

A.N. Prior's development and presentation of tense-logic and its context in June-August 1954

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Abstract

A.N. Prior (1914-69) presented his tense-logic for the very first time at the New Zealand section of the Australasian Association of Psychology and Philosophy, Second Philosophical Congress, Wellington 27th – 30th August, 1954. He introduced his ideas as a new and important approach to the discussion of basic aspects of time. It is, in fact, remarkable that Prior was able to write such an important paper at this time of his life. He had to face several challenges mainly due to TB in the family. However, this also makes it possible to follow several of his considerations since he wrote about all of it in his many letters to his wife Mary, who was then at the sanatorium with TB. We present a selection of the letters along with the presidential address itself.

Keywords: Tense-logic, modal logic, time.

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Chapter 1.

Introduction: A.N. Prior's introduction of tense-logic on the 27th August 1954 and the context of the event in June-August 1954

A.N. Prior (1914-1969) is the founding father of tense-logic. He introduced his important ideas on time and tense to an international research community for the very first time on the 27th of August 1954 at a conference at Victoria University, Wellington, New Zealand. The following offers a brief presentation and a discussion of the new paradigm that Prior suggested in 1954 as well as an account of Prior's personal situation at the time – with an emphasis on the challenges and the inspiration he experienced when he was preparing himself for the important event in Wellington.

The Prior family had to deal with several challenges during 1954. First, Mary was diagnosed with tuberculosis, and she had to stay at the sanatorium for several months. The children Martin (born 1944) and Ann (born 1949) also got TB and had to stay in isolation at home taken care of by a nurse and taught by teachers visiting them.

Furthermore, the family was advised to move to another house at a new location that was supposed to support their recovery better. This meant that Arthur Prior not only had to deal with the problems directly caused by TB, but he also had to take care of selling their house, buying a new and better house and moving from the old to the new house. In addition, he continued his work with teaching and research at Canterbury University College, Christchurch. Moreover, he was the president of the New Zealand Congress of Philosophy, and in this capacity, he had to give the Presidential Address at the New Zealand section of the Australasian Association of Psychology and Philosophy, Second Philosophical Congress, Wellington 27th – 30th August 1954.

It seems that Prior for a long time was rather uncertain regarding his choice of topic for the presidential address. He had for a long time wanted to develop a new logic that could treat the tenses of time in a satisfactory manner. In his letter to Mary on June 6, 1954¹ he wrote: "This afternoon also whacked out some stuff on this Time business – a short thing for

1 The letter is included in section 2.

Analysis cracking at Jack's idea that the flow of time is an illusion²; and the beginning of a long thing on 'The Syntax of Time Distinctions'³, which is going to be a classic."

On the other hand, he was probably uncertain whether his ideas were sufficiently mature and ready to be presented at the conference in Wellington. After all, he had just begun to work with this "long thing". He clearly had the strong belief that it was going to be a classic, but also had a number of open questions that should be answered before he could present his new paradigm in logic to a broader audience of scholars. It is obvious that several of the open questions had to do with modal logic, and in particular with the ancient argument of Diodorus which may be interpreted as an attempt to link time and modality.

We know about Prior's considerations from his many letters to Mary, in which he not only updated her on the state of things regarding the children and other matters in their home, but also shared his thoughts on logic and philosophy with her. This was meaningful because Mary herself was trained in logic and philosophy.

In a letter to Mary on the 29th of July, Prior stated that he would "definitely devote the larger part of my Wellington thing to a discussion of whether the question of Platonism v. nominalism is purely a verbal question"⁴. However, there is no such lecture or paper mentioned in the programme for the Second Philosophical Congress⁵.

Eventually, Prior decided to use the occasion at the conference in Wellington for the very first presentation of his tense-logic to an international research community. The title of his paper was "The Syntax of Time-Distinctions". It seems that it was some important discoveries reported in the letters to Mary dated 19th July, 23rd July, and 2nd August that made Prior decide to go for a presentation of tense-logic at the conference in Wellington. It even looks as if Prior tested and further

2 Editors' note: J.J.C Smart, also known as Jack Smart was an Australian Philosopher at the University of Adelaide. The reference here is to Smart's article *The River of Time* (1949) in *Mind*.

3 Editors' note: *The Syntax of Time Distinctions* (1958) was eventually published in *Franciscan Studies*. Arthur worked on it during the summer and presented it as the presidential address at the Second Philosophical Congress, held at Victoria University Wellington, New Zealand on the 27th August 1954.

⁴ Jakobsen et al. 2020, p. 185.

⁵ Jakobsen et al. 2020, p. 223.

developed his ideas in connection with his teaching in philosophical logic at Canterbury University College, Christchurch.

Right from the beginning Prior planned that the “long thing”, i.e., *The Syntax of Time-Distinctions*, should eventually be even longer. On June 6 he wrote: “Later on I may work on interaction between tense-logic and deontic logic, but that’s way up in the air at present –”. It appears that Prior was aware that he was about to introduce a new paradigm, a new approach to logic that ought to be further developed in various ways.

It is also remarkable that this new understanding of logic was introduced as a rediscovery of a way of doing logic well-known to ancient and medieval logicians. In general, Prior very much emphasised the importance of studying the history of logic. He found that the modern logician in many ways can benefit from the works of earlier generations of logicians.

