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The True Futures: A Couple of Case Studies

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Abstract

In this paper we compare Arthur Prior's well-known Ockhamistic semantics to an alternative semantics for future contingents. We show that the alternative semantics is able to distinguish between two different statements about counterfactual future possibilities, which are equivalent according to Prior's Ockhamistic semantics.

Keywords: Arthur Prior, Ockhamistic semantics, future contingents, The Thin Red Line.

1. Introduction

The present paper is concerned with certain branching time semantics, namely Arthur Prior's Ockhamistic semantics and a semantics taken from the paper [3]. Using various example statements, the two semantics are compared with the aim of making clear the role of true futures of counterfactual moments, that is, moments outside the true course of events, also called the true chronicle.

In the next section of this paper we give an account of Prior's Ockhamistic semantics. See also [10], p. 126 ff. Here truth of a formula is relative to a moment as well as a chronicle -- which is to be understood as truth being relative to a true future of the moment in question. We shall consider various example statements that make it clear that Prior's semantics takes into account the true future with respect to which truth is relative, but this semantics does not take into account true futures of counterfactual moments.

In the third section we present the semantics from [3]. A characteristic feature of this alternative semantics is that truth of a formula is relative to a moment as well as a chronicle function, that is, a function which assigns to each moment a chronicle. Of course, such a function has to satisfy certain mathematical requirements. Using a couple of example statements, we show that contrary to the Priorean semantics, this alternative semantics does give an intuitively appropriate account of true futures of counter actual moments.

1.1 Related work

Nuel Belnap and Mitchell Green's paper [1] initiated a lengthy and lively debate about branching time semantics based on chronicle functions, which they called Thin Red Line (TRL) functions. Also the semantics proposed in [3] has been discussed in a number of publications, see [9] and references therein. See also the very recent overview [12].

A specific topic in this debate is the use of models of time based on chronicle functions (or TRL functions) to analyze Molinism, a theological system aiming at reconciling divine foreknowledge with human free will. Such analyses have for example been put forward in [7]. See also the paper [5], which attacks the criticisms of Molinism and TRL models that have been raised in paper [11].

However, it is not the goal of the present small paper to initiate yet another round in the debate, but just to take a look at a couple of hitherto unpublished example statements that we think illustrate some interesting features of the semantics given in [3].

2. Prior's Ockhamistic semantics

In this section we shall give an account of a branching time semantics which was put forward by Arthur N. Prior[1] in [10], p. 126 ff. The aim with the semantics was to formalize some ideas of William of Ockham (ca. 1285--1349) concerning human freedom and divine foreknowledge. See [15]. Furthermore, we shall consider a pair of examples of statements from natural language that make it clear that Prior's semantics does take into account the true future with respect to which truth is relative.

To define Prior's Ockhamistic semantics, we need a set T equipped with a binary relation $<$. The elements of T are to be thought of as moments and $<$ as the earlier-later relation. It is assumed that $<$ is irreflexive and transitive, and furthermore, to account for the branching structure of time it is assumed that $<$ is backwards linear, that is, it is the case that

$$\forall t,u,v.\ (t < u \wedge v < u) \Rightarrow (t < v \vee v < t \vee t=v).$$

A pair $(T,<)$ is called a frame. An important feature of the semantics is a notion of ``temporal routes'' or ``temporal branches'' which are to be thought of as possible courses of events. We shall call such branches chronicles.[2] Formally, a chronicle is a maximal linear subset of $(T,<)$. The set of chronicles induced by $(T,<)$ will be denoted $C(T,<)$.

We also need a function V which assigns a truth value $V(t,p)$ to each pair consisting of an element t of T and a propositional letter p. A triple $(T,<,V)$ is called a model. Given a model, truth is relative to a moment as well as a chronicle to which the moment belongs. This is to be understood as truth being relative to a true future of the moment in question. We define the valuation operator V_{Prior} by induction as follows:

$V_{Prior}(t,c,p)$	iff	$V(t,p)$ where p is a propositional letter
$V_{Prior}(t,c,p\wedge q)$	iff	$V_{Prior}(t,c,p)$ and $V_{Prior}(t,c,q)$
$V_{Prior}(t,c,\neg p)$	iff	not $V_{Prior}(t,c,p)$
$V_{Prior}(t,c,Pp)$	iff	$V_{Prior}(t',c,p)$ for some $t'<t$
$V_{Prior}(t,c,Fp)$	iff	$V_{Prior}(t',c,p)$ for some $t'>t$ where $t'\in c$
$V_{Prior}(t,c,\Box p)$	iff	$V_{Prior}(t,c',p)$ for all $c'\in C(T,<)$ where $t\in c'$

[1] According to [8], p. 189, the notion of branching time should be attributed to Saul Kripke.

