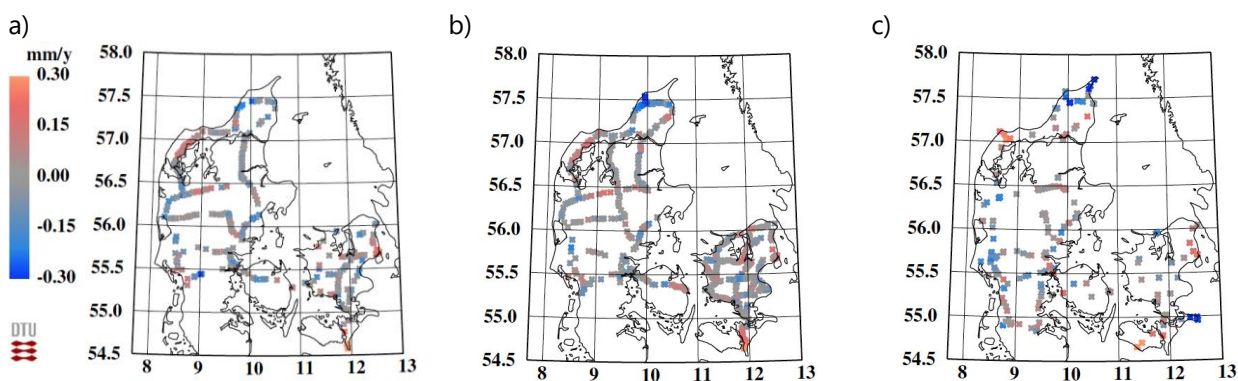


Figure 9. Post-fit residuals of uplift rates in mm/year from levelling campaign #1 to #2 (a), #1 to #3 (b), and #2 to #3 (c).

Finally, the reference models are added back to obtain the final uplift models. In Fig. 12 the final DKup24 model for uplift relative to the ellipsoid is shown. This model should be used when comparing with GNSS data. In Fig. 13 the corresponding model for uplift relative to the geoid is shown. This model should be used when comparing with levelling and when assessing sea level changes using tide gauge data.

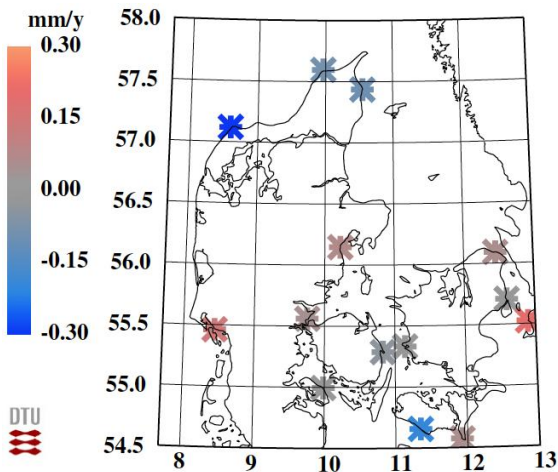


Figure 10. Post-fit residuals of uplift rates in mm/year derived from the tide gauge data.

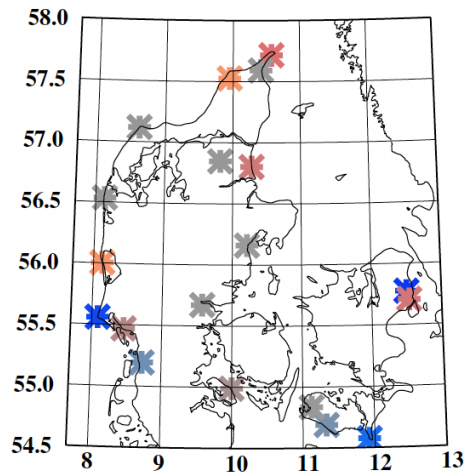


Figure 11. Post-fit residuals of uplift rates in mm/year derived from the GNSS data. (Colours as in Fig. 10)