In the following we present a selection of the letters from the period along with the presidential address itself, in which we have added translations of Prior’s Polish notation into the formalism that is now mainly used in philosophical logic.

In his presidential address Prior explains his approach to time according to which tenses should be taken seriously. Furthermore, he suggests a tense-logical formalism taking inspiration from modal logic. However, although he notes that the logic of “always in the future” to some extent works like “necessary” and “sometimes in the future” to “possible”, there are also clear differences. The main reason is that p does not in general imply “sometime in the future p ” whereas it is a general law of modal logic that p implies “possibly p ”. Things become even more complicated when the past is taken into account, since we have to allow for a potential asymmetry between past and future. In this work Prior is able to benefit a lot from his previous work with the Master Argument of Diodorus. This inspiration is evident not only in the presidential address itself but also in several of the letters.

In his presidential address Prior related his tense-logical approach to time to the earlier-later calculus as it is known e.g. in physics. Although he found the former approach to time more basic than the latter, both views are clearly important. In fact, the presidential address introduced a paradigm for the study of time which Prior worked with until his death in 1969 and which many philosophers, logicians, and computer scientists have developed further during the following decades. Within Priors framework a lot of studies have been carried out. Among other things

Prior's claim of the primacy of tense has been analysed carefully and a number of arguments pro and con his view have been put forward over the years.

Chapter 2.

Selected letters from Arthur to Mary Prior during June-August 1954

The 16 papers published in the following section should be seen together with the nine letters from the same period that can be found in (Jakobsen et al. 2020; listed on p. 182-183).

The selected papers can all be relevant as seen in relation to Prior important introduction of tense-logic at Victoria University, Wellington on the 27th August – either directly or indirectly as a useful contribution to a better understanding of Prior’s situation when he was preparing himself for this significant event.

For each letter he names of the persons who have done the transcription and produced the editors’ notes have been mentioned in the first footnote related to the letter.

The handwritten letters all belong to the Martin Prior collection⁶ currently kept at Aalborg University, Denmark. The letters will also be made available in The Nachlass of A.N. Prior⁷. Here the user may read our transcriptions with comments (such as the 16 letters in the following section), and it will also be possible to inspect scans of Prior’s handwritten original papers.

⁶ The Martin Prior Collection includes a number of letters between Arthur and Mary Prior from 1945 and later. The letters in the collection focus on cultural and philosophical matters of common interest. Both Arthur and Mary were qualified and interested in a number of cultural, political, historical, philosophical and theological issues.

⁷ See <https://nachlass.prior.aau.dk/>. The page numbers in the handwritten letters are indicated in {}.

2.1. Letter from Arthur to Mary Prior, June 6, 1954

23 Vernon Ter.
Hillsborough, Chch.
6/6/1954

Darlingest,⁸

It's been a cold, odd, but reasonably fruitful day. Was at Cuthbert's² this morning, signing various documents connected with tomorrow's transactions, and delivering him the keys. He mentioned that his son now has a temporary assistant lectureship with the History Dept. here, which is nice. I told him that Phillips⁹ was very discerning. Then went to the Electoral people and got the requisite forms; there's no need for you to worry about this; I can do it for you, and have. Tried to get hold of Gordon Troup¹⁰, but he was at Boy's High School or somewhere, so shall have to try another day. Bought some official rat poison from the City Council that Ratofax doesn't seem to be much good; unless the rats we have are numbered in dozens. And this afternoon saw Mrs. Kirk; she can't come Wednesday this week, and suggested Thursday, but I put her off, and she's coming 3 times next week. She seems very good and sensible, and is not taking much notice of the Correspondence School stuff, but coaching him in what they're doing at Opawa, and she thinks he's good. This afternoon also whacked out some stuff on this Time business – a short thing for Analysis cracking at Jack's idea that the flow of time is an illusion¹¹; and the beginning of a long thing on 'The Syntax of Time Distinctions'¹², which is going to be a classic. I'm waiting to hear

8 This letter has been transcribed by David Jakobsen and Martin Prior. It is located at the Aalborg University, Aalborg in The Martin Prior Collection.

2 Editors' note: Cuthbert was Arthur and Mary's lawyer and was helping them with buying a new house.

9 Editors' note: Neville Phillips (1916-2001) was a historian and university administrator. In 1954 he was Professor of History and Political Science and became Vice-Chancellor and Rector in 1966 of University of Canterbury.

10 Editors' note: This is quite likely a reference to Gordon Troup (1898-1977) who, according to the memory of Martin Prior, taught French at Canterbury University.

11 Editors' note: J.J.C Smart, also known as Jack Smart was an Australian Philosopher at the University of Adelaide. The reference here is to Smart's article 'The River of Time' (1949) in *Mind*.

12 Editors' note: *The Syntax of Time Distinctions* (1958) was eventually published in *Franciscan Studies*. Arthur worked on it during the summer and presented it as the

from the Phil. Quarterly about the Diodorus thing¹³; if they take it, I'll shoot the big thing over to the Journal of Computing Systems; if they don't I'll shoot Diodorus over to the JCS and tell them the big thing's coming. I have a short minor thing on Stoic Logic to submit to Franciscan Studies. Later on I may work on interaction between tense-logic and deontic logic, but that's way up in the air at present – main thing is to get these other things done.

