A Modal Translational Semantics in Prior's "Symbolism and Analogy"

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Abstract

This paper explores a modal semantics Arthur Prior developed in his 1957 lecture, "Symbolism and Analogy." Prior's semantics employs a translational scheme where certain modal axioms are translated as sentences in an easily understood language. Using Prior's semantics, we show that one can distinguish between modal logics like D, M, T, S4, and S5 without recourse to possible worlds. Finally, given the current conception of what a semantics ought to be, we consider whether Prior's modal semantics is indeed a semantics.

Keywords: deontic logic, modal logic, modal semantics, possible worlds semantics, temporal logic, translational semantics

1 Introduction

In 1957, Arthur Prior delivered a series of lectures, "The Logic Game," to introduce modern formal logic to a general audience. These lectures were aired by the New Zealand Broadcasting Service and serialized in *The Listener Magazine*. In the first lecture, "The Necessary and the

Possible" (1957b), Prior discussed the modal notions of possibility and necessity. In the second lecture, "Symbolism and Analogy" (1957c), he further discussed the niceties of modal logic using Jan Łukasiewicz's (Polish) notation – Prior's preferred symbolic notation. Finally, in the third lecture, "Many-valued Logics" (1957d), he discussed Łukasiewicz's three-valued logic, Ł3.¹

Prior's second lecture is particularly fascinating as he presents a way of interpreting necessity and possibility that does not use the now standard model-theoretic semantics that employs (accessibility relations among) possible worlds. Instead, he uses a translational scheme where the meanings of the axioms of some target modal systems are translated as sentences of the English language with ordinary and obvious modal principles (Prior, 1957c, p. 678). For instance, if "possibly" is interpreted as an alethic possibility, then the modal sentence "If p then possibly p" expresses the ordinary and obvious principle that *Whatever is so can be so* (Prior, 1957c, p. 675). However, if "possibly" means "it is morally permissible that," then "If p then possibly p" no longer expresses a true principle since it is false that whatever is actually done is morally permissible. Thus, while "If p then possibly p" is a true principle in an alethic interpretation of "possibly," it is not so in its deontic interpretation.

This paper explores the modal semantics that Prior developed in "Symbolism and Analogy." In the next section, we discuss the historical context of "Symbolism and Analogy" by highlighting Prior's related work that came immediately before and immediately after. We also show its place in the development of modal logic at that time. In the third section, we discuss Prior's modal semantics in more detail. We proceed by characterizing its language and formal machinery. Moreover, we show that such a semantics can distinguish between the standard modal systems like D, M, T, S4, and S5 without recourse to possible worlds semantics as is presently understood. Finally, in the fourth section, we conclude by considering whether Prior's translational semantics is indeed a semantics, given the current conception of what a semantics ought to be.

¹ The transcripts of Prior's "The Logic Game" lectures are available at *The Virtual Lab for Prior Studies*: <u>https://research.prior.aau.dk/home.php</u>.

2 The Historical Context of "Symbolism and Analogy"

From the early 1950s onwards, Prior was interested in modal logic. His fascination with the subject started with his teacher at the University of Otago, J. N. Findlay, who used C. I. Lewis and C. H. Langford's *Symbolic Logic* (1932) for a course in 1940 (Cresswell, 2019, p. 335). By the time he delivered the "Symbolism and Analogy" lecture in 1957, his interest in the subject was already at its peak.

Several breakthroughs happened in modal semantics, especially of the model-theoretic (aka truth-conditional) variety, in the mid-1940s through the early 1950s.² Among them were Rudolf Carnap's two works, "Modalities and Quantification" (1946) and *Meaning and Necessity* (1947), Robert Feys's "Les Systèmes Formalisés Aristotéliciennes" (1950), and G. H. von Wright's *An Essay in Modal Logic* (1951). These breakthroughs were precursors to the now standard possible worlds semantics developed independently by Saul Kripke in his "A Completeness Theorem in Modal Logic" (1959a) (a work that was inspired by Prior's "Modality and Quantification in S5" (1956) and which Prior himself refereed) and Jaakko Hintikka in "Modality and Quantification" (1961).³ In this kind of semantics, the modal proposition, "Necessarily p," is true just in case p is true in all (accessible) possible worlds (states of affairs), while "Possibly p" is true just in case p is true in some (accessible) possible worlds (states of affairs).

² For a discussion of the history of modal semantics before Kripke, see Copeland (2002) and Cresswell (2019).

