On A.N. Prior's Logical System Q

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Abstract

Through his philosophical and logical analysis in Time and Modality in 1957, Arthur Norman Prior proposed the logical system Q. In this paper, I logically characterise Q and, subsequently, study Q's deficiencies. I also review other works which have been carried out based on Q in recent decades.

Keywords: A.N. Prior, System Q, tense-logic.

1 A Logic of Today-Yesterday (TY)

The point of departure is a special focus on a logic of Today and Yesterday (I name this logic '*TY*'). Accept that we 'only' have two times. In particular, 'today' and 'yesterday' are our defined times in *TY*. Let the symbol p stand for some arbitrary proposition. We will have the following semantic conclusions in *TY*.

1. p is true at both times. In our logical system, the semantic value *True on yesterday* and the semantic value *True on today* are represented by $T_y - T_t$, where T stands for True, and y and t represent 'yesterday' and 'today', respectively.
2. p is true today and has been un-statable (let me say 'vague') yesterday. Then, the semantic value is: $V_y - T_t$, where V represents Vague and T represents True.
3. p is true today and has been false yesterday. Then the semantic value is $F_y - T_t$, where F and T represent False and True, respectively.
4. p is false today but has been true yesterday. Then the semantic value is $T_y - F_t$.
5. p is false today and has been vague yesterday. So the semantic value is $V_y–F_t$.
6. p is false at our both times. Therefore, the semantic value is $F_y - F_t$.

It shall be taken into account that p might (could) not have been made, or been expressed, yesterday. Therefore, $\neg p$ (the negation of p), $p \wedge q$ (the conjunction of p and some other arbitrary proposition q), $\Diamond p$ (or 'p at some time (i.e., at either yesterday or at today)'), and $\Box p$ at both times (i.e., at both yesterday and today)') might (could) not have been made, and been expressed, yesterday either.

Proposition. The sentences (i) 'p is true today and has been true yesterday.' and (ii) 'p is true today and has been vague yesterday.' are *designated*. *Analysis*. Accept that f is a formula. Then f can express a logical law if and only if f's concrete substitutions are true whenever they are statable (and are not vague).

Obviously, both (i) and (ii) are true today. Also, in (ii) the value of yesterday's might (could) have been true.

According to the offered six semantic values, table 1 presents the semantic values of conjunction ($\wedge$), negation ($\neg$), possibility ($\Diamond$), necessity ($\Box$), and the negation-of-possibly-not ($\neg\Diamond\neg$). It is remarkable that the only difference between the semantic values for p (Table 1: fourth column) and for $\neg\Diamond\neg p$ (Table 1: fifth column) is where p has the semantic value '$V_y - T_t$' (Table 1: second row). In fact, in that case, p is not true [at] both times, and today we can, necessarily, state that it is not p (i.e. we can deny). But yesterday it has been unstatable/vague, so p has the value '$V_y - F_t$'. But in the same case, where p has the value '$V_y - T_t$', then not-*p* has the value '$V_y - F_t$'. Therefore, not-*p* is not true at either time, and this fact—that not-*p* is not true at either time—is true today but has been un-statable/vague yesterday. This means that $\neg\Diamond\neg p$ has the value '$V_y - T_t$'. From this difference others will follow.

Definition. The symbol α expresses a law if and only if it takes one of the values '$T_y - T_t$' and '$V_y - T_t$'.

Note that a law cannot have the value 'F_y–T_t' (although the today's semantic value is true). Let α stand for a law. Accordingly, it can be concluded that $\neg\Diamond\neg\alpha$, always, expresses a law too (because it takes one of the values '$T_y - T_t$' and '$V_y - T_t$'). However, α is not a law (because it is impossible that α take the value '$V_y - T_t$'). More specifically, α can only have the value '$T_y - T_t$' (for all possible values of its variables) if α, itself, has had the value '$T_y - T_t$' (for all possible values of its variables). This, in turn, could only be valid if the variables in α were not capable of taking either the value '$V_y - T_t$' or '$V_y - F_t$'; for any formula of which any part takes one of these values must itself take one of them. Note that this follows from the fact that any function of what has been vague yesterday will itself have been vague yesterday.