It's really a good job you people dumped my old coat at the suit doctor's – if I hadn't been wearing my big new one I'd have been wet through by the time I got to Cuthbert's this morning (I was near enough to it as it was); and I don't think it's harmed the coat, except that it's got pretty damp. I've re-lit the ventilator specially to dry it, and will have a bath while I'm at it.

I suppose the time between now and Wednesday afternoon is really quite short as times go; but hell it do drag! There is an emptiness about life away from you that writing letters to you (and even having them from you, though that does always brighten things) can't fill; so my letters get empty too.

I was sorry for Martin's sake to put off Mrs. Kirk for Thursday; I think she's sensible with him and he likes her; but of course there was no way around it – we can't have our privacy eaten into then. Such thoughts suddenly make me alive to the fact that my hands are cold! Must post this and get into that bath and to bed.

I love you and love you and love you

XOXOXOX

- Skig

presidential address at the Second Philosophical Congress, held at Victoria University Wellington, New Zealand on the 27th August 1954.
13 Editors' note: Diodoran Modality (1955) *Philosophical Quarterly*, Vol 5., pp. 205-213.

2.2. Letter from Arthur to Mary Prior, June 17, 1954¹⁴

23 Vernon Tce
Hillsborough
17/6/54

Darling,

It's been a day full of oddments. This afternoon I've been minding the children with Miss B.¹⁵ taking her day off. They've been v. good, but Ann's temp. is up again. (99.5). Well, they'll be X-rayed tomorrow¹⁶. – When I came in with their tea I found them playing a delightful game of, I think, Martin's invention. Martin will go through some kind of motion meant to represent the name of some famous person, usually a musician, & Ann would interpret it, with much laughter; e.g. he would move his hand as if turning a handle (Handel), or as if doing the washing with Persil (Purcell), or waggle his tongue for 'Wagler' (Wagner), or hit the bed with his fist for 'Bash' (Bach), or move as if firing an arrow for the philosopher Aristotle; or push something along for Pushkin. (He asked me the other night what Pushkin's poetry was about, & I said I wasn't sure, but later on had a look at that book Valmai¹⁷ once gave us about 'The Poetry & Loves of Pushkin'; & decided it was not exactly for Martin). The NZ Insurance Co. chap called today to look over our furniture with an eye to our permanent policy. He said that they'd insure it for £800 without further inspection; & we could make it more if we could justify it by an inventory. But £800 is OK isn't it? He also asked if we wanted it insured against fire only or to take out a general household risk policy (against fire, burglary, storm & flood, &c.)

When the kids were playing that game tonight, Ann said 'Isn't Martin –' & then stopped because she couldn't think of {2} the word she wanted, but told me it was a word like 'distinguished'; & then she found it – 'disgusting'. I'm not quite sure what she thinks it means. Since their lights

¹⁴ Editors' note: This letter has been edited by Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University.

¹⁵ "Miss B." is Miss Brown who took over nurse from Helen Baird when she got married.

¹⁶ Martin and Ann had TB and were X-rayed at a clinic in town on a regular basis.

¹⁷ Valmai Moffett was a 'cello player and a friend of the family. Cf. Portrait of Valmai Moffett - Dunedin Public Art Gallery

have been out they've been singing various songs like 'Happy un-birthday to you'; & she said to me about this, 'Aren't we disgusting!'

I picked up your letter this morning. It was a lovely surprise to get it. I didn't realise when you spoke yesterday of 'those of us who are on walks' having to do this & that, what a sudden step forward you have had.

It's just 8, but I'll post this now – must have an early night tonight; tomorrow will be very full.

– I love you & love you & love you.

- Skig

XOXOXOXOX

2.3. Letter from Arthur to Mary Prior, June 21, 1954¹⁸

Senior Common Room,
Canterbury University College,
Christchurch
Monday, June 21, 1954,
5.40 p.m.

Darling,

It hasn't been a bad day. - Rang Cuthbert on way to work, & arranged for him to have a document ready for me tomorrow night about 5.15 p.m. Will discuss with him then the detailed procedure of the sale.

Danks was all in favour of the Ryle-Delphi idea for next term. He was also all in favour of an idea of mine that the subject should just be 'Philosophy'; I will be interesting to see what sort of questions the word provokes from the multitude. - I have written to Ryle accordingly.

The Panel on Friday is to consist of Percival; the Dean (of Christchurch: Ve. Martin Sullivan); Henderson of the Engineering School; & myself. Henderson is a friend of Pip Alley's¹⁹; I should say just as Red, but considerably abler (in expressing himself, anyway). So it shouldn't be a bad show.

I have punctured some sherry & some brandy, so that our choice of drinks on Thursday will be less restricted than it was last time.

When I saw Mrs. Lambourne yesterday I gathered that she regarded her brother as a bit on the young side to be buying a house, but that they have been saving like mad for it, that it was nevertheless her (Mrs. L's) suggestion that they should nip in after 18 Grange St.; & that his wife was particularly enthusiastic about the idea. It was encouraging that on the evening I came round he {2} seemed anxious that the deal should go through, & that I shouldn't sell it over his head; but I shall nevertheless myself be happy when we've got the thing down in black & white.

18 Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 11.

19 Philip John (Pip) Alley (1901 – 78), a pioneer in New Zealand of soil mechanics, establishing one of the first soil mechanics laboratories in the country. By the late 1950's, Pip Alley had become well known in Christchurch and at the University as being an avowed communist.