³ There is an open question as to which work has priority, however. Many acknowledge Kripke as having priority. Kripke (reluctantly) acknowledged the priority of Hintikka's work over his in the abstract, "Semantical Analysis of Modal Logic" (1959b). For Jack Copeland, however, "The first person to have announced completeness proofs for propositional M, S4 and S5 relative to a semantics explicitly interpreted in terms of the notion of a possible world appears to have been (Timothy) Smiley in 1957. The earliest completeness proofs for quantified systems weaker than S5 appear to have been obtained by Hintikka and Kripke, in a glorious photo-finish" (Copeland, 2002, p. 100).

Prior was familiar with these pre-Kripkean breakthroughs in modal semantics. He cited von Wright in several of his early works. For example, he cited *An Essay in Modal Logic* in "The Ethical Copula" (1951b), his 1954 presidential address to the New Zealand Philosophy Congress "The Syntax of Time-Distinctions" (later published in 1958), "Didoran Modalities" (1955a), and his logic textbook, *Formal Logic* (1955b). He alludes to "Professor Carnap" in his *Time and Modality* (1957a), a work that was published in the same year as "Symbolism and Analogy" was. *Time and Modality* developed from Prior's John Locke Lectures at Oxford in 1955/56, which was made possible through Gilbert Ryle's invitation (Prior, 1957a, p. vii). In this work, Prior showed how to interpret modal logic as a logic of time (Cresswell, 2019, p. 335).⁴ These works used Feys's notation for the modal notions and Łukasiewicz's notation for the logical connectives.⁵

What is curious about Prior's fascination with modal logic, however, was his seemingly oscillating attitude towards possible worlds semantics. In his unpublished manuscript, *The Craft of Formal Logic* (1951a), Prior has this to say about it:

For the similarity in behaviour between signs of modality and signs of quantity, various explanations may be offered. It may be, for example, that signs of modality are just ordinary quantifiers operating upon a peculiar subject-matter, namely possible states of affairs [...] It would not be quite accurate to describe theories of this sort as "reducing modality to quantity." They do reduce modal distinctions to distinctions of quantity, but the variables to which the quantifiers are attached retain something modal in their signification – they signify "possibilities," "chances," "possible states of affairs," "possible combinations of truth-values," or the like.

(Prior, 1951a, pp. 736–737)

⁴ Prior already explored this idea earlier in "The Syntax of Time-Distinctions" with the PF-calculus and *l*-calculus.

⁵ Though they were not cited in Prior's "In What Sense is Modal Logic Manyvalued?" (1952), the influence of Feys, von Wright, and Carnap can also be seen in this early work.

In this early work, Prior seems to be in favor of analyzing modalities in terms of quantification over possible states of affairs. Moreover, if "possible states of affairs" means "possible worlds," then considering the time when Prior wrote this, it seems that he could claim priority to the idea that modal notions reduce to possible worlds.⁶

However, in a later 1962 essay, "What is Logic?" (later published in 1976), Prior seemed to have a negative indictment about it. He writes,

Some writers have attempted to represent modal logic also as a kind of disguised quantification theory – perhaps with quantification over "possible states of affairs." I don't myself think that this will do either, though I won't develop my objections to it now. And quite certainly the logic of obligation and the logic of knowledge cannot be represented this way.

(Prior, 1976)

Here, Prior repudiates the idea of reducing modal notions to quantification over possible worlds. Perhaps his reason for this is metaphysical: quantification over possible worlds must not be seen as a fundamental metaphysical explanation (Copeland, 2016, pp. 3511–3512). That is, to analyze "Possibly p" in terms of the existence of at least one possible world where p obtains must not be taken as ontologically committing to the existence of possible worlds.

Curiously, however, in the same year that he wrote "What is Logic?," he also published "Possible Worlds" (1962). In this work, he employs the U-calculus that possible worlds to analyze modal notions.⁷ Perhaps Prior here is being a (Peircean) pragmatist in that "the only way to discover whether a given field can be handled as a logic, that is as the subject of a calculus, and how far it can be so handled, is to try it out and see what happens. You can't settle the question *a priori*" (Prior, 1976).

Perhaps we could classify Prior's oscillating views about possible worlds into early, middle, and later periods.⁸ Prior entertained a non-

⁶ Aneta Markoska-Cubrinovska (2016) develops this idea further.

⁷ Prior's "Possible Worlds" develops from "Interpretations of Different Modal Logics in the 'Property Calculus'" (1956), an earlier joint work with C. A. Meredith.