However, it shall be taken into consideration that ordinary propositional variables are not restricted in their possible values to those other than '$V_y - T_t$' and '$V_y - F_t$'.

Definition. The *logical system Q* can fundamentally be defined based on the logical operators '$\neg$', '$\wedge$', '$\Box$', and '$\Diamond$', as well as on ordinary propositional variables.

Proposition. There can be no laws of the form $\Box\alpha$ at all.

Table 1. Primary Semantic Values in *TY*

	p	$\neg p$	$\Diamond p$	$\Box p$	$\neg\Diamond\neg p$
Value 1	$T_y - T_t$	$F_y - F_t$	$T_y - T_t$	$T_y - T_t$	$T_y - T_t$
Value 2	$V_y - T_t$	$V_y - F_t$	$V_y - T_t$	$V_y - F_t$	$V_y - T_t$
Value 3	$F_y - T_t$	$T_y - F_t$	$T_y - T_t$	$F_y - F_t$	$F_y - F_t$
Value 4	$T_y - F_t$	$F_y - T_t$	$T_y - T_t$	$F_y - F_t$	$F_y - F_t$
Value 5	$V_y - F_t$	$V_y - T_t$	$V_y - F_t$	$V_y - F_t$	$V_y - F_t$
Value 6	$F_y - F_t$	$T_y - T_t$	$F_y - F_t$	$F_y - F_t$	$F_y - F_t$

In this it is like the Ł-modal system (see [14,21]); but it is unlike it in that in the Ł-modal system there are not only no laws of the form α but no individual true propositions of the form p (the values of such propositions are limited, in that system, to 'V_y–T_t', 'F_y–T_t', and 'T_y–F_t'). By the way, in the system Q, p does have the value '$T_y - T_t$' when p has it, and there are statements for which p can stand which do have this value, though there are no forms which have it for all values, including '$V_y - T_t$' and '$V_y - F_t$', of their constituent variables, see [16], i.e. Chapter V of *Time and Modality* [17].

Table 2 presents conjunction-based semantic values in our logical system. We focus on the logical conjunction $p \wedge q$, where p and q are two arbitrary propositions.

Table 2. Conjunction-based semantic values in TY

$\wedge$	$T_y - T_t$	$V_y - T_t$	$F_y - T_t$	$T_y - F_t$	$V_y - F_t$	$F_y - F_t$
$T_y - T_t$	$T_y - T_t$	$V_y - T_t$	$F_y - T_t$	$T_y - F_t$	$V_y - F_t$	$F_y - F_t$
$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - F_t$	$V_y - F_t$	$V_y - F_t$
$F_y - T_t$	$F_y - T_t$	$V_y - T_t$	$F_y - T_t$	$F_y - F_t$	$V_y - F_t$	$F_y - F_t$
$T_y - F_t$	$T_y - F_t$	$V_y - F_t$	$F_y - F_t$	$T_y - F_t$	$V_y - F_t$	$F_y - F_t$
$V_y - F_t$	$V_y - F_t$	$V_y - F_t$	$V_y - F_t$	$V_y - F_t$	$V_y - F_t$	$V_y - F_t$
$F_y - F_t$	$F_y - F_t$	$V_y - F_t$	$F_y - F_t$	$F_y - F_t$	$V_y - F_t$	$F_y - F_t$

Note that any of the propositions, p and q, can have 6 semantic values. Therefore, the conjunction $p \wedge q$ would have 36 semantic values. In particular, we will have the following rules:

- Let p's value be T and q's value be T. Therefore, the value of $p \wedge q$ will be T.
- Let p's value be F and q's value be F. Therefore, the value of $p \wedge q$ will be F.
- Let p's value be T and q's value be F. Therefore, the value of $p \wedge q$ will be F.
- Let p's value be V. Therefore, the value of $p \wedge q$ will be V (regardless of q's value).
- If the semantic values of today's and yesterday's are identical, then the semantic value of $p \wedge q$ will be the same.

- If p's value is $T_y - T_t$, then the value of $p \wedge q$ will be the same as q's value.