[2] Some authors call them histories.

So $V_{Prior}(t,c,p)$ amounts to p being true at the moment t in the chronicle c. A formula p is said to be Prior-valid if and only if p is Prior-true in any model $(T,<,V)$, that is, it is the case that $V_{Prior}(t,c,p)$ for any moment t and chronicle c such that $t \in c$.

For example, let us consider the formula $q \rightarrow HFq$ (where Hp is an abbreviation for $\neg P \neg p$). This formula expresses what is known as *the principle of retrogradation of truth*. From the above definitions it is obvious that $V_{Prior}(t,c,q \rightarrow HFq)$ for any t and any chronicle c with $t \in c$, this being the case since the tense operators H and F just moves the moment of evaluation back and forth along the chronicle c. Therefore $q \rightarrow HFq$ is valid with respect to this semantics.

Now, truth of a formula is relative to a moment as well as a chronicle to which the moment belongs. The moment with respect to which truth is relative, is the moment where the formula is interpreted. What about the chronicle with respect to which truth is relative? According to Prior, it is to be understood as a provisionally given true future of the moment with respect to which truth is relative. In [10], p. 126, he calls it a `prima facie assignment'. However, it should be noted that there is no technical justification for this difference in status between moment and chronicle parameters. In *Handbook of Philosophical Logic*, the following remark is made on Prior's Ockhamistic semantics:

> Prior explains it this way. On the Ockhamist approach formulas like Fp are given `*prima facie* assignments' at t; such an assignment is made by choosing a particular b in B_t [where B_t denotes the set of chronicles to which the moment t belongs]. His idea seems to be that there is something more provisional about the selection of b than about that of t; but this he does not articulate or defend very fully. In the technical formulation of the theory there is no asymmetry between moments and branches; it is just that two parameters need to be fixed in evaluating formulas.
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> [13], p. 143-144

It may be doubted whether Prior's Ockhamistic logic is in fact an accurate representation of the temporal logical ideas propagated by William of Ockham. According to Ockham, God knows the contingent future, so it seems that he would accept an idea of absolute truth, also when regarding a statement Fq about the contingent future - and not only prima facie assignments like $V_{Prior}(t,c,Fq)$. That is, such a proposition can be made true ``by fiat'' simply by constructing a concrete structure which satisfies it. But Ockham would say that Fq can be true at t without being relativized to any chronicle. For a recent discussion, see [9].

2.1 Example statement about the true future

We shall now consider a pair of examples of statements from natural language that make it clear that the true future with respect to which truth is relative is taken into account by Prior's Ockhamistic semantics. Suppose a coin is tossed. Consider the two statements

> The coin will come up tails. It is possible, though, that it will come up heads.

and

> The coin will come up heads. It is possible, though, that it will come up tails.

The examples differ only with respect to what they say *will* happen: The first example says that tails *will* happen and heads *might* happen whereas the second example says that heads *will* happen and tails *might* happen. We symbolize the examples as respectively

$$Fq \wedge \Diamond Fq'$$

and

$$Fq' \wedge \Diamond Fq$$

where q and q' stand for respectively `The coin comes up tails' and `The coin comes up heads'. As usual, $\Diamond p$ is an abbreviation for $\neg\Box\neg p$. Clearly, the two displayed formulae should not be counted as logically equivalent since they do not say the same about what *will* happen in the future of the moment with respect to which truth is relative. Indeed, it is easy to check that the formula

$$(Fq \wedge \Diamond Fq') \leftrightarrow (Fq' \wedge \Diamond Fq)$$

is not valid in Prior's semantics. Thus, the true future matters. We shall also consider the pair of formulae displayed above in the next subsection and in Subsection 3.1.