A new copy of the Australasian Jnl. of Philo. has arrived. I haven't had time to read it, but it contains a critical notice by John Mackie²⁰ of Passmore's Hume, & Ron Butler's review of Bernard Mayo's The Logic of Personality.

Ken Home was asking about Passmore's movements today; I gathered from him that the advertisement for the chair has appeared.

I must, in no time now, attend to Stage III.

- Heaps & heaps of love

- Skig

XOXOXOXOX

20 John Leslie Mackie (1917–1981). In 1946 he became a lecturer in moral and political philosophy at the university. In 1955, the year after this letter, he was appointed to the chair of philosophy and psychology at the University of Otago, New Zealand.

2.4. Letter from Arthur to Mary Prior, June 25, 1954²¹

23 Vernon Tce.
Hillsborough
Friday 25/6/54.

Darling,

It's late (8.25) & there's so much news of one sort or another that I don't know how I'll get it all down.

Had a talk to Henry Field²² & Gordon Troup²³ this morning about the possibility of a fund for legal aid to the Hulmes²⁴. I gathered from Henry F. that their need on that side isn't likely to be great. He thought Juliet clearly schizophrenic, & that there was little doubt that she (& the other girl too, apparently) wd. be declared of unsound mind, unable to plead, or something of that sort; & their lawyer is Gresson, who's a close personal friend of the Hulmes & almost certainly wouldn't lay on the fees thick. - But the background is in other ways worse than one realised. Henry & Hilda are apparently on the verge of separating (had this not happened, that wd. have taken place quietly when he'd gone to S. Africa) - for some time there have been in effect this two households at Ilam, one consisting of Henry & the other of Hilda & some chap named Parry. And the general impression appears to be that Hilda is as hard as nails & will get by, that it's Henry that's been hit all along the line. If there is already no way in in which he can help Juliet, he'll probably leave with Jonathan very shortly. And Henry {2} Field is not too confident of Hulme's own continuing sanity, but says that there seems nothing one can do without running the risk of making everything worse than ever.

Discussed other matters with Henry Field at morning tea - e.g. Elliott & Cohen; & also Martin's education. He thought it might be a good thing

21 Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 12.

22 Professor of Education at Canterbury University College.

23 Lecturer in French at Canterbury University College.

24 Mr. Hulme was Vice-Chancellor of Canterbury University College. His daughter, Juliet Hulme was convicted of murder later in 1954.

to get some schoolteachers in to do occasional supervision at something like 7/- an hour. (Timbrell himself might be a possibility).

Ran into Michael Shorter very briefly after lecture tonight; he had messages from Valmai²⁵. She's going to be out of town again till August or thereabouts, with the Nat. Orchestra. And she's not been well - gall bladder trouble - but hopes the trip will do her no harm. They have played that trio thing, & she thinks it might be a good thing later to do it over 3YA²⁶. - Michael's engineering talks seem to be going quite well. He's more or less branded the lions in their den & has been talking to them about mathematics, & there's going to be some sort of discussion hour next time at which they're preparing sundry teasers for him.

It's very difficult to say how the Delphin thing went off, though I think Martin Sullivan talked far too much & thereby cut down the number of questions that could be dealt with.

At Church on Sunday (at St. Martins) the thing's going to be run by students (it's WSCF²⁷ Sunday or something) & I've been asked to talk briefly {3} to the children about the university. So I've asked Miss Brown to stay home till lunch time to make this possible (while letting her [stay] away tomorrow afternoon as well as Sunday). - Round about July 16-21 Mrs. B. would like to go to some Centennial celebration at Waimati, though she won't go if we can't make arrangements. I think I'll just get Alison Wilson a list of the times during that period (a Friday afternoon to a Wednesday) that I have to be out, & see if she can find sitters that could manage them.

Must post now - heaps & heaps of love.

-Skig

xoxoxox

25 Cellist in the Christchurch orchestra.

26 A public service radio station in Christchurch.

27 WSCF stands for World Student Christian Federation.

2.5. Letter from Arthur to Mary Prior, June 27, 1954²⁸

CANTERBURY UNIVERSITY COLLEGE
CHRISTCHURCH, N.Z.
Sun. 27/6/54.

Darlingest,

It's about 6.15 - the kids have had their tea, but it's not time for their PAS²⁹ or bedding down yet. And I'll have this posted only tonight, & get another early night.

I have just composed a proper formal reply to the invite to Helen's wedding, which I shall post at the same time as I post this. I have added to it a postscript in the same style, viz. 'Professor A.N. Prior, Master Martin Prior, Miss Ann Prior, & Miss Helen Baird's bicycle, would be pleased to see Miss Helen Baird at any time at their residence at 23 Vernon Terrace, Hillsborough.'

I don't know if I told you that Michael has said yesterday that he'd be very glad to have those 2 old book-cases in our back room.

Now I'm going to quit. This here study's cold, & maybe I'll start [writing] more to you in bed, after I've attended to the kids & posted this.

Heaps & heaps of love.

– Arthur.

XOXOX

28 Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 13.