Similarly, implication-based and disjunction-based semantic values can be presented:

Table 3. Implication-based semantic values in *TY*

$\supset$	$T_y - T_t$	$V_y - T_t$	$F_y - T_t$	$T_y - F_t$	$V_y - F_t$	$F_y - F_t$
$T_y - T_t$	$T_y - T_t$	$V_y - T_t$	$F_y - T_t$	$T_y - F_t$	$V_y - F_t$	$F_y - F_t$
$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - F_t$	$V_y - F_t$	$V_y - F_t$
$F_y - T_t$	$T_y - T_t$	$V_y - T_t$	$T_y - T_t$	$T_y - F_t$	$V_y - F_t$	$T_y - F_t$
$T_y - F_t$	$T_y - T_t$	$V_y - T_t$	$F_y - T_t$	$T_y - T_t$	$V_y - T_t$	$F_y - T_t$
$V_y - F_t$	$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - F_t$	$V_y - T_t$
$F_y - F_t$	$T_y - T_t$	$V_y - T_t$	$T_y - T_t$	$T_y - T_t$	$V_y - F_t$	$T_y - T_t$

Table 4. Disjunction-based semantic values in *TY*

$\vee$	$T_y - T_t$	$V_y - T_t$	$F_y - T_t$	$T_y - F_t$	$V_y - F_t$	$F_y - F_t$
$T_y - T_t$	$T_y - T_t$	$V_y - T_t$	$T_y - T_t$	$T_y - T_t$	$V_y - T_t$	$T_y - T_t$
$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - T_t$
$F_y - T_t$	$T_y - T_t$	$V_y - T_t$	$F_y - T_t$	$T_y - T_t$	$V_y - T_t$	$F_y - T_t$
$T_y - F_t$	$T_y - T_t$	$V_y - T_t$	$T_y - T_t$	$T_y - F_t$	$V_y - F_t$	$T_y - F_t$
$V_y - F_t$	$V_y - T_t$	$V_y - T_t$	$V_y - T_t$	$V_y - F_t$	$V_y - F_t$	$V_y - F_t$
$F_y - F_t$	$T_y - T_t$	$V_y - T_t$	$F_y - T_t$	$T_y - F_t$	$V_y - F_t$	$F_y - F_t$

At this point I shall focus on the implication. Let p's value be 'V_y–T_t'. According to table 1 (and as discussed above), the semantic values of $\neg\Diamond\neg p$ and p are '$V_y - T_t$' and '$V_y - F_t$', respectively. Therefore, semantically we can conclude that '$V_y - T_t$' implies '$V_y - F_t$'. More specifically, the value of the implication '$V_y - T_t \supset V_y - F_t$' is '$V_y - F_t$' (that is not a designated value). It is interesting that the implication, semantically, takes the same value (i.e. '$V_y - F_t$') when p has the value '$V_y - F_t$'. In fact, in case p's value is false on today and has been un-statable on yesterday, the implications $\neg\Diamond\neg p \supset \Box p$ and $\neg\Box\neg p \supset \Diamond p$ will be semantically equivalent. Prior stresses the fact that these results correspond to the intuitive objections to the S5 type of tense-logic.

Now let p be the proposition 'I am both a logician and not a logician'. This proposition is false whenever it is statable. However, it is not always statable. In addition, $\neg p$ is not statable (i.e., is vague) when p is not (and so is not always true). Consequently:

– 'Not-always-not-p' (formally speaking: $\neg\Box\neg p$) is true when it is statable (i.e., has the value '$V_y - T_t$'); while since p is never true.
– 'Sometimes-p' (formally speaking, $\Diamond p$) is false when it is statable (i.e. has the value '$V_y - F_t$').

Correspondingly, based on table 3, the value of the implication '$V_y - T_t$' ⊃ '$V_y - F_t$' is '$V_y - F_t$'. Therefore, it shall be concluded that the implication has the undesignated value '$V_y - F_t$' when p has either the value '$V_y - T_t$' or '$V_y - F_t$' (obviously, in both cases the yesterday's value is vague).

It is worth mentioning that the afore-mentioned implications are Gödelian axioms (and that's why Prior has, in [16], mainly focused on them). According to [16], the other two Gödelian axioms, and the classical propositional calculus, are verified [as well]; but besides being more tedious to work out, this result is of less significance, for although whatever is falsified by these tables will be something we do not want in Q, we will not want all that they verify, since some formulae which they verify merely reflect the fiction that there are 'only' two times (and 'we really do not want to say that we only have two times' [but Prior has, only, supposed that we only have two times in order to make a logical background for defining and analysing system Q]).