⁸ Classifying Prior's views about possible worlds into early, middle, and later periods follows the discussions of Copeland (2016) and Cresswell (2016). Arguably, Prior's early period started during the writing of *The Craft of Formal Logic* in the late

metaphysical commitment to possible worlds during his early and later periods. However, during his middle period, he abandoned his initial idea outlined in *The Craft of Formal Logic*. For example, the idea that modalities are reducible to possible worlds is not evident in "The Syntax of Time-Distinctions," nor in *Formal Logic* and *Time and Modality*. In these works, he ventured into analyzing modal notions (and different modal logics) in terms of (different accounts of) quantification over times and not in terms of quantification over possible worlds. "Symbolism and Analogy" belong to this middle period.

3 Prior's Modal Semantics in "Symbolism and Analogy"

Arguably, the modal semantics that Prior developed in "Symbolism and Analogy" takes off from the main points of "The Syntax of Time-Distinctions" and *Time and Modality*. This supposition is reasonable given the temporal proximity of these three works. In this section, let us see how this semantics works, particularly how different ways of interpreting (or translating) the modal notions of possibility and necessity imply different modal logics. Let us begin by elucidating the language of the target modal system using Prior's preferred Polish notation.

Our target modal system comprises of atomic sentences, primitive logical connectives, and modal notions. Let p, q, r, ... stand for atomic sentences. Let C and N stand for the primitive logical connectives, material conditional (if then) and negation (not), respectively. And let *M* stand for the modal notion of possibility while *L* for necessity. Compound sentences in this system are recursively defined in the standard way. Thus, CpNq means "If p then not-q;" *LM*p means "It is necessarily possible that p;" and CNpNLp means "If not-p then p is not."

¹⁹⁴⁰s to 1951. His middle period started during the writing of *Formal Logic* in the early 1950s and culminated in *Time and Modality* until the late 1950s. His later period started in the early 1960s with "Possible Worlds" and lasted until his death in 1969. There are exceptions here, of course; e.g., his (1956) work with Meredith seems to be part of his later period, though it is within his middle period given this classification.

Let us now turn to the other logical connectives in this system. Let A and K represent the Boolean connectives of (inclusive) disjunction (either or but not both) and conjunction (and), respectively. These connectives are defined in terms of C and N as Apq =df CNpq; Kpq =df NCpNq. On the other hand, let E represent material equivalence (if and only if) and is defined as: Epq =df KCpqCqp

There are some basic rules of inference in this modal system – rules adopted from a two-valued propositional logic, e.g., uniform substitution and contraposition, but the rule of detachment (*modus ponens*) is the main inference rule used in this system. "The rule of detachment simply states that if we already have some implication as an axiom or as a theorem previously proved, and if we also have the implying formula, we may infer the implied formula as a new theorem" (Prior, 1957c, p. 675). That is, if Cpq and p, then q. Using these basic rules of inference, we deduce theorems from axioms and other theorems.

Finally, let us turn to how "*L*" and "*M*" are interpreted in this system. For this, let us first consider a straightforward alethic interpretation or translation of the modal notions. For Prior, there are "ordinary and obvious" modal principles, which are elementary truths of modal logic. One such principle is *What is so can be so*, which is "an elementary truth that we learn when we are learning to talk" (Prior, 1957b, p. 627). More formally, the familiar modal axiom can capture the first ordinary and obvious modal principle: CpMp, or the M-axiom.

Given CpMp, we now have our first modal logic M. From this and another ordinary and obvious modal principle that *What must be cannot not be*, we may derive the definition of *L*. If *L*p means "p must be so", then it is an elementary modal truth that if p must be so, then p cannot not be so. More formally, CLpNMNp. On the other hand, we may also define *M* in terms of *L* given the ordinary and obvious modal principle that *What can be is what need not not be*. That is, CMpNLNp. Thus, given the alethic interpretation of modal notions and these ordinary and obvious modal principles, we can show that *L* and *M* are interdefinable in the following way: *L*p =df NMNp; *M*p =df NLNp.

Since *L* and *M* are interdefinable, we can show that the modal axiom CLpp, i.e., the T-axiom, follows from CpMp. Here is a simple proof of this.

- 1. CpMp (M-axiom)
- 2. CpNLNp (definition)

- 3. CLNpNp (contraposition)
- 4. CLpp (uniform substitution)

Given this, we now have two modal logics based on the alethic interpretation of *L* and *M*. From the ordinary and obvious modal principle that *What is so can be so*, we have the M-axiom and modal logic M. From this, we derive the T-axiom and the corresponding modal logic T. However, the axioms of other modal logics, S4, S5, and D, are, for Prior, controversial modal principles (Prior, 1957c, p. 675). They require other ways of interpreting *L* and *M* to make them ordinary and obvious modal principles. Let us first start with the S4-axiom.