2.2 Examples about true futures of counterfactual moments

We saw above that Prior's semantics can distinguish between two different statements about the true future. We shall now consider two statements that Prior's semantics *cannot* distinguish between, namely two different statements about the true future of a counterfactual moment. The statements in question are essentially obtained by embedding the two statements from the previous subsection in another statement that moves the moment of evaluation to a counterfactual moment.

The scenario is similar to a scenario considered by Belnap and Green in [1]. Suppose a coin (coin number one) is tossed. If it comes up tails, another coin (coin number two) is tossed. Consider the two statements

> Coin number one will come up heads. It is possible, though, that it will come up tails, and then *later* (*) coin number two will come up tails (though coin number two could come up heads).

and

> Coin number one will come up heads. It is possible, though, that it will come up tails, and then *later* (*) coin number two will come up heads (though coin number two could come up tails).

The examples only differ with respect to what they say *will* happen in the future of a counterfactual moment: At (*) the first example says that tails *will* happen and heads *might* happen whereas the second example says that heads *will* happen and tails *might* happen. We shall symbolize the example statements as respectively

$$Fp' \wedge \Diamond F(p \wedge Fq \wedge \Diamond Fq')$$

and

$$Fp' \wedge \Diamond F(p \wedge Fq' \wedge \Diamond Fq)$$

where p and p' stand for respectively `Coin number one comes up tails' and `Coin number one comes up heads', and analogously, q and q' stand for respectively `Coin number two comes up tails' and `Coin number two comes up heads'.

Note that the formulae can be obtained by substituting each of the formulae $Fq \wedge \Diamond Fq'$ and $Fq' \wedge \Diamond Fq$ displayed in Subsection 2.1 for the propositional letter r in the formula

$$Fp' \wedge \Diamond F(p \wedge r)$$

where they are evaluated at a counterfactual moment. Intuitively, we find that the two formulae displayed above should not be counted as logically equivalent since they do not say the same about what *will* happen at (*). However, the two formulae are equivalent with respect to Prior's Ockhamistic semantics, that is, the formula

$$(Fp' \wedge \Diamond F(p \wedge Fq \wedge \Diamond Fq')) \leftrightarrow (Fp' \wedge \Diamond F(p \wedge Fq' \wedge \Diamond Fq))$$

is valid in this semantics. Here is a proof of the left to right direction of the bi-implication: Let a model $(T,<,V)$ be given such that $V_{Prior}(t,c,Fp' \wedge \Diamond F(p \wedge Fq \wedge \Diamond Fq'))$ where $t \in c$. Then $V_{Prior}(t,c,Fp')$ and $V_{Prior}(t,c,\Diamond F(p \wedge Fq \wedge \Diamond Fq'))$, that is, there exists a chronicle c' containing t and a moment $t' \in c'$ where $t' > t$ such that $V_{Prior}(t',c',p)$ as well as $V_{Prior}(t',c',Fq)$ and $V_{Prior}(t',c',\Diamond Fq')$. The latter implies that there exists a chronicle c'' containing t' and a moment $t'' \in c''$ where $t'' > t'$ such that $V_{Prior}(t'',c'',q')$. But $V_{Prior}(t',c',p)$ implies $V_{Prior}(t',c'',p)$ because p is a propositional letter, and moreover, $V_{Prior}(t',c',Fq)$ implies $V_{Prior}(t',c'',\Diamond Fq)$ and $V_{Prior}(t'',c'',q')$ implies that $V_{Prior}(t',c'',Fq')$. Collecting this information, we get $V_{Prior}(t,c'',F(p \wedge Fq' \wedge \Diamond Fq))$ and hence $V_{Prior}(t,c,Fp' \wedge \Diamond F(p \wedge Fq' \wedge \Diamond Fq))$ as desired. The right to left direction is completely symmetric.

We note that the proof hinges on p being a propositional letter, or to be more precise, it hinges on the semantics of the propositional letter, $V_{Prior}(t',c',p)$ being independent of the choice of chronicle, which is a characteristic feature of Prior's Ockhamistic semantics – in Prior's words, propositional letters have no trace of futurity in them, cf. [10], p. 124. In the present case it fits nicely with p standing for `Coin number one comes up tails' which obviously does not depend on the future.

