Łukasiewicz, Bocheński, and Feys: Their Impact on the Early Prior

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Abstract

We investigate how Arthur Prior became a tense-modal logician, and which moderns influenced him in his early thinking about modality. His unpublished manuscript *The Craft of Formal Logic*, written in the period 1949–51, is in effect a record of his rather isolated apprenticeship as he trained himself in formal logic, during the two years before the commencement of his torrent of publications on modality. We analyse sections of this rich record of his logical development, especially those dealing with modal logic, and we extract a detailed account of the pattern of influences exhibited in *The Craft*. *The Craft* reveals that Prior's first encounters with modern symbolic modal logic were the pioneering explorations by Bocheński, Feys, and Lewis. Von Wright was also an early influence. It was through Bocheński's writings that Prior learned of Łukasiewicz's approach to modality, and Łukasiewicz's work quickly became a beacon for Prior. The roles of Lewis and von Wright appear to have been smaller than those of Łukasiewicz, Bocheński, and Feys—hence our focus on these three figures. As well as biographical material on these three outstanding logicians, we include numerous previously unpublished passages from *The Craft* in order to establish the nature and extent of their impact on Prior.

Keywords: Biography, history of logic, modern origins of modal logic, history of tense logic, Arthur Prior, Jan Łukasiewicz, Innocenty Maria (Józef) Bocheński, Robert Feys, Georg Henrik von Wright, Clarence Irving Lewis, Prior's *The Craft of Formal Logic*, modal notation, translational modal semantics, possible worlds, Polish logic, multi-valued logic, Warsaw School of Mathematical Logic.

1 Introduction: Time and modality

Prior's earliest mention of a logic of time-distinctions is in the penultimate chapter of his unpublished manuscript *The Craft of Formal Logic (Prior 1949–51).* He began writing *The Craft* in 1949, initially as a Dictionary of Formal Logic, and completed the 800-page manuscript in December 1951. It was intended to be his first book on formal logic and Oxford University Press agreed to publish it if he would give greater emphasis to modern logic. However, he ended up writing a completely different book, *Formal Logic* (Prior 1955).

That earliest mention came on page 750 of *The Craft.* Prior remarked that there are varieties of modal predicate to be set alongside the ordinary or 'alethic' modes of necessity and possibility. (In this he followed von Wright (von Wright 1951b).) Then, after noting that Peter of Spain classified adverbial distinctions of time as modes, he said:

> That there should be a modal logic of time-distinctions has been suggested in our own day by Professor Findlay.
>
> (Prior 1949–51: 750)

Findlay's paper 'Time: A Treatment of Some Puzzles' had appeared in the *Australasian Journal of Psychology and Philosophy* in 1941. The suggestion Prior spoke of was scarcely more than a passing comment, made in a footnote: '[O]ur conventions with regard to tenses are so well worked out that we have practically the materials in them for a formal calculus', Findlay had remarked (1941: 233). His footnote gave the barest outline of what would become Prior's project. Finday said, 'The calculus of tenses should have been included in the modern development of modal logics', and he suggested some possible theorems of such a calculus—for example:

> (x).(x past) future; i.e., all events, past, present and future, *will* be past.
>
> (Findlay 1941: 233)

Much later, as Prior engaged with Diodorus Chronos's 'Master Argument' for determinism, Findlay's footnote pushed its way to the front of his mind. Famously, Mary Prior (his second wife) remembered 'his waking me one night, coming and sitting on my bed, and reading a footnote from John Findlay's article on time, and saying he thought one could make a formalised tense logic'.[1] The story of what happened next—the calculus of tenses he set out in his paper 'Diodoran Modalities', and the further developments that he revealed in his Presidential Address to the 1954 New Zealand Congress of Philosophy in Wellington—is now well-known (Prior 1958, Øhrstrøm and Hasle 1995, Copeland 1996, 2020). We are going to describe the overture to all of that: our topic is how Prior had arrived in the position of being able to recognize and act upon the suggestion contained in Findlay's footnote.

Tense logic is, so to speak, modal logic in disguise. In the logic of pure futurity—first presented in 'Diodoran Modalities'—the modal possibility operator is reinterpreted as a future-tense operator and is written *F*. In his Wellington Address, Prior added a second reinterpretation of the possibility operator, a past-tense operator written *P*. Thus was the first *bi-modal* logic born. As well as these two versions of the possibility operator, the logic presented in the Wellington Address also contained two analogues of the necessity operator, one defined as –*F*– (it will always be the case that) and the other as –*P*– (it has always been the case that). It was Prior's knowledge of modal logic that led him to tense logic.

In order, therefore, to paint a full picture of the genesis of Priorean tense logic, one requires an account of how Prior acquired his expertise in modal logic. By contextualising Prior's earliest writing on modal logic, in *The Craft*, we aim to supply this account. We will also furnish some information on three leading pioneers of modal logic all of whom influenced Prior profoundly, as he wrote *The Craft* (and all of whom are somewhat neglected in the Anglophone literature). They are Łukasiewicz and Bocheński, both Poles, and Feys, a Belgian.

[1] Quoted in Kenny 1970: 336.

The Craft is a fascinating document, in effect a record of Prior's rather isolated apprenticeship as he trained himself in formal logic, during the two years before he began submitting his own work on modal logic—soon a torrent of publications—to the *Journal of Symbolic Logic*, *Analysis*, the *Australasian Journal of Philosophy*, and elsewhere. The torrent began with his "On Propositions Neither Necessary Nor Impossible", discussing Bocheński, Łukasiewicz, and Leśniewski, and submitted to the *Journal of Symbolic Logic* in November 1951, and then his "Lukasiewicz's Symbolic Logic" and "In What Sense is Modal Logic Many-Valued?", both discussing Łukasiewicz (Prior 1952a, 1952b, 1953a).

As the references in *The Craft* reveal, Prior's first encounters with modern symbolic modal logic were the pioneering explorations by Bocheński, Feys, and Lewis: Bocheński in his chapter 'La Logique de la Modalité' of his *La Logique de Théophraste* (Bocheński 1947a)—and also, a little later in the process of writing *The Craft*, the important material on modality in his *Précis de Logique Mathématique* (Bocheński 1948a)—and Feys in his article 'Les Systèmes Formalisés des Modalités Aristotéliciennes' (Feys 1950). Prior became familiar with Lewis's work in the book *Symbolic Logic* (Lewis and Langford 1932). Von Wright's *An Essay in Modal Logic* was also a significant influence (von Wright 1951a). It was through Bocheński's writings that Prior learned of Łukasiewicz's approach to modality, and Łukasiewicz's work soon became paramount among these early influences on Prior. He later wrote in the Preface to *Time and Modality*: '[W]hile I differed radically from the late Professor Łukasiewicz on the subject of modal logic, my debt to him will be obvious on almost every page' (Prior 1957a: vii-viii).

Cresswell has suggested that 'Łukasiewicz was not a friend of modal logic; principally because he thought it led Aristotle into error' (Cresswell 2021: 4)—but, friend or not, Łukasiewicz certainly cultivated the field, spending much of his professional life devising modal logics. Indeed, his stated aim in the early work on the multi-valued logics for which he is famous was to express modal concepts in extensional terms, as we shall explain.

In this account of Prior's influences, we focus on Łukasiewicz, Bocheński, and Feys, and we introduce them in that order, since (although Prior encountered Bocheński's writings first) Łukasiewicz—a towering presence in modal logic—was a powerful influence on the much younger Bocheński, as he was also on Feys. The influence of Lewis and von Wright on Prior was ultimately much less than that of Feys and the Poles. In Section 6, where we detail the pattern of influences exhibited in *The Craft*, we will note the points at which work by Lewis and von Wright impacted Prior.

Łukasiewicz and Bocheński, two rather colourful heroes of modal logic, both hailed from that golden era of Polish logic, in the years before the Second World War—along with many other great names of 20th century logic, including Chwistek, Jaśkowski, Kotarbiński, Kuratowski, Lejewski, Leśniewski, Lindenbaum, Słupecki, Sobociński, Tarski, Wajsberg, and others. In Louvain, Feys—an important innovator in modal logic and also an expositor—was himself deeply influenced by the work of the Poles. Prior, excitedly metamorphosing into a formal logician on an isolated island between Australia and Antarctica, somehow managed to put down a taproot into Polish logic.