29 A large tuberculosis pill.

2.6. Letter from Arthur to Mary Prior, June 28, 1954³⁰

CANTERBURY UNIVERSITY COLLEGE
CHRISTCHURCH, N.Z.
Mon. 28/6/54

Darling,

It's been quite a pleasant day, - at college this morning Father O' Brien dropped round, to tell me that his boys were having examinations today & tomorrow, so that he had a bit of time on his hands, & wd. be glad of a yarn. So he came round (to Vernon Tce.) this afternoon, & is coming to lunch tomorrow. This afternoon he told me that the report of my utterances on Friday had created something of a sensation at his seminary, my statements on the proofs, as reported, being particularly puzzling, not to say startling. So I gave him the full details, wherewith he was comparatively well satisfied.

Bill Heybourn³¹ was also round to see me. He's applying for some sort of job with the catchment board, & thought that some sort of character testimonial from me wd. be useful, as I knew Jobbins. I suggested that as I'm really not competent to speak of his qualifications for the position in question³², it might be better if I rang up Jobbins & spoke to him personally & this I shall do.

{2} This morning Martin didn't get anything done but reading, but this afternoon we got down to things properly before Fr. O'Brien came, & he did 2 English lessons & some mental arithmetic. He made many grimaces of distaste, but got a bit interested as we went on. He had to say about a poem whether he liked it or not, & why. It was one by Alfred Noyes about a foolish duckling, & he didn't like it, so I got him to write that down. As to why he didn't like it, his first reaction was that it was childish, but we didn't write down that; then he said he didn't like poems about animals

30 Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 14.

31 A neighbour from Cashmere Hills.

32 Added in the margin: As I wouldn't be personally known to most of the people.

that talked, so he wrote that down. The next lesson was picking out the nouns in a paragraph; & this he did quite perfectly, even though they included abstract nouns like 'run' (in he broke into a 'run') & 'shout', & nouns like 'earth' & 'stone' ('made of e. & s.'). So I complimented him on this & drew attention to the peculiarities of these ones, & talked about Categories for a minute or two. In the arithmetic {3} he did the addition perfectly, but wasn't hot on the subtraction, & in too the middle of the latter he was visibly tiring, so I let it go at that.

I went round about 4 to see Timbrell about coming round. He can't do it himself but will try & dig someone up & I'll see him again at the end of the week.

Mrs. Blockley can come in the mornings when Sister is at Waimate.

Lorna's brother (or rather her sister-in-law, who I got on to) knows nothing about her presence in Chh., & looking up her letter, I see that what she has is '25th June or later'.

I rang Bob to talk about the house-cleaning, but he was out. Today's mail included a nice letter from Rod. Chisholm³³, which I enclose. I'll send him my Logic Notes.

The plumber came today & installed the hot water in the wash house. Electrician will be along tomorrow to put in point.

The address you wanted is {4} as follows: -

R. C. Gooderidge
NZ Manager, Oxford University Press
POBox 185
(or 17 Grey St.)
Wellington C1

I enclose also Gaius's³⁴ letter.

- I love you & love you & love you
- Skig

XOXOXOX

33 Roderick Milton Chisholm (1916-1999) was an American philosopher at Brown University.

34 Transcription not clear.

2.7. Letter from Arthur to Mary Prior, June 29, 1954³⁵

As from 23 Vernon Tce.
Tues. 29/6/54

Darling,

I am in fact writing this in Miss Labatt's room in lieu of lecturing to Stage I, whom I have left doing a test. As you see, a lot of mail has come, on which I venture the following comments: -

(1) I have apologised to Gillian for not replying to her on the form provided & forwarding it to you instead.

(2) Don't leave Mother's letter unread. The marked portion is extremely interesting, & I had not heard about it before.

What stiff upper lips we do keep! I have always felt that I have been grumpy, and you have been wonderful cheerful, & now you tell me vice versa.

As usual I do wish I could remember Ann's middle-of-the-night story, but I hardly can. She just rambles on & on, with do most wonderful fancy. Last night's was about a person called {2} 'Elbie' who was imprisoned in a blind, but escaped, & it fell to pieces. Part of the story was that she wanted to build a house. - 'She wanted to build it will straw; but NO; then she wanted to build it with bricks; but NO; then she wanted to build it with plastic; but NO ...' I asked her what she knew about that made of plastic; & she said electric light switches.

Just along the road from us there lives one Fox, who works on the Press & knew me in the days when I did. The children & I were talking to him when out for a walk the other day. And this afternoon when Ann & I were walking along he came past in his car & said Hullo, & I asked Ann who it was. She said 'Mr. Fox', & I asked her how she remembered his name. She said, 'I will remember it for ever, like an elephant'.

Martin still creeps like a snail most unwillingly to school; however, we're getting a few things done, {3} & occasionally he gets briefly interested in what he's doing.

³⁵ Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 15.

It was a pleasant lazy session with Father O'B. today. He said my explanations of Friday's effort had satisfied his colleagues, & I said I hoped he hadn't defended me too assiduously as I was by no means a Thomist, though philosophical theology was probably my next interest to logic. (I told him of Father Thomas's³⁶ translation to Oxford, & remarked how regrettable it was that so good a logician was now descending to the teaching of theology). We argued the toss about the divine unrelatedness for a little, but most amicably.

Will have to get back to that class; must post this on the way home, as there's a seminar tonight.

I love & love you & love you
- Skig

XOXOXOXOX

PS. Father O'B. was interested in {4} our pictures. Re that little John-the-Baptist one that Colin gave us & that is in our bedroom; I thought I was early Italian, & he was very surprised & thought it was modern, & I wondered if I wondered if I had made a hopeless gaffe & assured him I know nothing of the subject & would ask you. What is it? - He liked your own pictures, esp. the one with that jam jar of water in it.