The S4-axiom tells us that *CLpLLp*, i.e., "if necessarily p then necessarily necessarily p". This modal principle is not an ordinary and obvious modal principle if *L* and *M* are interpreted as alethic modalities since it might be necessarily true that water is H2O but it is not necessarily true that water is H2O is necessarily true. The former necessity might be of a metaphysical sort while the latter is of a logical sort. Nor is the S4-axiom easily derivable from the ordinary and obvious principle that *What is so can be so*. Given the rules of inference of our modal system, we cannot derive the S4-axiom from either the M or the T axiom.

However, if we were to interpret the modal notions in terms of temporal notions, then we could have a way of validating the S4-axiom.⁹ Suppose *Mp* means "p either is or will be true" and *Lp* means "p is and always will be true". Given this temporal interpretation, *CLpLLp* is now translated as "If p is and always will be true, then it is and always will be true that p is and always will be true," which makes the S4-axiom an ordinary and obvious modal principle since if 2 + 2 = 4 is and always will be true, then it is and always will be true. Moreover, this temporal interpretation of *L* and *M* still preserves the ordinariness and obviousness of both the M and T axioms. On the one hand, if it is true that I am having coffee right now, then it is either or will be true that I am having coffee right now that 2 + 2 = 4.

⁹ Prior used the similar strategy in "The Syntax of Time-Distinctions" and *Time and Modality.*

This temporal interpretation of the modal notions, however, does not validate the truth of the S5-axiom. In fact, this interpretation makes it false. The S5-axiom tells us that *CMpLMp*, i.e., "If possibly p then necessarily possibly p." Given our current temporal interpretation of the modal notions, this axiom is translated as "If anything is or will be true, then it is and always will be true that it is or will be true" (Prior, 1957c, p. 675). This, however, is false. Consider the sentence "If it is possible that pigs fly then it is necessarily possible that pigs fly." Given the temporal interpretation that there are flying pigs either is or will be true, but it is surely not the case that it is true and always will be true that there are flying pigs. Moreover, we cannot derive the S5-axiom (and its corresponding S5 modal logic) from the M, T, and S4 axioms.¹⁰ Thus, some other interpretation must make the S5-axiom an ordinary and obvious modal principle.

To show this, Prior interprets *M*p as "p is or has been or will be true" and *L*p as "p is and always has been and always will be true."¹¹ Given this interpretation, *CMpLMp* now becomes the true modal principle: "If p is or has been or will be true, then it is and always has been and always will be true that p is or has been or will be true." Moreover, this interpretation will also validate the M, T, and S4 axioms. For instance, "If p is so, then p is or has been or will be true" and "If p is and always has been and always will be true, then p is so" are both true modal principles; so is "If p is and always has been and always will be true, then it is and always has been and always will be true that p is and always has been and always will be true."

Let us take stock of what we have discussed so far. We have seen three ways of interpreting the modal notions, *L* and *M*. The straightforward alethic interpretation that translates *M*p as "Possibly p" and *L*p as "Necessarily p" validates both the M and T axioms but not the S4 and S5 axioms. The temporal interpretation that translates *M*p as "p either is or will be true" and *L*p as "p is and always will be true" validates S4, M, and T but not S5. Finally, the other temporal interpretation that translates *M*p as "p is and always has been and always will be true" validates all four modal systems. Let us now turn to modal logic D, the weakest of the modal logics that Prior considered in "Symbolism and Analogy."

¹⁰ The proof of this can be seen in (Prior, 1955a, p. 208).

¹¹ Compare with (Prior, 1957a, p. 18)

Prior notes that

In ordinary speech the words "must" and "can" are notoriously ambiguous; sometimes they express necessity and possibility, but as often as not their import is not modal but moral: "you must do this" means not "you cannot help but doing it" but "you are obliged to do it"; and "you can do this" means not "nothing prevents you" but "you are allowed."