In conclusion, Prior's Ockhamistic semantics cannot distinguish between the two statements. The following section concerns a semantics that *does* distinguish between these statements.

3. An alternative semantics

In what follows, we revisit a formal semantics originally given in [3] which - we argue - does give an intuitively appropriate account of true futures of counterfactual moments.

A notable difference between this alternative semantics and the Priorean Ockhamistic semantics is that in the alternative semantics, the involved notion of possibility just refers to what might happen *now* whereas possibility in the Priorean sense refers to what might happen *from now on* in general. In other words, in the alternative semantics $\Box q$ means that q is true no matter what happens *now* whereas in the Priorean semantics $\Box q$ means that q is true no matter what happens *now and in the future*. These alternative notions of possibility and necessity may be called `immediate possibility' and `immediate necessity'.

In what follows, we present the mathematical machinery. Now, in the Priorean semantics, truth is relative to a moment and a chronicle to which the moment belongs. In this alternative semantics, truth is relative to a moment and a chronicle function. A *chronicle function*[3] is a function C which assigns to each moment a chronicle such that the following two conditions are satisfied:

(C1) $\forall t.\ \ t \in C(t)$.
(C2) $\forall t,u.\ (t<u \wedge u \in C(t)) \Rightarrow C(t)=C(u)$.

The first condition says that the present moment is actual. The second condition says that any moment which *will* be actual has the same true future as the present moment. The definition of truth of a formula as relative to a moment and a chronicle function is to be understood as truth being relative to a moment as well as a true future of each moment. The chronicle function C with respect to which truth is relative, is assumed to

[3] The idea of a chronicle function was introduced in the paper [6] by Vaughn McKim and Charles Davis, which has been overlooked by many authors.

be normal at the moment with respect to which truth is relative. Given a moment t, the chronicle function C is said to be *normal*[4] at t if and only if

$$\forall u<t.\ C(u) = C(t).$$

We let $N(t)$ denote the set of chronicle functions which are normal at the moment t. Without restricting attention to chronicle functions normal at a given moment, the intuitively valid formula $q \rightarrow HFq$ would not be valid.

How is the modal operator $\Box$ interpreted? A key feature of the semantics is that $\Box p$ is true at t with respect to C normal at t if and only if p is true at t with respect to all chronicle functions C' normal at t such that C' differ from C at most at the moment t and its past. Formally, the chronicle functions C and C' are said to *differ at most at t and its past* if and only if

$$\forall u.\ C(u) \neq C'(u) \Rightarrow (u \leq t).$$

We let $\Delta_P(t,C)$ denote the set of chronicle functions which differ from C at most at t and its past. We now define the valuation operator V as follows:

$V(t,C,p)$	iff	$V(t,p)$ where p is a prop. letter
$V(t,C,p \wedge q)$	iff	$V(t,C,p)$ and $V(t,C,q)$
$V(t,C,\neg p)$	iff	not $V(t,C,p)$
$V(t,C,Pp)$	iff	$V(t',C,p)$ for some $t'<t$
$V(t,C,Fp)$	iff	$V(t',C,p)$ for some $t'>t$ where $t' \in C(t)$
$V(t,C,\Box p)$	iff	$V(t,C',p)$ for all $C' \in N(t) \cap \Delta_P(t,C)$

A formula p is said to be valid if and only if p is true in any model $(T,<,V)$, that is, it is the case that $V(t,C,p)$ for any moment t and chronicle function C normal at t.

It is straightforward to check that the formula $q \rightarrow HFq$ is valid with respect to our semantics. In fact, note that all standard axioms of linear tense logic are valid, and also, all rules of linear tense logic preserve validity. In this sense all minimal reasoning principles of linear tense logic are adhered to.

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[4] The notion of a normal chronicle function can be traced back to Thomason and Gupta's paper [14]. Thanks to Peter Øhrstrøm for pointing this out.