³⁶ Ivo Thomas, a logician and Blackfriar priest.

2.8. Letter from Arthur to Mary Prior, July 4, 1954³⁷

23 Vernon Tce.
Hillsborough
Sun. 4/7/54.

Darling darling,

It was a funny almost listless sort of time we had together this afternoon, you with your exhaustion & me with my cold & bunged-up ears. But like you I wouldn't have missed it for anything (& there's something silly about saying 'like you', but it's always nice to know again, even if one knows it all the time, that you love me too).

Nothing worth recording has happened since I got home. - We've had tea, & the kids are embedded, & I've been sorting out the odd book to take along to Henry Field's MA class tomorrow - 'Philosophy of the 1950's'. Really it's amazing what a lot of 1st class stuff has come and in Philosophy in the 1950's. I've excluded things of purely technical interest like Łukasiewicz's Aristotle's Syllogistic (but have put in an issue of the JCS³⁸ as a showpiece). There's dozens of collections - I've put in Logic & Language II; Semantics & the Phil. of Language; From a Logical Point of View; Translations from Frege; & Philosophers Speak of God. And full-sized books in all lines - I've put in Hare's Language of Morals³⁹ & Wiener's Human Use of Human Beings⁴⁰ & Galbie on Peine & Goodman's Structure of Appearance⁴¹ (Wittgenstein's inaccessible). And a flow of good stuff in the {2} periodicals (& numerous new periodicals too - I've put in the JCS⁴² & the Brit. Jnl. of Phil. of Science).

Hope the concert goes off well again & that you're not too done in after it.

I love you & love you & love you
- Arthur.

37 Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 19.

38 Editors' note: Journal of Computing Systems

39 Editors' note: The Language of Morals, published 1952 by the British philosopher R.M. Hare.

40 Editors' note: Norbert Wiener, The Human Use of Human Beings: Cybernetics and Society, first published 1949.

41 Editors' note: Nelson Goodman, The Structure of Appearance, first published 1951.

42 Editors' note: Journal of Computing Systems

2.9. Letter from Arthur to Mary Prior, July 5, 1954⁴³

23 Vernon Tce.
Hillsborough
Mon. 5/7/54.

Darlingest.

As I write , you'll be having your concert at Cory⁴⁴ Partons in Dunedin till the end of the week, so I couldn't show him Jack's letter. But I asked Logie about the physics chain, & he said it had been advertised & gave me a schedule, which I posted forthwith to Jack. (If Ward's going to apply he will have to do I smartly, as closing dates end of month).

The Burmese chap was a bit disappointing, as the addressed the Staff Club mostly about the activities of the WSCF. I believe, though, he gave a very good broadcast last night.

The way with then Henry Field's class tonight went off very pleasantly tonight.

I've been reading Benson Mates's Stoic Logic⁴⁵, & I think I have a solution of one of the historical problems he leaves unsolved, & can make a fair guess at another. I'll write him a letter about it.

I've also taken out of the library a book on 'Symmetry' by Herman Weyl, which looks odd but interesting. It's got plenty of pictures anyway. If it's any good I'll bring it up on Wednesday. There may be something in it for Martin also. It's got everything in it from carbon atoms to Moslem mosaics & woman's hats (no underclothing though).

{2}⁴⁶

43 Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 20.

44 Cory - transcription unclear.

45 Benson Mates published his book Stoic Logic in 1953 which was based on his Ph.D. thesis.

46 This page is written on the back side of page 1, but is numbered as page 3. It seems it should have been marked as page 2.

And I'm reading properly that book of Wiener's that Jack Smart was so keen on, The Human use of Human Beings. I've interested Henry Fields⁴⁷ in it, & it may also have stuff I can use in this Civil Liberties thing.

Weyl, by the way, seems to be tremendously impressed by D'Arcy Thompson's Growth and Form.

There isn't much more time, so I think I'll post this. Sorry it's again so scrappy. I guess you'll be having a good rest after your concert when you get it.

See you Wednesday.

- Love & love & love

- Skig

XOXOXOXOX

47 Henry Fields, professor of Education.

2.10: Letter from Arthur to Mary Prior, July 11, 1954⁴⁸

23 Vernon Tce.
Hillsborough
Sun. 11/7/54.

Darlingest,

It's about ¼ to 7, & before long I'll be giving the kids their medicine & packing them up for the night. And then, or not long after, I'll post this & turn in myself. - I have 3 late nights ahead of me, & some full days. Tomorrow night's the Phil. Club, Tuesday will be the Stage III Seminar, & Wednesday the Civil Liberties affair. And I've 2 lectures tomorrow morning & one tomorrow evening; must see Michael about a few things, & get some PAS for Ann; probably Lorna's coming in afternoon; Tuesday, thank goodness, will be pretty free of lectures, though there's a Stage I in the evening, & I'll have to see how Alison Wilson's got on collecting sitters for the period Miss B⁴⁹. will be away. Wednesday will be full again, with the Prof. Board & all; but Thursday I'll be able to start taking it easy. - It's miserable not being able to see you Wednesday, but I suppose the less one thinks about {2} that the better.