2. Łukasiewicz, 1878-1956

Jan Łukasiewicz was born in Lwów (today's Lviv in Ukraine). He studied mathematics and philosophy at Lwów University, and received a PhD in 1902 for research on induction and deduction (Łukasiewicz 1903, Polkowski 2019: 12). He explained that his interest in symbolic logic was aroused by reading Russell:

> He [Russell] was trying to explain the concept of an ordered collection to me in a rigorous, mathematical way, instead of bothering me with philosophical nonsense … That put me on the path to mathematical logic.
>
> (Łukasiewicz 2013: 67)

Łukasiewicz was appointed Professor at the University of Warsaw in 1915, shortly before the Austro-Hungarian Empire began to disintegrate into nation-states and Poland headed towards independence (Łukasiewicz 1994: 133). Interested in modality and wanting to express possibility in formal terms, he began developing his many-valued logics in 1917, saying in a lecture the following year:

> I have shown that in addition to true and false propositions there are *possible* propositions, to which objective possibility corresponds, as something third to existence and non-being. This gives rise to a system of *three-valued logic*, which I worked out in detail last summer.
>
> (Łukasiewicz 1918: 4)

His first papers on three-valued logic came out in 1920 (Łukasiewicz 1920a, 1920b). Then in 1924 he devised the parenthesis-free logical symbolism that Prior would fall in love with more than 25 years later—'la notation polonaise', Feys called it, Polish notation (Łukasiewicz 1929, 2013: 29, Feys 1950: 479). It has been suggested that the modal part of Polish notation was due to Bocheński (Cresswell 2021: 4), but Łukasiewicz was using his trademark 'M' for possibility (from the German 'möglich') by 1930, many years before Bocheński's entry into modal logic (Łukasiewicz 1930: 52).

Łukasiewicz was the driving force behind a logic seminar that ran in Warsaw from 1926. This attracted a brilliant collection of mathematicians and philosophers, working on what they called 'logika matematyczna', mathematical logic. The group that gathered around Łukasiewicz, Leśniewski and Tarski became known as the Warsaw School of Mathematical Logic. Besides research into many-valued logics, they were responsible for a wide range of logical innovations, including matrix formulations of propositional calculi, Leśniewski's protothetic, and Tarski's theory of truth.[2] Sadly the group's activities came to an end in 1939, when German bombs fell on Warsaw and the members scattered.

Łukasiewicz remained in Warsaw during the war, giving lectures in the 'uniwersytet podziemny' or underground university. But as the Soviet army drew close in 1944 he fled, ending up in Ireland in 1946. Embarking on a life of voluntary exile, at age 68 and with poor English, it must have seemed to him that the best times were over. Yet in Dublin he enjoyed a most productive decade, becoming Professor of Mathematical Logic at the Royal Irish Academy in 1948 and publishing extensively on logic in English, including his much-admired book *Aristotle's Syllogistic* (Łukasiewicz 1951a, 1957). The mature work done in Dublin is arguably his finest. Prior described 'On Variable Functors of Propositional Arguments' (Łukasiewicz 1951b) as 'quite the most exciting contribution that has been made to symbolic logic in English for a very long time' (Prior 1952a: 43).

Prior wrote to Łukasiewicz at about this time, encouraged by Bocheński, with whom he was corresponding—'The proof you give in your second letter is amazingly simple', Bocheński wrote. 'Could you not send it directly to Łukasiewicz (Dublin, Fitzwilliam Square 57, Ireland, Europe)?' (Bocheński 1951b). Thereafter Łukasiewicz and Prior

[2] Some results were summarised in Łukasiewicz 1930 and Łukasiewicz & Tarski 1930.

exchanged several letters,[3] and when Łukasiewicz's copy of *Formal Logic* arrived from the Clarendon Press, he wrote:

> I am most grateful to you that you have employed my symbolic notation which, as I hope, will be now popular in the whole English-speaking world … Your textbook will be most useful for all students of symbolic logic.
>
> (Łukasiewicz 1955)

A year later Łukasiewicz died in Dublin—shortly before Prior was due to travel there from Oxford to meet him for the first time. Łukasiewicz's extensive handwritten memoir, composed during 1949–1950, remained unpublished for more than sixty years. The following short extracts, most of which are translated into English for the first time, give a sense of the man and his milieu.[4]

On influences during his student years in Lwów (1897–1902):

> The first volume of Husserl's *Logical Investigations* made a big impression in Lwów, especially on me … But the second volume … was an equally big disappointment. It contained once again the obscure philosophical language that repelled me from German philosophers. I was surprised that such a difference could exist between two volumes of the same work. Later I realised that it was not Husserl who had spoken to me in the first volume … but someone far greater, whom Husserl had used in his book—namely Gottlob Frege.
>
> (Łukasiewicz 2013: 65-66)

Łukasiewicz's first book (Łukasiewicz 1910), titled *O zasadzie sprzeczności u Arystotelesa* (On the Principle of Contradiction in Aristotle), made his name and also brought him into the orbit of Leśniewski:

> I met Leśniewski in Lwów in 1912 … One afternoon someone came to the front door … bowed and asked politely, 'Does Professor Łukasiewicz live here?' I replied that it was so. 'Perhaps the gentlemen himself is Professor Łukasiewicz?', the stranger asked. I

[3] Łukasiewicz's letters to Prior are dated 1953, 1955 and 1956.

[4] Łukasiewicz's memoir has so far been available only in Polish (Łukasiewicz 2013), although alternative translations into English of our first two extracts have appeared previously.

> replied that it was so. 'I am Leśniewski and I have come to show you the proofs of an article I wrote arguing against you.' I invited him inside. It turned out that Leśniewski was publishing … an article criticizing some of the views I had expressed in my book *O zasadzie sprzeczności u Arystotelesa*. His criticism was written with such scientific precision that it left me no room for counterarguments.
>
> (Łukasiewicz 2013: 17-18)

Soon Łukasiewicz and Leśniewski were fighting side by side, campaigning for the new science of logika matematyczna. They were immensely successful. By 1939, there were five Professorships in Mathematical Logic across Poland (compared to only one in the rest of Europe, held by Scholz in Münster), and furthermore Polish high-school students were being versed in propositional and quantificational logic (Woleński 1995: 378-379). In 1930, Chwistek had been appointed to the Professorship in Mathematical Logic at Lwów, on Russell's recommendation (ibid: 336). Russell said later: 'I used to know of only six people who had read the later parts of [*Principia Mathematica*]. Three of these were Poles' (Russell 1959: 86).

The Warsaw School began to form not long after Łukasiewicz rejoined the University of Warsaw in 1920, following a spell as a high-ranking official in the Ministry of Education:

> Late autumn 1920 … I started lecturing at the University of Warsaw, where I was to teach until 1939 … I remember that at this time Leśniewski was on his best form, creating his ontology. Tarski was a student then, and very soon afterwards he also announced his first scientific results. That was the beginning of the Warsaw School of Mathematical Logic.
>
> (Łukasiewicz 2013: 28-29)

The School was well developed by the time Bocheński and Feys came into contact with Polish logic. Łukasiewicz said that Bocheński saw logic as a 'weapon':

> [T]hanks to my scientific work I made contacts among priests [and] gained students … interested in the history of medieval logic, like Salamucha, and through him the Dominican Father Bocheński.

> These priests also became interested in mathematical logic, seeing it as a weapon with which to defend religion from attacks.
>
> (Łukasiewicz 2013: 72)

Łukasiewicz explained that he first met Bocheński in 1934, in Prague, at the Eighth International Congress of Philosophy. Bocheński later described Łukasiewicz as 'rather small, neatly dressed and rather shy', continuing: 'He wrote once that a scientific paper should be also an aesthetic achievement—written in perfect language, beautiful … And I would say that his own writings are a model in that respect: they are really beautiful' (Bocheński 1994: 1-2).

> The representatives of the so-called Vienna Circle attended, as well as a few Poles, among others Ajdukiewicz and myself. The paper I gave on the history of propositional logic later appeared in print in German … Professor Scholz called it 'the most beautiful 20 pages on the history of logic' … At this congress we first met our Father Bocheński, with whom we would later have very close connections.
>
> (Łukasiewicz 2013: 34-35)

Feys and Łukasiewicz met, perhaps for the first time, in 1938, at a congress in Zürich *Les Entretiens de Zurich sur les Fondements et la Méthode des Sciences Mathématiques* (Zürich Conversations on the Foundations and Method of the Mathematical Sciences):

> The heart and soul of the congress was Gonseth, a professor at the Federal Polytechnic in Zürich. Invitees from other countries were the mathematicians Lebesgue and Fréchet from Paris, logicians Jørgensen from Copenhagen, Skolem from Oslo and Feys from Leuven … From Zürich there were also Bernays and Dürr … In my lecture 'Logic and the Problem of the Foundations of Mathematics' I presented my and my students' results in the area of many-valued systems of propositional logic. The comments during the discussion suggested that what we called many-valued modal logic was not suitable for mathematics, because there were no modal propositions in mathematics. This later led me to create non-modal many-valued systems which, during and after the Second World War, I managed to apply to the arithmetic of natural numbers.
>
> (Łukasiewicz 2013: 37)