3.1 The example statements revisited

In this subsection we return to the example statements discussed in Subsection 1.1 and Subsection 2.2. More precisely, we return to the bi-implication

$$(Fp' \wedge \Diamond F(p \wedge Fq \wedge \Diamond Fq')) \leftrightarrow (Fp' \wedge \Diamond F(p \wedge Fq' \wedge \Diamond Fq))$$

composed of two formulae saying different things about what will happen in the true future of a counterfactual moment. We shall demonstrate that the bi-implication is invalid in the semantics considered in the previous subsection. According to the notion of possibility involved in this semantics, the left hand side formula in the bi-implication is true at the left-most moment of the chronicle function equipped model in Fig. 1 if and only if the formula $F(p \wedge Fq \wedge \Diamond Fq')$ satisfies the condition that it is true at the left-most moment of at least one of the following two chronicle function equipped models in Fig. 2.

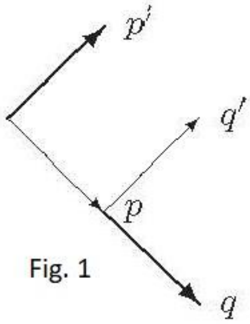

Fig. 1

Fig. 2

Clearly, this condition is satisfied. Analogously, the truth of the right hand side formula in the bi-implicationcan can be reduced to a condition regarding the truth of the formula $F(p \wedge Fq' \wedge \Diamond Fq)$. But this time the condition is not satisfied. So the bi-implication is not valid in the semantics of the previous subsection.

The fact that the two example statements above are distinguished by the new semantics makes clear that true futures of counterfactual moments do matter in this semantics. In this sense our semantics makes more distinctions than Prior's semantics, that is, it is more fine-grained.

Also, according to our semantics, the occurrences of `possible' in the example sentences refer to the possible outcomes of the first toss (that is, heads or tails) rather than all the possible sequences of outcomes of tosses (that is, heads, tails followed by heads or tails followed by tails). This is made clear by the fact, cf. above, that the occurrences of `possible' are interpreted by quantifying over exactly two chronicle functions, namely one for each possible outcome of the toss of coin number one. The outcome of the toss of coin number two is, on the other hand, kept fixed. We find that this is in accordance with our intuitive understanding of the example statements. Also, it corroborates the claim made earlier that the notion of possibility involved in our semantics refers to what might happen *now* rather than what might happen *from now on* in general.

There are other formulas than the bi-implication above that are valid in Prior's Ockhamistic semantics, but not in ours, for example the following.

$$F\Diamond Fp \rightarrow \Diamond FFp$$

The formula is not true in the left-most moment of the chronicle function equipped model in Fig. 3. We are aware that many other authors, for example [4], find that this formula is intuitively valid, but on the other hand, the invalidity of the formula in our semantics is in line with the fine-grainedness of the semantics, allowing the emergence of possibility in time.

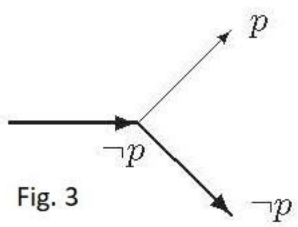

Fig. 3

Since there are formulas valid with respect to Prior's Ockhamistic semantics, but not with respect to ours, our prime example being the bi-implication displayed in the beginning of the present subsection, we conclude that Prior-validity does not imply validity in the sense of the previous subsection.

However, the converse is the case: If a formula is valid in our semantics, then it is also valid in Prior's Ockhamistic semantics, or formulated contrapositively, if a formula is invalid in Prior's Ockhamistic semantics, then it is also invalid in our semantics, see the paper [2] for a proof. The proof itself is quite technical, but the overall strategy is straightforward: For each Priorean model $(T,<,V)$ we define a new model $(T^*,<^*,V^*)$ equipped with a chronicle function C^* with the following property: For any $t \in T$ and $c \in C(T,<)$ where $t \in c$ there exists a $t^* \in T^*$ at which C^* is normal such that $V(t,c,q)$ if and only if $V^*(t^*,C^*,q)$ for any formula q. Thus, whenever a formula is true in a Priorean model (together with a moment and a chronicle), there exists a model (together with a moment and a chronicle function) in our new semantics in which the formula in question is true.

Intuitively, our semantics have more models (that is, potential countermodels), than the Priorean semantics, one reason being that there is generally an abundance of different ways to equip Priorean models with chronicle functions. Of course, this is in line with our semantics being more fine-grained.

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