2 The Matrix M_Q

The logical analysis of *TY* and its six-valued (i.e. (i) $T_y - T_t$, (ii) $V_y - T_t$, (iii) $F_y - T_t$, (iv) $T_y - F_t$, (v) $V_y - F_t$, and (vi) $F_y - F_t$) semantics have been the first step towards structuring a matrix with an 'infinite number of elements' which would give us the exact 'many-valued' equivalent of 'what we can know as system Q'. In fact, the 6-valued logic *TY* is an approximation of an infinite matrix (I name the matrix 'M_Q'), where the sequence for a proposition contains two values (i.e. yesterday's and today's values). The Matrix M_Q is analysed as follows:

1. **The Structure of the Matrix**. M_Q is structured based on the symbols '*T*' (that expresses Truth), '*V*' (that expresses Vagueness), and '*F*' (that expresses Falsity). Each of M_Q's elements may be associated with an infinite sequence of the symbols *T*, *V*, and *F*. The last symbol (which expresses today's value) must be either *T* or false *F* (because today's proposition is certainly statable).
2. **Designated Sequences in Matrix**. The designated sequences are the sequences that do not contain *F*s. In other words, there is no falsity in a designated sequence. As pointed out above, in *TY*, we have a law if and only if we have one of the values '$T_y - T_t$' and '$V_y - T_t$'. These are, in fact, designated sequences in *TY*.
3. **Negation in Matrix**. The sequence for $\neg p$ is defined as follows: $\neg(v_1 v_2 v_3 \ldots) = (\neg v_1)(\neg v_2)(\neg v_3) \ldots$, where any v_i stands for a semantic value. Suppose that we only have one single value. Therefore, (i) the semantic value of 'the negation of *T*' is *F*, (ii) the semantic value of 'the negation of *F*' is *T*, and (iii) the semantic value of 'the negation of *V*' is *V*.
4. **Conjunction in Matrix**. Based on the sequences for *p* and for *q* (as some other proposition), the conjunctive term '$p \wedge q$' is defined as follows:
   Accept that we only have two single values (one for *p* and one for *q*). The conjunction-based values based on a pair of single values are represented in table 5.

Table 5. Conjunction-based values based on a pair of single values in M_Q

∧	T	V	F
T	T	V	F
V	V	V	V
F	F	V	F

5. **Necessity in Matrix.** Based on the sequence for p, the necessary term '*p*' can be analysed in M_Q. More specifically:
   - If the sequence for p is only structured based on *T*s then *p* will be certainly structured based on *T*s.

– If the sequence for p contains any vague ingredient, then in the sequence for p these Vs will surely keep their place unaltered. In addition, all other places will be occupied by Fs.
– If the sequence for p has no Vs but contains Fs, [whether it also contains Ts or not,] the sequence for p will be structured based on Fs.
– **Possibility in Matrix.** Based on the sequence for p, the possible term '$\Diamond p$' is analysable in M_Q. More specifically:
– If the sequence for p is structured based on Fs, the sequence for $\Diamond p$ will be the same.
– If the sequence for p is structured based on Fs and V s (and there is no T), the sequence for $\Diamond p$ will be the same as the sequence for p.
– If the sequence for p contains Vs and Ts (it does not matter if it contains Fs or not), then, in the sequence for $\Diamond p$, the Vs will keep their place unaltered. Also, all other places will be occupied by Ts.
– If the sequence for p does not contain Vs but contains Ts (it does not matter if it contains Fs or not), then the sequence for $\Diamond p$ will only be structured based on Ts.

3 Logical System Q: Whatness and Whyness

This section itemises the most important logical characteristics of the logical system Q.

1. Q is a three-valued logic admitting the existence of contingent beings. It means that Q is a logic in which one could intelligibly, and rationally, state that some beings are 'contingent' and some are 'necessary'.
2. The logical structure of Q has been characterised by (and is presentable in the form of) the matrix M_Q.
3. Prior had the idea that variable domains might lead us to truth-value gaps (even at the level of propositional logic) (see [7]), although he did not pursue this approach in his published material. This Prior's idea can be regarded as his main motivation for designing the system Q.