Fleeing from the Soviet advance on Warsaw, Łukasiewicz and his wife eventually reached Brussels, in December 1945, after enduring sixteen terrible months as refugees. There they had a change in fortunes:

> [A]n Englishman of Irish descent came from London to Brussels … to persuade us to go to Ireland. He told us that the Irish Government was willing to help Polish academics by giving them positions in scientific institutions in Ireland … My wife and I … decided we would accept, since there were no other options at all.
> (Łukasiewicz 2013: 88-89)

3. Bocheński, 1902–1995

Józef Maria Bocheński was born in 1902, near Kraków, to a wealthy family (Bocheński & Parys 1990, Kozak 1997). In 1928, he took his vows in the Dominican order. Bocheński was a warrior monk. He fought until the ammunition ran out in the brief Polish–German war of 1939 and then—after being captured by the Germans but escaping through a toilet window—he managed to find his way across Europe to the UK, where he served as a chaplain and training officer in the Polish Army. 1944 saw him again on the European mainland, in the thick of the fighting around the Italian city of Monte Cassino. An intellectual warrior too, whose interests spanned several fields, he joined the University of Fribourg in 1945, soon becoming Professor of Philosophy. He lived with his fellow monks in the Albertinum, an 18th-century building in the centre of the small Swiss town, and this became his home for the rest of his long life. There he pursued his researches in logic, philosophy, and religion—as well as waging intellectual war on the Soviets from his study, which doubled as a studio for his many radio broadcasts.

The Polish parliament called Bocheński an 'outstanding researcher, priest, and patriot' and declared 2020—the twenty-fifth anniversary of his death—the Józef Maria Bocheński Year.[5] The official announcement also hailed him 'as a foremost exponent of an approach in philosophy known as analytic philosophy' and noted that his 'works have sold over 1 million copies worldwide'.[6]

Bocheński's first extensive foray into modal logic was his *Z historii logiki zdań modalnych* (On the History of the Logic of Modal Propositions).

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[5] https://www.polskieradio.pl/395/7791/Artykul/2386810,Polish-senators-pay-tribute-to-philosopher-Bochenski

[6] Ibid.

This was his habilitation, at Kraków's Jagiellonian University, and it was published in book form in Lwów, after receiving the Dominican imprimatur on 1 March 1938 (Bocheński 1938). Bocheński's summary of it (Bocheński 1937) offered 'a survey of the history of the formal logic of modal propositions from Aristotle to Ockham'. As well as Aristotle and Ockham, he covered Theophrastus, the Stoics, Averroes, St. Albert the Great, Aquinas, Peter of Spain, Pseudo-Scotus, and others. He referred to 'Łukasiewicz's brilliant study' of propositional logic (Bocheński 1937: 682, Łukasiewicz 1930).

Then in 1947 came *La Logique de Théophraste* (The Logic of Theophrastus), his first major publication at Fribourg (Bocheński 1947a). When he began the research for this book, 'only three fragments of Theophrastus's logic were known' (according to Kaczyński), and Bocheński 'discovered about a hundred other fragments in Byzantine commentators' (Kaczyński 2003: 19). In the preface, Bocheński explained that the book was written in 1937, while he was teaching in Rome at the Angelicum, for a series that Łukasiewicz was editing. It had reached the production stage when the German bombardment of Warsaw destroyed the entire print run, and also the manuscript (Bocheński 1947a: 5). Luckily, Bocheński found a batch of incomplete proofs in Rome, in 1944, and reconstructed the book. Thirty-six pages were devoted to modal logic, with compact derivations in Polish notation.

A year or so later came his scholarly and helpful *Précis de Logique Mathématique* (Bocheński 1948a).[7] Then, hard on the heels of this ninety-pager, he published his major work *Ancient Formal Logic* (Bocheński 1951a), followed by *Formale Logik* (Bocheński 1956a). *Ancient Formal Logic* was in English (a translator, identity unknown, must have been involved) and *Formale Logik* (Formal Logic) in German, with a 1961 English translation by Prior's good friend Ivo Thomas, under the title *A History of Formal Logic* (Bocheński 1961).

While Prior was writing *The Craft*, he plucked up courage to write to Bocheński. His outward letters have not yet been found, but Bocheński's replies survive, three of them. The following extracts, full of humour and worldliness, are revealing of the man.[8]

[7] Some give the date of publication as 1949, but the only date in the book is 1948, the date of its imprimatur.

[8] We thank Tomasz Grabowski OP of the Wydawnictwo Polskiej Prowincji Dominikanów, Poznań, Poland, for allowing us to publish these excerpts from Bocheński's letters.

Fribourg, October 26, 1951

Dear Professor Prior,

I received both your letters and beg, first of all, to apologize for being as late as I am in answering you ...

I would like to tell you that I did very much appreciate your kindness. We are, all of us, very isolated, being few and scattered. It is a real pleasure to hear that a Collegue[9] so far away is interested in the same problems you are working at and that he finds ones little writings may be of some use. I imagine you must be there still more isolated than we are in Switzerland. I have got at least Bernays and Dürr in Zürich – above all Bernays whom I believe to be a very great thinker. I shall always be glad to receive news from you; the letters you wrote me were very instructive to me ...

Apologies for my Babu-English; I hope you will be able to understand it.

The next surviving letter to Prior came from the US, while Bocheński was a visiting professor at Notre Dame during 1955–1956. It mentions Łukasiewicz's notation, which Prior had seen in Bocheński's work some years previously and fallen for heavily.

October 26, 1955

Dear Professor Prior,

I am writing this letter with a feeling of shame and guilt, for being so much unpolite toward you during so many months. What happened was this: having been delayed by overwork and some peculiar circumstances in answering you, I felt so much uneasy about the matter, that I did not dare to write to you at all ... What prompted me finally to write ... were not only remorses of conscience but also the book of yours, I got to-day here, I mean your 'Formal Logic'. I immediately read large portions of it, and I must

[9] We have left mis-spellings uncorrected.

say I am very enthusiastic about that work. Most certainly the most up-to-date and most useful textbook in existence …

I had here a sort of Peanian rebellion among my hearers (many of whom are professor teaching logic) against my using the 'CCCC's', people suspecting me of being biased by my nationality.[10] But when your book was produced, it appeared that you are more of a 'CCCC-logician' than I am, as I offer both terminologies; also your justification of its [sic] is convincing. The 'rebellion' was impressed ....

And here are, finally, some news from the USA ... Sobociński is here; he was thrown from a College, being not enough 'classically' minded; I am trying to make him a position ... Scholz, in Europe, is still – by a sheer miracle – alife. Dürr produced (a rather dürr [dry]) textbook of logistics. Lorenzen converted to intuitionism. Tarski is now in Europe on a 'research-trip' … A new paper[11] appeared in Eastern Germany (we have none in Western Europe, and Sobocinski's thing[12] is done now – a scandal of the 'free world', that we are left with the JSL alone as against two Eastern papers).

Well, I still hope you will accept my apologies and not feel to much offended ... Any way, believe me, I am very repentant and sorry.

The third and last surviving letter, written the following year, also came from Notre Dame. It alludes to Prior's visit to Oxford to deliver the *John Locke Lectures*, and mentions Bocheński's love of flying small planes and driving fast cars.

Notre Dame, Indiana, March 15, 1956

Dear Professor Prior,

[10] In Łukasiewicz's parenthesis-free notation, C*pq* is written for 'If *p* then *q*'.
[11] Presumably 'journal' was intended.
[12] The *Journal of Computing Systems*. Edited by Sobociński, this ran from 1952 until 1954.

> It was very kind indeed of you to write me such a long and passionately interesting letter ...
>
> I most certainly do agree with you that a book should be written on temporal logic in middle ages. But everything should still be written ... And here I am touching a sorry subject. After Moody took to cattle-breeding in Texas and Boehner died, there is really nobody I would know in the US who would seriously work in history of medieval logic. To give you an idea ... Rev. McY luncheons with me (a professor of logic); I: 'the Aristotelian syllogism usually begins with an "if"'. He: 'It is not my opinion'. I: 'My dear fellow, this is not a question of opinions; here are the Analytics of Ross, let us see'. He: 'O! Greek; I do not think it is useful to read Aristotle in Greek; I always noticed that people who do so pervert the sense of Aristotle. I do not understand Greek' ...
>
> I feel that the state of History of Logic as a whole is very bad .... [N]ow America looks toward Oxford ... So if you were able to incite at least some people to do some work there, this would be important. Another point is the following: if you get an invitation for the U.S., do accept ... I can assure you that this is a most interesting country, full of logicians (crowds of logicians everywhere, and good ones) ...
>
> Personally I am driving around in my powerful American car and flying ... This letter is terribly egoistic, but I think this is the way letters should be. Many thanks again and best wishes for a full success in Oxford.