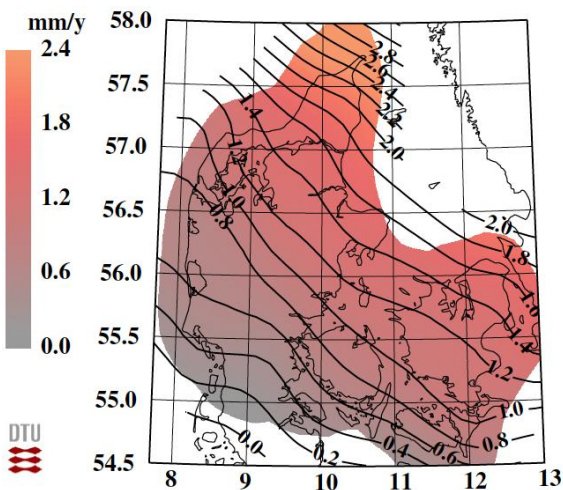


Figure 12. DKup24 model uplift rates relative to the ellipsoid in mm/year.

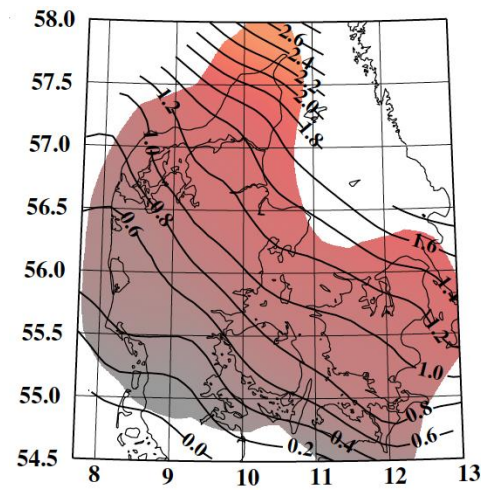


Figure 13. DKup24 model uplift rates relative to the geoid in mm/year. (Colours as in Fig. 12)

Most Danish islands are covered by the uplift models. However, the island of Bornholm located in the Baltic Sea, is not. In the town of Tejn, a permanent GNSS reference station was established in

2010. The processing of data from this station resulted in an uplift rate of 0.90 mm/year as reported by Khan (2017). The reference GIA uplift model at this location gives 0.79 mm/year, so the difference is quite small and probably not significant. Hence, the reference models may be used on Bornholm. From Tejn in the North to the South tip the GIA rates decrease to 0.57 mm/year. The corresponding change in the geoid is about 0.26 mm/year across the island.

4.1 Comparison with the NKG2016LU model

As explained in the introduction Vestøl et al. (2019) have derived the empirical uplift model NKG2016LU. Though similar approaches are applied, there are differences in the modelling approach (as described in Chapter 2) and in the data selection. In this investigation, data from all three national levelling campaigns has been used. Also, tide gauge information and data from more GNSS stations have been used.

Initially, the NKG2016LU model is assessed relative to the NKG2016GIA model. The differences are shown in Figure 14. Comparing with Figure 7 reveal that the pattern of the empirical parts of the NKG2016LU and the DKup24 models are similar. However, the range of those empirical parts are about 2-3 times larger in the DKup24 model. Then difference between the DK24up and the NKG2016LU models were computed. Those differences show (Figure 15) negative values in the Northwest and positive values in the Southeast. Hence, the DKup24 model provides smaller rates in areas in the Northwest and larger rates in the Southeast, fully consistent with the differences described above.

The main differences between the two models occur in the Northwestern part as well as in the Southeastern part of the region. In both areas, the additional information from the 2nd levelling and from tide gauges may have provided additional information to the DK24up model. Also, the different modelling approaches may have resulted in different error filtering and smoothing.

To quantify the assessment the models are compared with the observations. For the DKup24 model, mean values and standard deviations of the differences are shown in Table 4. For the NKG2016LU model the “abs” model was compared with the GNSS rates and the “lev” model with the levelling and the tide gauge rates. In addition, the unknown biases, u_0 and u_{MSL} , estimated to be 1.05 mm/year and 1.62 mm/year, respectively, were included. The results are shown in Tabel 5.