4. Relying on his philosophical views, Prior proposed Q as a 'correct' (or 'actualist') modal logic (see [2,3]). The main reason is that Q has a 'natural' semantics. In my opinion, Prior was especially interested in thinking about the philosophical-logical (even more than formal-logical) aspects of Q. In my view, Q could especially support Prior's philosophical analysis of the interrelationships between 'tense logic' and '[ordinary] modal logic'. For instance, based on his philosophical conception of Q, he expresses that "if tense-logic is haunted by the myth that whatever exists at any time exists at all times, ordinary modal logic is haunted by the myth that whatever exists, exists necessarily" (see [16]). This valuable statement is one of the most philosophical products of Q.
5. What distinguishes Q from other similar/relevant logics is its account of sentences which contain names for individuals not existing in a given world (see [9]). All such sentences are said to be undefined, truth-valueless, and vague. This might be called the gap convention, in analogy to the falsehood convention which states that atomic sentences are false in case of empty reference.
6. In his philosophical-logical analysis of 'time and modality', Prior believed that Q was a reasonably strong modal logic which would nevertheless lack sceptical and indecisive principles like, e.g., and $\neg\Diamond\neg(p \supset q) \supset (\Diamond p \supset \Diamond q)$. Prior also believed that Q could be combined with a normal quantification theory without yielding the sceptical and indecisive principles in the mixed field.

4 What are Q's Deficiencies?

According to [16], this section analyses Q's problems and deficiencies. Accept that the proposition 'S exists' (where *S* represents a Subject) means 'There are facts about *S*'. Regarding '*S* exists', we can interpret that '*S* [necessarily] exists' and, 'There are [, necessarily,] facts about *S*'. So, how should it be possible that *S* should not exist?

Let us now focus on the proposition 'It has been possible that *p*'. Then, it can be interpreted that 'it is true if and only if *p* could [, possibly,] be true.'. Therefore, *S*'s non-existence is something that cannot have been possible (and in fact, *S*'s non-existence is something that have been, surely, impossible). Consequently, since '*S* does not exist', the proposition 'There are no facts about S' is not a thing that could be/have-been true.

4.1 Q as an Ordinary Modal Logic

Let us interpret Q as an ordinary modal logic. I shall draw your attention to the following deficiencies:

1. The ordinary modal logic Q can offer a way of keeping the clear definition of the proposition 'S exists' (and of 'S [, necessarily,] exists.') as well as of the proposition 'There are facts about S' (and of 'There are [, necessarily,] facts about S'). However, it should be a fact that 'S does not exist' really is an impossible supposition (note that 'S does not exist' is equivalent to 'There are no facts about S'). Let P represent a Predicate. Formally speaking: $\neg\Diamond\neg\exists P(P(S))$ ought to be a modal theorem (in Q). But $\exists P(P(S))$, when S's existence is necessary, does not follow, if $\neg\Diamond\neg$ does not entail $\Box$.
2. The absence from Q of the rule to infer α from α can be recognised as a Q's significant deficiency. Suppose that it is not necessary that α. So how can α be a law in our logical system? In addition, how can the logical operator of 'necessity' act as a stronger notion than that of exemplifying a logical law?
3. Q does not commit us, as Łukasiewicz's Ł-modal system would if we treated it as an ordinary modal logic, to the view that there are not any true assertion of necessity.

4.2 Q as a Logic of Possibility/Necessity

Let us now interpret Q as a logic of necessity and possibility. Such an interpretation can have its own peculiarities and abnormalities. Here I focus on an example.

One formula which is 'falsified' by the tables is $\Diamond p \supset \Diamond(p \vee q)$. In fact, 'If p is possible, then either-p-or-q is possible'. According to this logical description, one may believe that the possibility of the disjunct p entails the possibility of the disjunction $p \vee q$. But let us now suppose that p stands for the proposition 'Only S1 exists', and suppose that p is possible. Also, let q be the proposition 'S2 does not exist.'. Here, p is possible. But the question is 'whether it is possible that the disjunction 'either p or q' (and, in fact, the proposition 'Either only S1 exists or S1 does not exist.')

be true?' The oddity of it is that the disjunction is vague unless 'S2 exists'. Therefore, the disjunction is only statable (i.e. is either true or false) if both parts of it (in particular: (i) 'S2 does not exist.' and (ii) 'Only S1 exists') are false. Consequently, in this case, 'possibly either-*p*-or-*q*' is false although 'possibly *p*' is true.