In Fribourg, Bocheński drove a green racing-type Mini Cooper, perhaps attracted by the car's success in the famous Monte Carlo Rally.[13] He was notorious for his racing starts at traffic lights, as well as his free and easy attitude toward road rules and speed limits. Searching for the right description of him, one of his fellow monks in the Albertinum said: *méchant*.

[13] With thanks to Giovanni Sommaruga for information.

4. Feys, 1889-1961

Robert Feys was born in the Belgian town of Malines (Mechelen), near Brussels. Not long after the First World War he chanced across Whitehead and Russell's *Principia Mathematica*. By then he already had a PhD from the Institut Supérieur de Philosophie in Louvain, awarded in 1909, and had fought as a rifleman in the Belgian army from 1914 to the end of the war. His PhD dealt with aspects of experimental psychology, but under the *Principia*'s influence he turned to symbolic logic. 'Feys was won over', said his colleague De Raeymaeker, 'he had found his way … and ended up devoting himself entirely' (De Raeymaeker 1961: 372-373). He described Feys as: 'Modest, rather shy, somewhat inclined to distraction, he was affable, of exquisite tact, always ready to help, anxious not to offend anyone' (ibid.: 374). Feys' eventual logical foci were modal logic, Gentzen-style natural deduction and sequent calculi, and Schönfinkel's combinatory logic, as well as the history of logic. He is probably best known for his book *Combinatory Logic* with Curry (Curry and Feys 1958)—the fruit of Curry's 1950-51 visit to the University of Louvain, where Feys had been appointed to a chair in 1944. Feys was a force in European logic, and in 1958 co-founded the journal *Logique et Analyse*—in the second volume of which Prior published his 'Notes on a Group of New Modal Systems' (Prior 1959).

Feys' first logical article, 'La Transcription Logistique du Raisonnement: son Intérét et ses Limites' (The Logistic Transcription of Reasoning: its Importance and Limits) offered a pleasing semi-formal approach, and included a prescient analysis of the four Aristotelian modalities in terms of what Feys called 'des cas possibles', possible cases (Feys 1924). He distinguished between possible propositions, which are true in at least one possible case; necessary propositions, which are 'true in the totality of possible cases'; impossible propositions, which are 'not true in any case'; and contingent propositions, where the class of cases in which the proposition is true 'does not coincide with the totality of possible cases' (Feys 1924: 321). 'Here', he said, 'we have precisely the four types of modal proposition of Aristotelian logic' (ibid.).

In a later, more refined, presentation of this idea, Feys wrote: 'It is natural to interpret "*p* is necessary" as "*p* is true in all cases", "*p* is possible" as "*p* is true in some case"' (Feys 1950: 497-498). His work was a predecessor to modern possible worlds semantics (Copeland 2002: 101-102). Cases, Feys explained, can be regarded as 'any kind of situation … in which the propositions are borne out or not borne out', saying that the exact 'nature of these cases can be left indeterminate as long as it is just a matter of their suggesting an axiomatic structure' (Feys 1950: 497). Strikingly, he also noted that one can 'identify them with moments of time' (ibid.).

Feys, a canon, had much more than his religion in common with Łukasiewicz and Bocheński. De Raeymaeker spoke of his demand for 'precision and rigour in thought', and his abhorrence of 'all empty verbosity' (De Raeymaeker 1961: 374). Feys' 1924 paper made no reference to work by the Warsaw group[14], who were then scarcely known outside Poland; but by 1937, in his exposition 'Les Logiques Nouvelles des Modalités' (The New Logics of Modalities), Feys was explaining his reason for shying away from Łukasiewicz's symbolism:

> Lukasiewicz's notation without punctuation would demand a great deal of effort from a reader who is not used to it.
>
> (Feys 1937: 521)

In the second part of his exposition, however, published the following year, there is a tentative 'If we adopt Lukasiewicz's notation', followed by some sparing use of it (Feys 1938: 220). By the time he wrote his masterful 1950 paper 'Les Systèmes Formalisés des Modalités Aristotéliciennes' (Formal Systems of Aristotelian Modalities), he was making no apologies for his thoroughgoing use of 'la notation polonaise' (Feys 1950).

As Prior wrote *The Craft*, Feys 1950 paper made a great impression on him, especially its quantificational account of modality. Prior strongly endorsed the 'parallelism between modality and quantity', as Section 6 relates (Prior 1949–51: 724).

5. Prior's entry into the Polish world

In 1946, Prior was appointed to a temporary lectureship in philosophy at the picturesque Canterbury University College, situated in Christchurch, a smallish city in New Zealand's South Island. The vacancy Prior filled was created by Karl Popper's resignation. At first Prior was, as he put it, 'the only philosopher about the place'. Later he described his initial years there in a letter to Hugh Montgomery (a furniture manufacturer who attended Prior's Advanced Logic course in 1959—that

[14] Nor did the final part of this article, published the following year, mention Polish work (Feys 1925).

year taught by his students Jonathan Bennett and Robert Bull—and then changed careers, becoming a temporary lecturer at Canterbury in 1962 and eventually Chair of Philosophy at Auckland University):

University of Manchester, 23 May 1961

Dear Mr. Montgomery,

… About the difficulties in progressing in formal logic on your own – I had this to cope with too, when I started teaching at Canterbury after the war. Philosophy and Psychology were then united: the prof. was a Psychologist, and I was the only philosopher about the place, and hadn't done anything officially in the field for years. One can only do plenty of reading, I think – I read various 'classics' and was delighted with them – especially, to begin with, J. N. Keynes's Formal Logic (4th edition ...) and W. E. Johnson's Logic, and then I got stuck into Principia Mathematica Volume I; then I got Bochenski's Précis de Logique Mathématique, and was fascinated by the Polish notation, and corresponded with Bochenski and Łukasiewicz; Łukasiewicz's and Tarski's books (Ł's Aristotle's Syllogistic and Tarski's Introduction to Mathematical Logic and now his Logic, Semantics and Metamathematics – and one can also now get, through Blackwell's, Ł's Elementy Logiki Matematycznej, in which the symbols are so illuminating that the fact that the text is incomprehensible doesn't much matter) – these chaps, I was going to say, give the subject a formal precision it hasn't in anyone else.

As this letter indicates, it seems it was Bocheński's *Précis de Logique Mathématique* that ultimately recruited Prior to Polish notation. But the *Précis* was not the first of Bocheński's works he encountered: references to the *Précis* appear in later parts of *The Craft*, while earlier parts mention Bocheński's *La Logique de Théophraste* and also his edition of Peter of Spain's *Summulae Logicales* (Bocheński 1947b). Several journals taken by the Canterbury University College library—and doubtless read regularly by Prior—carried reviews of these three works, including the *Journal of*

Symbolic Logic, the *Journal of Philosophy*, and *Dominican Studies*.[15] (*Dominican Studies*, first published in 1948, described itself as 'A quarterly review of theology and philosophy'—Prior's favourite subjects. Quite likely it was at Prior's request that the college library subscribed to *Dominican Studies*. No other library in New Zealand appears to have taken it.) Bocheński's first article in *Dominican Studies*, 'On the Categorical Syllogism' (Bocheński 1948b)—in which he mentions his *Théophraste*—may have been the first of his works to catch Prior's interest. Kenny said, probably on the strength of information either from Prior himself or Mary Prior: 'In 1950 [Prior's] attention was caught by an article of Bocheński in *Dominican Studies*' and 'Bocheński's work drew his attention to the merits of Polish symbolism' (Kenny 1970: 332). By 1950, Bocheński had two articles in *Dominican Studies*. The second of these (Bocheński 1949) discussed *Principia Mathematica* and used Russellian notation, with no mention of either Łukasiewicz or his symbolism, so Kenny's statement seems to point towards the first article (Bocheński 1948b), with its liberal use of Łukasiewicz's notation, as Prior's entrée into Polish logic. Nevertheless, Prior did not refer to this article in *The Craft*, aside from a passing reference, in the Introduction, to Bocheński's 'contributions to the English periodical *Dominican Studies*' (Prior 1949–51: 50). 'On the Categorical Syllogism' was perhaps a stepping-stone to the *Théophraste* and Bocheński's other writings. However, we have found no independent corroboration of Kenny's statement, and cannot exclude the possibility that Prior's acquaintance with the *Théophraste* or the *Summulae* pre-dated his encounter with Bochenski's work in *Dominican Studies*.