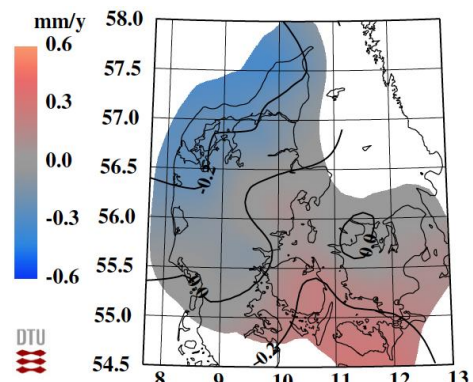


Figure 14. Difference between uplift rates of the NKG2016LU and the NKG2016GIA models in mm/year.

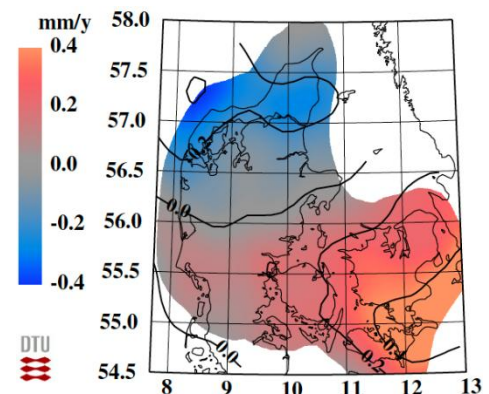


Figure 15. Difference between uplift rates of the DKup24 and the NKG2016LU models in mm/year.

Data set	N	DKup23		NKG2016LU	
		Mean	St.dev	Mean	St.dev
Lev 1 – 2	307	0.00	0.10	0.00	0.23
Lev 1 - 3	537	-0.05	0.09	0.07	0.23
Lev 2 – 3	301	0.01	0.16	0.04	0.25
TG	14	-0.02	0.16	-0.12	0.34
GNSS	19	0.01	0.18	0.06	0.21

Table 5. Statistical parameters of post-fit residuals from levelling, from tide gauge data (TG), and from GNSS data compared to the DKup24 and the NKG2016LU models in mm/year.

The comparison with the observations (Table 5) shows that both models agree well with the GNSS rates. Both set of GNSS rates are computed using ellipsoidal heights in ITRF2008. The comparison with the levelling shows somewhat different results. The mean values are centred around zero, but the standard deviations are about twice as big for the NKG2016LU model. The comparison with the tide gauge rated show similar results. Again, the additional information from the 2nd levelling and from tide gauges included the DK24up model were not considered in the NKG2016LU mode. In Vestøl et al. (2019) an overall accuracy of 0.25 mm/year of the NKG2016LU model is reported which is fully consistent with the results obtained in this comparison. The overall accuracy of the DKup24 model is found to be around 0.2 mm/year; quite similar, though the DKup24 model fits the observations much better. I should be noted that it is not a comparison with independent data. However, the additional data sources used for DKup24 provide more redundancy in the estimation. Hence, the better fit to the data suggests that the accuracy of the DKup24 model is better than 0.2 mm/year.

5 Summary and perspectives

Geodetic data from three national levelling campaigns has been integrated with data from continuously operating GNSS stations and coastal tide gauge stations in a computation of land uplift rates. The results are two models: One model describing uplift relative to the ellipsoid and one describing uplift relative to the geoid. The errors of the models are estimated to be around 0.2 mm/year which may project heights 100 years into the future within a few centimetres, hereby extending the use of the current height system DVR90 in a dynamic manner. In the peripheral areas larger discrepancies between the data sources hint that the models are less accurate here.

In transforming coordinates in a recent international terrestrial reference frames such as ITRF2020, to the national ETRS89 realization in the Nordic region, the procedures recommended by NKG may be used. According to Häkli et al. (2023) the NKG2016LU has been adopted as the vertical velocity field. The procedures have been working very well in general. However, eventually the errors in the velocity fields will reach considerable values, e.g. an error of 0.25 mm/year will reach 1 cm over 40 years. In parts of Denmark, the DKup24 model reveals local uplift rates that are not recovered fully by the NKG2016LU model. Hence, the DKup24 model may be applied here and provide a better transformation.

It is recommended to update the time series and to adopt newer reference frames such as ITRF2020 in future calculations. Also, newer data sources based on SAR may contribute to future improvements of the uplift models.

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Access to the models. The models are made available by Knudsen, Engsager & Khan (2024) at https://data.dtu.dk/articles/dataset/The_DKup24_uplift_model_for_Denmark_/27277848

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