(Prior, 1957c, p. 678)

This "moral" use of the modal notions presents another way of interpreting *L* and *M*. In this interpretation, *M*p means "it is permissible that p" and *L* means "it is obligatory that p." This moral interpretation validates the D-axiom that *CLpMp*, i.e., "If p is obligatory, then p is permissible." Moreover, it preserves the interdefinability of *L* and *M*. For instance, if something is obligatory, its omission is not permissible (Prior, 1957c, p. 678). Likewise, if something is permissible, its commission is not obligatory. But while this is so, the moral interpretation makes the other four modal axioms false. For instance, the M-axiom is no longer true since it now implies the false principle that everything actually done is morally permissible. Thus, this moral interpretation does not validate the "stronger" modal axioms like M, T, S4, and S5.¹²

4 Is Prior's modal semantics a semantics?

In the last section, we have seen Prior's modal semantics that provides different interpretations of the modal notions. These different interpretations, in turn, provide ways of translating (and thus validating) different modal axioms (principles) of different modal logics, which may include or exclude each other. For instance, we have seen that an alethic interpretation validates modal logics M and T, while a temporal interpretation may validate S4 and S5. Finally, a moral interpretation validates D but not the other four modal logics. And this is all done with

¹² It is curious to note that Prior did not discuss the distribution axiom (aka the K-axiom) in "Symbolism and Analogy." Perhaps this is so because he did not think that CLCpqCLpLq is an ordinary and obvious modal principle. He did consider it in "Possible Worlds," where he referred to it as Aristotle's law (Prior, 1962, p. 37).

no recourse to the notion of possible worlds. As a concluding remark, let us now turn to whether Prior's modal semantics is indeed a kind of semantics.

When we come to look at Prior, especially the modal semantics he developed in his "middle period," we see that whenever he seems to be doing what we call "semantics," he is usually doing what we might call "translation" (Cresswell, 2016). What we call semantics now refers to model-theoretic accounts where the meaning of a sentence is determined by its truth conditions. However, this is not what Prior did in "Symbolism and Analogy" and, for that matter, in his "The Syntax of Time-Distinctions" and *Time and Modality*. What Prior was doing in these works is a kind of translation semantics, where the meaning of sentences in a given language is determined by translating it to another language (Lepore & Loewer, 1981). That is, instead of cashing out the meaning of some sentence p in terms of the predicate "is true in (at)," it is cashed out in terms of the predicate "translates as" or "is interpreted as" (Copeland, 2016).

The difference between these two semantical devices is easy to demonstrate. For example, in possible worlds semantics, the meaning of the modal sentence "*L*p" is cashed out in terms of p being true in/at all accessible possible worlds. And the difference between modal systems is determined by the given accessibility relation between these worlds. For instance, if the accessibility relation is reflexive, then it validates modal logics M and T. If it is reflexive and transitive, then it validates S4. If it is reflexive, transitive, and symmetric, then it validates S5.

On the other hand, in Prior's translational semantics, the meaning of *L*p depends on how *L* is understood. If it is understood in alethic terms, then it simply means "Necessarily p;" if it is understood in temporal terms, then it may mean "p is and always will be true" or "p is and has been and always will be true;" finally, if it is understood in moral terms, then it means "p is obligatory." The difference between these interpretations implies a difference between modal logics. For example, the moral interpretation implies modal logic D but not M, T, S4 or S5. This means that both possible worlds semantics and Prior's modal translational semantics deliver the same result, viz., a way of interpreting the modal notions and distinguishing between different modal logics.

Despite this, however, the former semantics is now standard, while the latter has largely been neglected.¹³

The rise to prominence of possible worlds (model-theoretic) semantics and the decline of (Prior's) modal translational semantics may be attributed to the now commonplace mantra that *Semantics with no treatment of truth conditions is not semantics* (Lewis, 1970). Since translational semantics merely offers a way of translating a sentence from an object language to a sentence from another object language, it follows that translational semantics is not real semantics. For instance, while we may translate "Lp" as "p is obligatory," such translation does not provide ways of validating whether "Lp" is true.

The truth-conditions view of semantics, however, is ill-founded.¹⁴ Consider Prior's modal translational semantics. Even if it does not straightforwardly specify the conditions where "*Lp*" is true, it nonetheless provides ways of determining what translation of "*Lp*" validates a given modal principle. For instance, while translating "*Lp*" in moral terms validates the D-axiom, it does not validate the M-axiom.

That being said, it seems just fair to appreciate and assess the theoretical merits of Prior's modal translational semantics in its terms and not just as a matter of historical curiosity. One thing going for Prior's semantics is that it is metaphysically lightweight compared to possible worlds semantics. It carries no commitment to possible worlds, avoids semantic ascent, and does not assign abstract semantic values to (sub)sentential elements of the language (Copeland, 2016).

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¹³ Copeland (2016) and Cresswell (2016) seem to provide conflicting accounts of why this is so.

¹⁴ As Copeland (2016) argues, the main arguments for this view are either circular or faulty.

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