The eventual effect on Prior of his discovery of Bocheński's work was profound. In the *Théophraste* Bocheński enthused: 'We will use the symbolism invented by Monsieur Lukasiewicz, the most convenient of all the symbolisms that we know and the most appropriate for fully rigorous operations' (Bocheński 1947a: 12). In 'On the Categorical Syllogism' he wrote of the 'simplicity and beauty' of Łukasiewicz's system, saying 'The choice of Łukasiewicz's symbolism and method is due to the fact that this is the most rigorous symbolism and method we know, indeed the only one which allows a complete formalisation of a deduction' (Bocheński 1948b: 36). He continued 'This is due to the fact that the system is bracket and pointless'—meaning by 'point' the

[15] The reviews were: Anon. 1948, and Thomas 1948a (the *Summulae)*; Moody 1948, Nelson 1948, and Thomas 1948b (the *Théophraste*); and Cooley 1950, and Thomas 1950 (the *Précis*).

Russellian dot. Church, in a review of ‘On the Categorical Symbolism’, said bluntly that Bocheński ‘is mistaken, in the reviewer’s opinion, when he writes that it is necessary for rigor to use this notation rather than one involving parentheses or brackets’ (Church 1950: 140). But such was the raw appeal of the Polish way of doing things that Prior took up Łukasiewicz’s notation and was soon using it in his first technical publications.

Prior made use of Polish notation even more broadly than Łukasiewicz himself, utilizing it in tense logic and beyond. Where did its attraction lie for Prior? ‘I can recall pretty clearly how he defended it from the early 1950s’, said Prior’s colleague and friend Hughes:

> (1) [Polish notation] could be written (at least for Propositional Calculus) using an ordinary typewriter (the least important point, but a bit more important in 1951 than it would be today). (2) He [Prior] once wrote to me [in October 1956] that the thing that appealed to him most of all about the Polish symbolism was ‘the way in which, syntactically, it puts first things first’; in the formula Cpq [if p then q], he said, ‘p and q are “arguments” <u>governed</u> by the operator or functor C; C is what <u>constructs</u> the statement out of the statements p and q, and the Ł [Łukasiewicz] symbolism brings this out while Peano-Russell does not’. (3) He thought that the lack of need for brackets in Polish was not merely an economy but a positive merit. Propositional logic is about operations on propositions, so one needs symbols for propositions and symbols for operators; but one <u>shouldn’t</u> need anything else, and the fact that in notations like Russell’s brackets are essential he thought a real defect.
>
> (Hughes 1993)

It was not only the Poles’ notation that appealed to Prior. Reading their work, he realised with delight that formal precision is possible in philosophy. He wrote in the Introduction to *The Craft*:

> Largely, it would seem, under the inspiration of Professor Jan Lukasiewicz, some of the Polish exponents of ‘logistics’ have turned their attention to the history of formal logic, and have found in modern symbolism a surprisingly powerful instrument for making clear what the ancient and medieval logicians were driving at in their most puzzling passages. Professor Lukasiewicz himself,

> being one of the foremost living authorities on the 'calculus of propositions', has naturally concentrated upon the logic of the Stoics; but the classical Aristotelians have been brought under review too, and simple notations have been devised for the symbolic presentation of Aristotle's own syllogistic system, bringing out clearly its character as a logical 'calculus'. At the same time, Polish Catholic writers have seen in 'logistics' the legitimate heir of the logic of the Middle Ages, and have employed the former, as the latter was employed by the Schoolmen, for the precise statement of theological and metaphysical arguments. The most easily accessible illustrations of this line of development are the more recent writings of the Dominican I.M. Bochenski …
>
> (Prior 1949–51: 49-50)

The University calendars from those years track the impact on the Canterbury philosophy syllabus of Prior's discovery of Bocheński's work. Not only did Bocheński have a tremendous influence on Prior but also, through Prior's teaching in the 1950s, on the first generation of New Zealand logicians.[16] In 1950, Prior's first- and second-year logic courses looked traditional enough, both using Susan Stebbing's book *A Modern Introduction to Logic* (Stebbing 1930). Bocheński's *Précis* appeared in 1951, in Prior's second-year logic course and also his course in Advanced Logic. 'Despite the language difficulty, I have found this a first-class textbook to accompany lectures to New Zealand students', Prior declared the following year (Prior 1952a: 35). In 1953, he made the *Précis* a textbook for even his first-year students, while his second years got Bocheński's *Ancient Formal Logic*. His bold experiment of using the *Précis* at first year was evidently not a complete disaster, since he continued the arrangement the following year. He also stuck with *Ancient Formal Logic* as a second-year text, and added Łukasiewicz's book *Aristotle's Syllogistic*, as well as listing three articles by Łukasiewicz.

6. How Bocheński, Feys, and Łukasiewicz shaped Prior as a modal logician: An analysis of *The Craft*

The Craft, Prior's earliest significant work on formal logic, is studded with references to Bocheński, at first to *La Logique de Théophraste* and his

[16] Annual Calendars of Canterbury University College, 1946-1957, Macmillan Brown Library, University of Canterbury; also at http://popper-prior.nz.

edition of Peter of Spain's *Summulae Logicales*, and then, in later parts of the manuscript, Prior begins to refer also to the *Précis*. His mentions of Łukasiewicz's work are also plentiful, but initially these are all second-hand, via Bocheński (and in one case, on page 370, via Chwistek 1948). It is not until page 784, a mere twenty-two pages before the end of *The Craft*, that Prior starts referring directly to writing by Łukasiewicz (Łukasiewicz 1948, 1951b). Feys does not appear until page 439, and as the discussion of modality develops, Feys' 'Les Systèmes Formalisés des Modalités Aristotéliciennes' is the inspiration for some of the most interesting aspects of Prior's thought. Lewis and von Wright play smaller but still significant parts.

A feature of *The Craft* is its many detailed comparisons of different symbolisms, including the notations of Boole, Feys, Lewis, Łukasiewicz, Peano, Peirce, Quine, and Russell–Whitehead. Prior evidently recognized the importance of a good notation and was shopping around. As he reaches the final two chapters of *The Craft*—which are the most technical—he plumps wholeheartedly for Łukasiewicz's notation (with additions by Feys). He uses this even when presenting axioms and theorems from *Principia Mathematica*, the bastion of Peano–Russell–Whitehead notation. In a rare statement addressing the reader directly, Prior enthuses: 'If the reader compares the formulae occurring in what follows with those occurring in Professor Quine's presentation of the proof, he will have an admirable illustration of the advantages of the Polish notation' (Prior 1949–51: 780).

The first several hundred pages of *The Craft* expound Aristotle and his successors. Bocheński was one of Prior's guides in this energetic exploration of ancient highways and byways. The first reference to Bocheński's work (other than in the Introduction, as noted above) is on page 128, where Prior relies on Bocheński's *Summulae* for information about Peter of Spain, and then a few pages later comes the first reference to Bocheński's *Théophraste*, when Prior notes approvingly that Theophrastus appears to have dropped 'the Aristotelian "indefinite" proposition' (Prior 1949–51: 132-133). Further references make it clear that Prior had these works by Bocheński at his elbow as he wrote. Then, on page 354, in the middle of a discussion of Theophrastus and Boethius, Prior suddenly embarks on an introduction to Łukasiewicz's symbolism. He gives no reference, least of all to a work by Łukasiewicz, and his source must have been Bocheński. By this stage he may even have been familiar with the *Précis*: these few pages concerning Polish notation seem to have been spliced into the manuscript at some later time in its composition. (The first explicit reference to the *Précis* comes more than four hundred pages later, on page 776.)

> In Principia Mathematica, 'If P then Q' is represented as 'p ⊃ q' … An extremely neat symbolism in which no brackets are used, the meaning being fixed entirely by the order of the symbols, has been constructed by Professor Lukasiewicz. Here 'If P then Q' is represented as 'Cpq'; 'Both P and Q' as 'Kpq'; 'Either P or Q', interpreted non-exclusively, as 'Apq'; 'Either P or Q', interpreted exclusively, as 'Jpq'; and 'Not p' as 'Np'. 'If P then if Q then R' becomes in this symbolism 'CpCqr', and 'If P then it is not the case that if Q then R' becomes 'CpNCqr'. It is possible to 'read off' these symbols into English without mistakes so long as we employ a form of speech in which some word is used before a particular kind of compound as well as in the middle of it – e.g. so long as we are careful to say, not 'P implies Q' but 'If P then Q'; not just 'P and Q' but 'Both P and Q'; and not just 'P or Q' but 'Either P or Q' (perhaps saying 'Eether' for the exclusive form). Thus we may read 'CKCpqCqrCpr' as 'If both if p then q and if q then r then if p then r'. It is also necessary to understand that the symbols 'K', 'A' and 'J' are not designed to join more than two propositions at once, though the propositions joined may themselves be of any degree of complexity. Thus 'P and Q and R' cannot be represented by 'Kpqr', but must be turned into either ('eether'!) 'Both both p and q, and r' ('KKpqr') or 'Both p, and both q and r' ('KpKqr'). By these means it is possible to give very compact expression to all the propositions listed by Boethius.
>
> (Prior 1949–51: 354-355)

Prior continued:

> Compactness is not, however, the only advantage of such a symbolism. It puts a quick end to the dispute which has occupied logicians almost since they were first aware of the existence of the form 'Either P or Q', as to whether this means 'Either P but not Q or Q but not P' or 'Either P or Q or both'. For we can now say that whatever meaning it may be most natural to associate with the word 'or', the symbol 'v' or 'A' may bear any meaning which we like to give it, and we choose to let 'p v q' and 'Apq' mean 'Either P or Q or both' (and to let 'Jpq' mean 'Either P but not Q or Q but

not P', i.e. 'AKpNqKqNp'. 'Apq' may be similarly translated into the 'J' form as 'JJpqKpq', 'Eether eether P or Q or both P and Q').