4.3 Q as a Tense Logic

By (i) utilising tense-logical interpretation and (ii) hypothesising that at one time it was true that 'only S1 existed.', we can use the same counter-example for the formula $\Diamond p \supset \Diamond(p \vee q)$. Then 'at some time *p*' is, surely, true but 'at some time either-*p*-or-*q*' is not. In fact, the proposition 'Either only S1 exists or S2 does not exist.' can, only, be statable (and be either true or false) when 'S2 exists', and then both parts of it are 'false'.

The semantic values, of *p* and *q*, which (by the tables) refute $\Diamond p \supset \Diamond(q \vee p)$ are 'T_y–F_t' for *p* and 'V_y–F_t' for *q*. Accordingly, formally we have: $\Diamond(\mathrm{T_y}–\mathrm{F_t}) \supset \Diamond((T_y - F_t) \vee (V_y - F_t)) = \Diamond(T_y - F_t) \supset \Diamond(V_y - F_t) = (T_y - T_t) \supset (V_y - F_t) = V_y - F_t$. Informally interpreting, these are, in fact, the cases in which '*p* is false now' but '*p* was true yesterday', and '*q* is false now' but '*q* was vague yesterday'. Actually, '*p* and *q* are both false today', and the only time at which *p* was true was one at which q (and therefore, the whole disjunction) was vague.

5 Further Developments of Q

Prior, in [18], gives some postulates which he conjectured would suffice for a modal logic which takes account of this possibility. As pointed out above, by introducing the matrix M_Q in 1957, Prior had fundamentally characterised the logical system Q. Later on, in 1964, Bull offered an axiomatisation of Prior's modal calculus Q, see [6]. Accordingly, in [19], Prior by drawing upon the result of Bull's analysis of that modal calculus, stated that the modal logic in question was called Q, and its axiomatisation was as what he and Bull offered in [19] and [6], respectively.

It is worth mentioning that Kripke believed that it was probable that it was Prior's work on many-valued matrices in [17] which gave him the idea that a possible world's model could be converted to a many-valued matrix (an idea that Kripke developed in [12]), see [7].

In 1973, Ruzsa, in his [20], dealt with some interesting extensions as well as some generalisations of the modal propositional calculus Q. [20] offers a strong mathematical analysis of, as well as semantical modelling of, the language of the Q systems.

After a few years, Correia, in [8], introduced Priorean Strict Implication (PSI) as a logical system for a strict implication operator. PSI was semantically analysed based on 'partial Kripke models' (without accessibility relations). According to [8], Prior's system Q and its [sub-]related systems were shown to be the fragments of PSI or of a mild extension of it.

In 2005, Akama and Nagata introduced a version of three-valued Kripke semantics for Q, which aimed to establish Prior's ideas based on possible worlds, see [2,3]. In addition, they investigated some important formal properties of Q and proved the completeness theorem of Q. A little later, the same authors, in their [5], attempted to modify Q as a 'temporal logic'. According to [5], although a temporal version of Q was suggested by Prior, the 'subject' had not been 'fully' explored in the main literature. Therefore, [5] attempted to work on the development of a three-valued temporal logic (so-called Q_t) and also offered its axiomatisation and semantics.

It seems that the logical system Q, after being axiomatised and being [semantically] developed, has provided a strong ground for logical analysis of future contingents (and, in fact, of contingent statements about the future). More specifically, [5] argued that their three-valued temporal logic Qt could provide a smooth solution to the problem of future contingents. Later on, in 2008, Q_t could provide a backbone for formal-logical analysis of future contingents. In such a framework, relied on the Prior's insight (mainly based on "future contingents" and [17] (ch. V)), [4] and [1] focused on the interesting problem of future contingents. The problem of future contingents is interwoven with a number of issues in theology, philosophy, logic, semantics of natural language, computer science, and applied mathematics, see [15].

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