Some time very shortly I must do a lot of clearing up - papers in my bedroom & study, bills to pay & the like. - I've been slipping a bit on the non-smoking lately, but will go back to rigour again tomorrow. When I left you today I remembered that I'd left my purse behind & wondered if I'd have cash on me to buy more smokes - found I had 10½^d & some stamps, & the man at the shop took a stamp, so I got my 10 de Reszke's; I won't get more till tomorrow night, & will help off then bt. 9&5. Must have one now, though!

Have finished medieval proofs of existence of God with Stage III - will be starting tomorrow on medieval theories of His nature, & raising the Hartshornian⁵⁰ difficulties à propos of some of Thomas's stuff - Old

48 Editors' note: This letter has been edited by Andreas M. Müller and Martin Prior. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 24.

49 Miss Brown, their nurse.

50 Charles Hartshorne, American philosopher who would be visiting NZ in this period. He developed a modal proof for the existence of God drawing on Anselm's ontological argument.

Hartshorne's good, but one trouble about his point of view {3} is that he's one of these jokers who don't believe in Things but only in Processes (plus Universals).

- Time's about up; must go to them kids. They're packed up now; but I don't suppose I've really heard the last of them for the night - Really, though, I mustn't go on so; just filling this letter with all the things I have to do. It's just like that Speech the other night, honey; it's just that it's on my mind. & when I'm writing to you in a hurry I can only put down what's on my mind, whatever it is - you're on my mind too, but because you're harder to write about my pen pushes you into a corner, so to speak; but you're always there. Sitting or lying & looking at you today as you talked {4} I remembered one day at Dave Mawson's when we were engaged & I was doing just the same - sitting & looking at you as you talked. And listening too; I think you think I don't listen because I look. And at Dave's you were talking about things that had been happening or had happened sometime at St. Margaret's or the like, just as today you were telling me about things that had happened at the San. And just like I loved you then, I love you now, & love you & love you & love you.

- Skig

XOXOXOXOXOXOX

2.11: Letter from Arthur to Mary Prior, July 19, 1954⁵¹

23 Vernon Tce.
Hillsborough
Mon. 19/7/54.

Darling,

I have found it very difficult to get down to my chores today because, I have been occupied with my equivalent of one of your pictures, viz. chasing & formalising an old logical point. You'll gather what the point is from the enclosed informal draft. Formalising the thing has revealed a nasty catch in it - on p. 4, the falsity (& consequent impossibility) of B does not follow from the truth (& consequent necessity) of A. I think I can mend this hole, but it makes the argument insufferably complicated. My problem is briefly this: - Let it be supposed that there is a shell at the bottom of the sea, which may have been seen before (this is the factor I've not allowed for on p.4), but which is not being seen now and never will be.

Now I want a proposition which is in the past tense, {2}

- (i) is false, and will be false for all future time, and
- (ii) will be entailed by 'The shell is being seen now', uttered either now or at any future time.

The proposition which I suggest on p. 4 is

'The shell will be seen' was true. (i.e. has been true).

But this unfortunately will not be false, if the shell has in fact been seen at some time before now; i.e. it meet requirements (i) & (iii), but not requirement (ii). What I need is, I think, something like

'The shell will be seen' has been true since t,

when 't' is some neutral way of referring to the present time (i.e. something like '8.35 p.m., 19/7/54').

⁵¹ Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 32.

I've also been working on something much easier, which has yielded a straightforward solution. Diodorus defined 'It is possible that p' as 'Either it is now the case that p or it will be the case that p', and I've been working out which of the various systems S4, S5 &c. this would yield. The answer is that it yields S4, i.e. if we define 'possible' this way, then it's {3} law that what is possibly possible is actually possible, though it isn't a law (as it would be in S5) that what is possibly impossible is actually impossible (nor is it a law, as it would be in Łukasiewicz's new system, that what is possibly false is possibly impossible - the thing is pure S4).

And now the bloody gong's nearly gone, & I'll have to post this after giving you nothing but Logic. Hope that's not too damned awful. I do love you, honey, & love you & love you.

- Skig

xoxoxox

2.12: Letter from Arthur to Mary Prior, July 20, 1954⁵²

23 Vernon Tce.
Hillsborough
Tues. 20/7/54

Darling honey,

Didn't go down to College today till I had to for the evening lecture (spent morning writing article on 'Diodoran Modalities', I think for Journal of Computing Systems, & digging up stuff at Grange St.⁵³; afternoon holding the fort), & when there found, beside yours, (a letter from some bloke Penang, of Kumara, Westland, addressing his claims to be a Prophet (& incidentally prophesying that our house would catch fire & all of us be killed) & complaining (to the Vice-Pres. of the Civil Liberties Council) of his wrongful confinement in a mental hospital in 1935 (& enclosing a letter on the same subject to the Govenor-General - this is the second time Sir Willoughby & I have been coupled). Does Joy know this bloke (or doesn't everybody in Westland know everybody else)? And (b) a letter from Stuart M. Brown, Jr., on behalf of the editorial board of the Philosophical Review, accepting for publication in the same our article on 'Erotetic Logic', & enclosing the referee's comments. I'll bring all these along, & dig out a copy of the article to bring along too, tomorrow. It is not necessary for us to act on the referee's suggestions, but if we want to do anything to the article in the light of them (& {2} some things may turn out to be worth doing), we are asked to do so pronto.