(ibid.: 356-357)

In his chapter on compound propositions and truth-functions, Prior discusses Łukasiewicz's three-valued logic, a topic that is mentioned briefly by Bocheński in his *Théophraste* and with more detail in his *Précis*. (As with the material on Łukasiewicz's symbolism, these pages appear to have been added subsequently.)

> Again, truth-functions may arise in systems which admit other truth-values beside the usual two. Suppose, e.g., that we have a 'three-valued' logic in which truth is represented by 1, falsehood by 0, and the supposed tertium quid by ½.
>
> (ibid.: 392-393)

A few pages later, he considers a potential application of three-valued logic: 'May not our hesitation as to whether to call 'If grass is green then 2+2 = 4' a true proposition or a false one, be due to the fact that it is neither?' (ibid.: 404). But he argues not:

> [I]t can be easily shown that a three-valued logic does not really make the interpretation of 'If P then Q' as a truth-function any easier. For the point about a truth-function, however many truth-values we consider, is that when the values of its arguments are given, the value of the function is unambiguously determined. Now with 'If grass is green then 2 + 2 = 4', where both components are true, we may be tempted to put down the value of the whole as 'neither true nor false'; but in another case, such as 'If all men die then all good men die', where again both components are true, we would say unhesitatingly that the conditional is true too … So even if we employ extended truth-tables … we do not know whether to put '½' or '1' beside the possibility 'p = 1, q = 1'. We may therefore dismiss three-valued systems as a solution of this particular problem.
>
> (ibid.: 404-405)

Strict implication appears on page 411, in the course of Prior's discussion of Diodorus on conditionals, and he endorses Lewis's proof of what we nowadays call the principle of explosion:

> The more probable interpretation of Diodorus, however, is that which assimilates his conception of 'following' to what Professor C. I. Lewis in our day has called 'strict' implication, which holds when the combination of the truth of the first proposition with the falsity of the second is not only false but logically impossible, e.g. when it is a 'contradiction' in the Wittgensteinian sense. Or to put it another way, 'strict' implication holds when the material implication of the second proposition by the first is not only true but logically necessary, e.g. when it is a Wittgensteinian 'tautology'. But this kind of implication has its paradoxes also … 'It both is and is not day' strictly implies 'I am conversing', since the whole combination 'It is day and it is not day and I am not conversing' is made self-contradictory by the contradiction between its first two parts. Professor Lewis argues that this is not the paradox which it may at first appear to be, for from any given self-contradictory proposition we can infer any proposition whatever by quite ordinary modes of inference. To take the given example, 'It both is and is not day' implies, to begin with, 'It is day', by the rule 'If both P and Q then P'; and by the same rule it implies 'It is not day'. Now 'It is day' implies 'Either it is day or I am conversing', by the rule 'If P then either P or Q' (if a proposition P is true, then it plainly follows that at least one of the propositions P, Q, is true). But if we combine 'Either it is day or I am conversing' with 'It is not day' (the second proposition inferred from the original contradiction), we obtain 'I am conversing' by a simple tollendo ponens. [Here he gives a footnote reference to Lewis and Langford, *Symbolic Logic*, pp. 250-251.]
>
> (Prior 1949–51: 410-12)

Feys enters Prior's discussion on page 439. Prior rejects exportation for strict implication, echoing his source (Feys 1950, 6.54 and 6.541). He then explains that Feys extended Łukasiewicz's notation to strict implication:

> We have seen that from the fact that 'Both P and Q' strictly implies R, we cannot infer that P implies in any way that Q strictly implies R; the most we can infer is that P strictly implies that Q materially implies R … We may express all this symbolically, either by using the form 'p ⥽ q', meaning 'P strictly implies Q', which Professor Lewis has grafted on to the symbolism of Principia Mathematica, or by using the form 'C′pq', with the same meaning, which

> Professor Robert Feys has more recently grafted on to the symbolism of Lukasiewicz. [Here Prior gives footnote references to Lewis and Langford, <u>Symbolic Logic</u>, Ch. VI, Sect. 14.26 and 15.84; and Feys, 'Les Systèmes Formalisés des Modalités Aristotéliciennes', Sect. 6.54 and 6.541.]
>
> (Prior 1949–51: 439)

Prior postpones the introduction of quantifier notation until his chapter 'Compound and Complex Propositions, and the Theory of Quantification' and he explains Łukasiewicz's quantifiers Π and Σ on page 526. He extends Łukasiewicz's symbolism with a negative quantifier 'Ωx' (Nothing), this re-appearing later in his article 'Negative Quantifiers' (Prior 1953c).

> In the symbolism of Lukasiewicz, simple singular propositions and propositional functions are represented as in <u>Principia Mathematica</u> (or with 'f' and 'g' for 'φ' and 'ψ'), but 'Πx' replaces '(x)', 'Σx' replaces '(∃x)' and the prefixes 'C', 'A', 'K' and 'E' are used, as in the logic of propositions, instead of the connectives '⊃', 'v', '.' and '≡', and 'N' instead of '~'. Thus 'If Enoch is human then Enoch is mortal' might be symbolised in this system by the formula 'Cφaψa', or 'Cfaga'; 'If anything is human it is mortal' by 'ΠxCφxψx', or 'ΠxCfxgx'; 'Everything is either not human or mortal' by 'ΠxANφxψx', or 'ΠxANfxgx'; and 'Something is at once human and not mortal' by 'ΣxKφxNψx' or 'ΣxKfxNgx' … [W]e add to Lukasiewicz's symbols the form 'Ωxfx' to mean 'Nothing is F' …
>
> (ibid.: 526-527)

Notation for the necessity and possibility operators does not appear until Prior's penultimate chapter, 'Modality'. In its structure, this chapter somewhat resembles Feys' 'Les Systèmes Formalisés des Modalités Aristotéliciennes', in that Prior and Feys both give an overview of modality in Aristotle and tackle the question of how to formalise matters; and moreover Prior touches on several issues that Feys raises (including exportation, as noted above). But Feys' approach was more intensely technical than Prior's—at this stage Prior was growing his technical wings and just beginning to appreciate the power of the symbols he was describing. Following his introduction to Aristotelian modality, and before introducing specifically modal symbols, Prior makes a pregnant observation: 'it will be seen that the relations between "possible", “impossible” and “necessary” exactly parallel those between “some”, “no” and “every”’, adding: ‘And this parallelism between modality and quantity can be developed in great detail’ (ibid.: 724). It was a parallelism Feys had also emphasised (Feys 1924, 1950)

> Let us employ the symbolism of Lukasiewicz. We know that if we use the symbols ‘f’, ‘g’, ‘h’, etc. for monadic predicates, ‘x’ for a variable name, ‘a’ for a fixed name, ‘Πx’ and ‘Σx’ for universal and particular quantifiers, and ‘C’, ‘K’, ‘E’ etc. as in the logic of propositions, we may express in this symbolism a variety of logical laws. For example, the formula ENΠxNfxΣxfx expresses the law that if and only if not everything is not F then something is F (non omnis non = aliquis); the formula CfaΣxfx expresses the law that if A is F then something is F; the formula CKΠxCfxgxΠxCgxhxΠxCfxhx expresses the syllogistic principle that if every F is G and every G is H then every F is H. If we take any law from the logic of quantified monadic predicates expressed in this way, but take ‘fx’, ‘gx’, etc. to represent complete propositions instead of matrices, and ‘Πx’ and ‘Σx’ to mean ‘It is necessary that’ and ‘It is possible that’ respectively, and ‘fa’ to mean ‘It is a fact that fx’, what we now have will still be a logical law – a law in the logic, not of quantified monadic predicates, but of modally qualified propositions. [Here Prior gives a footnote reference to Feys, ‘Les Systèmes Formalisés des Modalités Aristotéliciennes’, Sect. 16.3.] To take the first two of the above examples, with ‘fx’ meaning ‘grass is green’, ENΠxNfxΣxfx will now mean ‘If and only if it is not necessary that grass should not be green, then it is possible that grass should be green’ … and CfaΣxfx will now mean ‘If grass is in fact green then it is possible that grass should be green’.
>
> (Prior 1949–51: 726-727.)

Remarking ‘It is not, of course, convenient in practice to put quantifiers to this double purpose’, Prior considered the ‘special symbols for modal expressions … used by modern logicians’ (ibid.: 727). He contrasts Lewis’s notation ‘◊’ and ‘~◊~’ with the symbols for possibility and necessity that Feys uses:

> Professor Lewis has the symbol ‘◊p’ for ‘It is possible that P’, ‘It is impossible that P’ being represented simply as ‘~◊p’ … and ‘It is

> necessary that P′ as ‘~◊~p’ … Professor Feys has ‘Mp’ for ‘It is possible that P′ and ‘Lp’ for ‘It is necessary that P′. In Professor Feys’s symbolism ‘If and only if it is not necessary that not P then it is possible that P′ would be written ENLNpMp, and ‘If it is the case that P then it is possible that P′ would be written CpMp. C′pq, ‘That P, strictly implies that Q′, is defined as NMKpNq, ‘It is not possible that both P and not Q′.
>
> (ibid.: 727)

Modal notation, then as now, was a Tower of Babel. Łukasiewicz had introduced M for possibility, but in his early work he had no independent symbol for necessity, using NMN (Łukasiewicz 1930). Bocheński introduced ‘Sp’ to mean ‘It is necessary that p’ (Bocheński 1947a: 12), and Prior himself dabbled with using S for necessity (Prior 1953a, 1953b).[17] From about 1951, Łukasiewicz was using Γ for necessity and Δ for possibility (Łukasiewicz 1953a, 1957: v, 2013: 95-96),[18] and in 1954 he began using L in place of Γ and M in place of Δ for ‘typographical reasons’ (Łukasiewicz 1954a: 125). If Feys was the first to introduce L then his choice was natural enough, given that M was already in play for possibility, and N was reserved for negation. However, we have not so far excluded the possibility that L as a symbol for necessity was used earlier in Polish circles.

Prior follows Feys in *The Craft*, writing L for necessity and M for possibility (Prior 1949–51: 734-736). While his use of modal notation is quite sparing in *The Craft*, matters soon changed, and Part III of *Formal Logic* is generously spattered with occurrences of L and M (Prior 1955a). L and M were also used in the (posthumous) second edition of Łukasiewicz’s *Aristotle’s Syllogistic* (Łukasiewicz 1957, edited by Lejewski). Łukasiewicz’s ironic remark that his symbolic notation would be ‘popular in the whole English-speaking world’ became true to a degree when Hughes and Cresswell used L and M in their famous textbook—but grafted onto the orthodox propositional notation that Łukasiewicz rejected.

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[17] He explained (1953b: 321, fn. 7) that he was using Bocheński’s S for NMN rather than Feys’ L because L ‘suggests *logical* necessity, and we shall see that it is important to distinguish the necessity expressed by “NMN” in this system from logical necessity’.

[18] The earliest document we have seen in which Γ is used for necessity is a handwritten note by Łukasiewicz’s Irish colleague Meredith, dated 2 February 1953 (Meredith 1953).

In *The Craft*, Prior next enters into a substantial philosophical disagreement with Łukasiewicz—the moral of which, developed more fully in Prior's subsequent article 'On Propositions Neither Necessary Nor Impossible' (Prior 1953a), is that the modal operators are not truth-functional, not even in a many-valued logic as Łukasiewicz held. Łukasiewicz maintained that modal logic can be formed from a many-valued propositional calculus, by introducing a new operator M and a truth-table definition of it (Łukasiewicz 1930: 66-67). Interestingly, even by this point in the composition of *The Craft*, Prior appears still not to have read Łukasiewicz first-hand. When tackling Łukasiewicz's view that modal operators are many-valued truth-functions, it is Bocheński's account of Łukasiewicz's position to which Prior refers (Prior 1949–51: 728; Bocheński 1947a: 99-100).

> The employment of these symbols has laid bare a rather more recondite point of parallelism between modality and quantity. In his 'proto-thetic' or logic of propositions, the Polish writer Lesniewski has a theorem which he formulates symbolically as CKϕpϕNpϕq. This means, roughly, that if anything is true both of a given proposition P and of its contradictory, then it is true of any proposition whatever. If the functional symbol 'ϕ' is understood as standing for a truth-function of which P is the argument or one of the arguments, this theorem is certainly true. For example, if we let our 'ϕ' be N, we have CKNpNNpNq, 'If both not-P and not-not-P then not-Q', which is simply a variant of the law that a contradiction implies any proposition whatever. Again, if we let 'ϕp' be 'Cpr', we have CKCprCNprCqr, 'If both if P then R and if not P then R, then if Q then R'. This is true also, for if both P and not P materially imply R, then R is true, and so is materially implied by any proposition at all. The theorem, in fact, when thus limited in its scope, amounts to this, that if anything is true both of the truth-value denoted by 'P' and of the truth-value denoted by 'not-P' then it is true of the truth-value denoted by any proposition at all; which follows from the fact that the truth-values denoted by 'P' and by 'not-P' are the only truth-values there are. ('If both not P and not not P then not Q', for example, means that if the truth-value denoted by 'P' is the False and the truth-value denoted by 'not-P' is also the False, then the False is the only truth-value that any proposition at all can denote, for under these circumstances it would be the only truth-value there is). But Professor Lukasiewicz

has pointed out that if we let the 'ϕ' in Lesniewski's theorem be a modal operator, we are liable to obtain curious results. [Here Prior gives a footnote reference to *La Logique de Théophraste*.] For example, if we let it be M, we obtain CKMpMNpMq, 'If both P and not-P are possible then any proposition at all is possible.' This is a fantastic contention; for it is plainly possible, at least logically, both that every man should be mortal and that not every man should be mortal, but many other things, e.g. that there should be some man who is not a man, are as plainly not possible. The obvious conclusion to draw from this (though it is not the one drawn by Professor Lukasiewicz) is that Lesniewski's theorem is not applicable to modal functions … The … theorem itself is false when its 'ϕ' is allowed to represent modal functions as well as truth-functions.

(Prior 1949–51: 727-729)[19]

This disagreement over whether modality is truth-functional resurfaces again in the next chapter of *The Craft*, where Prior criticises Łukasiewicz quite sharply (see below). The present chapter moves on to an outline of Prior's own positive account of modality (which again footnotes Feys' 'Les Systèmes Formalisés des Modalités Aristotéliciennes'). His account stands in stark contrast to Łukasiewicz's view that (as he summarised his position in a letter to Prior (Łukasiewicz 1953b)) 'the best and the most reasonable approach to [the modals] is to treat them as "truth-functions"'. In their correspondence, Łukasiewicz emphasised several times to Prior that truth-functionality and many-valuedness were necessary conditions for formalising modality, and accused Prior of 'another philosophical prejudice, viz. that there exist only two truth-values', saying 'With this prejudice you will be never able to build up a reasonable system of modal logic' (Łukasiewicz 1956). According to Prior, however, the modal operators are not truth-functions but quantificational constructions. The following passages from *The Craft* (elements of which Prior included in his first publication on modal logic (Prior 1952b)) contain the first appearance of what would later become his finely-developed translational approach to modal semantics (Copeland 2016):

[19] Rybaříková argues that Prior's ontological position was later influenced by Leśniewski's 'calculus of names' via the work of Leśniewski's students Sobociński and Lejewski (Rybaříková 2023: 98-100). (Leśniewski had died in 1939.) Another conduit from Leśniewski to Prior was Słupecki (Prior 1957: 63, Słupecki 1955).

> In the symbolism of Lukasiewicz, the form 'Everything has something which is F to it' would be expressed by the formula ΠyΣxfxy, while the form 'There is something which is F to everything' would be expressed by the formula ΣxΠyfxy. If we re-interpret 'f' as a monadic predicate instead of a dyadic one, and keep 'x' as an ordinary variable on which its quantifier (in this case 'Σ') acts in the ordinary way, but treat the quantifier of 'y' as a sign of modality (in this case, since it is 'Π', of necessity), the same formulae ΠyΣxfxy and ΣxΠyfxy will express respectively the propositions 'There is bound to be something which is F', and 'There is something which is bound to be F'. (In the symbolism of Professor Feys, these would be expressed as LΣxfx and ΣxLfx.)
>
> (Prior 1949–51: 733)

Here Prior is again echoing Feys' arguments in 'Les Systèmes Formalisés des Modalités Aristotéliciennes'. As previously mentioned, Feys argued that modal propositions can be treated as quantifications, and suggested that '*p* is necessary' be interpreted as '*p* is true in all cases', and '*p* is possible' as '*p* is true in some case' (Feys 1950: 497-498). In a section titled 'Modality as a Form of Quantity' (starting on page 736), Prior continues to develop his account along Feysian lines, but replaces Feys' enigmatic term 'case' with ontologically more robust terminology. (He had already used the term 'possible world' on page 462, in a discussion of Boole, saying 'What Boole was after might perhaps have been plainer if he had said something like this: There is one "hypothetical Universe", which contains the totality of what we might call possibilities, or if you like, "possible worlds"'.)

> For the similarity in behaviour between signs of modality and signs of quantity, various explanations may be offered. It may be, for example, that signs of modality are just ordinary quantifiers operating upon a peculiar subject-matter, namely possible states of affairs … It would not be quite accurate to describe theories of this sort as 'reducing modality to quantity'. They do reduce modal <u>distinctions</u> to distinctions of quantity, but the variables to which the quantifiers are attached retain something modal in their signification – they signify 'possibilities', 'chances', 'possible states of affairs', 'possible combinations of truth-values', or the like.
>
> (Prior 1949–51: 736-737)

Prior summarises the approach a few pages later: 'modal predicates are quantifiers operating upon entities which already have a modal character ("possible states of affairs", "chances" etc.)' (ibid.: 744). A few months later he re-expressed the idea in a different way, saying 'the modal value of a proposition is the set of possible states of affairs in which, and in which only, the proposition in question is true' (Prior 1952b: 140); and in a subsequent article in 1962, which incorporated Geach's concept of what we now call 'accessibility' (Copeland 2002: 119-120), Prior said simply: 'That P is possible, is the case in world A if and only if in some world that can be reached from A, it is the case that P' (Prior 1962: 37). In the same paper he proposed a tense-logical interpretation of Geach's accessibility idea: 'we might reach one world from another … simply by the passage of time (one important sense of "possible state of affairs" is "possible outcome of the present state of affairs")' (Prior 1962: 36). Returning to *The Craft*, a few pages further on he emphasises: 'There is everything to be said [for the] moderate view that we may not only use devices developed in the study of quantity to throw light on modality, but also vice versa' (Prior 1949–51: 747). These words point towards one of the most distinctive features of his later philosophy, his view that the study of quantification over possible worlds or instants of time could usefully illuminate the study of modality and tense, and yet such quantifications are ultimately to be interpreted *in terms of* modality and tense, which are primitive notions.

As Prior's discussion of modality moves on, von Wright enters the picture, in the course of a cameo exposition of Prior's conception of 'quasi-modal' operators (ibid.: 749). (Bocheński had recommended von Wright's work, in his first letter to Prior (Bocheński 1951b).) The deontic and epistemic modes are leading examples of quasi-modals—as well as Prior's yet-to-be-invented tense operators, which he describes as quasi-modal operators in *Papers on Time and Tense* (Prior 1968: 138).

> These representations make it plain why Professor von Wright, for example, should describe quantifications as 'existential modes' to be set alongside the ordinary or 'alethic' modes. Professor von Wright points out that there are other groups of modal predicates also, such as the 'deontic' modes which appear in Ethics. [Here Prior gives a footnote reference to von Wright, 'Deontic Logic', in *Mind*, January 1951.] Propositions about the permissibility, non-permissibility or obligatoriness of actions may be easily expressed in 'dictum and mode' forms … Professor von Wright also mentions

> the 'epistemic' modes occurring in such forms as 'It is established that X is Y', 'It is established that X is not Y', 'It is not established that X is Y', etc.; and one could think of innumerable others … There is a hint of a large field here …
>
> (Prior 1949–51: 748-749, 752)

Prior's final chapter, titled 'The Logical Calculus', is the most technical—Prior the formal logician is emerging as from a chrysalis. By the time he wrote this chapter, Prior had finally got his hands on some of Łukasiewicz's publications, and he refers directly to several works, including *Aristotle's Syllogistic* (Łukasiewicz 1951a). After an introduction to the concepts of axiom, rule and definition—in 'the symbolism of Lukasiewicz' (Prior 1949–51: 778)—Prior launches into a section 'The Axiomatisation of the Aristotelian Theory of the Syllogism'. He had first encountered Łukasiewicz's axiomatic treatment of Aristotle in Bocheński's *Précis* and, enchanted by it, he was taking his students through the derivations in 1951. He wrote in *The Craft*:

> Contemporary Polish logicians, by combining the Aristotelian and the Leibnizian methods of reducing the number of axioms, have shown how all the Aristotelian syllogistic formulae may be derived without using more than 4 axioms employing the special symbols …
>
> (Prior 1949–51: 776)

In a footnote he continues:

> This is the procedure adopted in Bochenski's Précis de Logique Mathématique, §10. Some other possibilities are discussed in a paper [Łoś 1946] reviewed by W. Bednarowski in Mind, Oct. 1949, pp. 544-5. See also §§15-18, 25 and 26 of Lukasiewicz's Aristotle's Syllogistic.
>
> (ibid.)

It was Łukasiewicz's axiomatic treatment of traditional logic that fully brought home to Prior the power of modern symbolic methods. Following the section on Aristotle in Prior's final chapter comes an excited romp through propositional logic, focussing on the question of minimal axiomatisations. Prior refers approvingly to Łukasiewicz's articles 'The Shortest Axiom of the Implicational Calculus of

Propositions' and 'On Variable Functors of Propositional Arguments' (Łukasiewicz 1948, 1951b).

He returns to his fundamental disagreement with Łukasiewicz over the nature of modality, writing of an 'error' in the second of these papers: he criticises Łukasiewicz for 'substituting modal operators' for the variable functor δ, rather than restricting the substitution to operators 'which form truth-functions'. But he adds that this 'does not affect the validity and importance of Professor Lukasiewicz's use of this type of symbol to simplify the axiomatic foundations of truth-functional logic', and he describes the paper as giving 'a quite remarkable simplification' (ibid.: 784-785).

Prior ends *The Craft* by returning to the Feysian connection between modality and quantification and by presaging a move he would soon himself make, as he reinterpreted the modal operators as tense operators:

> [I]f the logician is prepared to forget for a moment the meanings which he usually assigns to his formulae, he may find that they are capable of other interpretations that are of equal logical interest, and this gives him an admirable means of bringing out structural similarities between different parts of his subject (the logic of quantity and the logic of modality, for example).
>
> (ibid.: 806)

7. Conclusion

Bocheński's work was Prior's first window onto the landscape of Polish logic. With his ambition to use 'logistics' for the 'precise statement of theological and metaphysical arguments', Bocheński opened Prior's eyes to the possibilities of modern symbolism. ('We wanted to make logic a work tool', Bocheński said; 'What would Saint Thomas have done today? He would have used mathematical logic' (Bocheński & Parys 1990: 22).) Bocheński was also a reliable and encouraging guide as Prior began—voraciously—to explore logic's history.

Feys' work on modality provided Prior with important insights and a congenial modal notation. The connection he learned from Feys between modality and quantification ran like a golden thread through his mature thought.

Łukasiewicz gave Prior the propositional and quantificational notation he would use to express his logical ideas for the rest of his life. Along with Bocheński, Feys, and others, especially Russell, Łukasiewicz led Prior into the world of symbolic logic.

Łukasiewicz's work (and Bocheński's exposition of it) made clear to Prior the fundamental importance of propositional logic. 'It seems that Aristotle did not suspect the existence of another system of logic besides his theory of the syllogism', Łukasiewicz had written, '[y]et he uses intuitively the laws of propositional logic' (Łukasiewicz 1951a: 49). Propositional logic is barely mentioned until the final chapter of *The Craft*, whereas *Formal Logic* (Prior 1955) begins with a thorough introduction to the subject—and Prior says on page 3 that the logic of propositions is 'basic, and the rest of logic built upon it'. In his 1952 review article 'Łukasiewicz's Symbolic Logic' (Prior 1952a), Prior quoted with approval Łukasiewicz's statement that 'the logic of the Stoics, the inventors of the ancient form of the propositional calculus, was much more important than all the syllogisms of Aristotle' (Łukasiewicz 1951a: 131).

Prior's 1952 review—which discussed Łukasiewicz 1948 and 1951b as well as 1951a—was one of his first papers to make extensive use of symbolism. As soon as he became familiar with Łukasiewicz's notation, axiomatic methods, and matrix methods, he adopted them all, repeatedly emphasizing their advantages. Łukasiewicz's detailed investigations into the history of logic also resonated with Prior. What he admiringly wrote in his review of Łukasiewicz's work would soon become true of Prior himself:

> Professor Lukasiewicz, having done very distinguished work as a mathematical logician in the modern style, is at the same time interested in the history of his subject … and contrives both to use modern techniques to bring out more clearly what the ancients were driving at, and to learn from the ancients useful logical devices which the moderns have in general forgotten.
>
> (Prior 1952a: 37)

In Prior's work, the spirit of the great Warsaw School lived on.

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