Reckon I better post this now. Sorry it's so literary, but it's good news about the article. The George-Packer business sounds sticky & rotten.

Love & love & love

- Skig

52 Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 33.

53 The Prior family was in process of moving to Vernon Terrace.

P.S. - Took kids to Bob Rossiter's this afternoon; despite the cold they wanted ice-creams, so I got them some. Bob R. said 'Won't you have one yourself, sir?'; I shivered; & Martin said 'You're advertising.' And on way home Ann said, 'If that man had been on the radio he would have said, "This program comes to you from Bob Rossiter Limited, Christchurch."'

A.

xoxoxoxoxoxoxox

2.13: Letter from Arthur to Mary Prior, July 23, 1954⁵⁴

23 Vernon Tce.
Hillsborough
Fri. 23/7/54.

Darling honey,

Have been busy as hell today, but EUREKA - I have found a sure-fire & quite simple & formalisable way of completing Diodorus's 'Master-Argument', & am incorporating same in my JCS⁵⁵ article. The proposition I'm after, in terms of my last letter on the subject, is

'The shell will be seen' has always been true.

I discovered it when lecturing to Brian & Jon⁵⁶ this morning.

I went past the San⁵⁷ to work this morning, & saw Margaret whoever-it-is & some other sheila walking down to the river, & gave your crayons to the other sheila to give to you, which I hope she did. God I do hope John McL. was reassuring this morning, & that that old C.⁵⁸ McIntyre isn't on to anything. My tongue is just hanging out for that letter from you tomorrow, & I hardly dare to rejoice in what good comes my way, such as finding the key to that old Greek riddle, until I know that it isn't drowned in distressing news from the San. {2}

Sawyer⁵⁹ has told me of a modern echo of the Diodoran contention that what is possible either is or will be true in the kinetic theory of gases - apparently its assumed in that that sooner or later the molecules will occupy all possible positions.

Enclose Mike Conway's photos, which have arrived at long last.

Love & love & love
- Skig
xoxoxox

54 Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 35.

55 Journal of Computing Systems.

56 Probably Jonathan Bennett.

57 Sanatorium.

58 Initial is very unclear here.

59 W.W. Sawyer - Professor of Mathematics.

2.14: Letter from Arthur to Mary Prior, July 25, 1954⁶⁰

23 Vernon Tce.
Hillsborough
Sun. 25/7/54.

Darling Polly,

It seems an age since I last wrote to you, though in fact it was only a couple of nights ago. - The doctors' joint verdict on what is to be done with you leaves one hanging in mid-air a bit. I marvel at the good sense with which you reflect that if the lavages reveal a cavity it's all to the good that it should be found out - me, I find it tempting to put off even my own routine X-ray for another 3 mths! (I'm not going to, of course - I go in on Tuesday).

It is mildly pleasant - nothing can be more than that while we don't know the state of your lungs - to be involved in all this literary activity just now. It casts a sort of faint but pervasive niceness over things. I've been thinking further about the Philosophical Quarterly versus the Journal of Computing Systems as the first destination of the Diodorus thing, & am pretty well decided on the PQ⁶¹. Nothing has happened here to speak of since I left you; but Alison Wilson plans to come again next Sunday & bring the twins. I gave the children omelette & Dutch loaf for tea. - Tomorrow I'm taking wads of stuff to typists, & I've arranged to see

Gillian⁶² in the afternoon about Stage I Logic papers. Tuesday afternoon there's my X-ray; Wednesday the Prof. Board meeting & a seminar in the evening; Friday morning the kids get X-rayed. But what my tongue hangs out for is Wednesday's {2} ½-hour with you. - I've a pile of small oddments to attend to that I've put off, & must work in somehow; the propogandising for John Mackie⁶³ with Parton.

60 Editors' note: This letter has been edited by Andreas M. Müller, Martin Prior and Peter Øhrstrøm. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 36.

61 Philosophical Quarterly, as referred to earlier.

62 Gillian Quentin-Baxter (later married Jonathan Bennett).

63 At this time professor at the University of Oxford.

This is a dead sort of letter, & that's rather how I'm feeling, mitigated by a small vein of thankfulness for small literary mercies. Will write you something decent some other time.

- Love & love & love

- Skig

xoxoxoxoxox

2.15: Letter from Arthur to Mary Prior, July 28, 1954⁶⁴

CANTERBURY UNIVERSITY COLLEGE
CHRISTCHURCH, N.Z.
Tues. 28/7/54. 65

Darlingest,

- It's been a queer sort of day. - First, mail. A further communication from Stuart M. Brown, Jr., this time with somebody else's article in it, which he asks me if I'd referee for them. I don't like to turn them down, & don't think I will; but I can't understand for the life of me why they sent this article to me. It's one on 'The Comparative Happiness of Groups', & is a defence of a 'census' method of estimating group happiness.

Then I spent a lot to this afternoon discussing high politics. - Proposed B about the Rectorship with a variety of people. Siemon, Allen & Garrett are agin it; Parton for it; Crowther inclined to be for. General agreement that if B is adopted, it's a good idea to get someone in from outside to run the department for 3 years, rather than let the burden fall {2} on the senior lecturer. Main difficulty about B is perhaps to get anyone to take it on. Hy. Field might, and he wouldn't be bad at this end of the job, but would be bad at the Wellington end, because too much disposed to agree with Beeby. Crowther has the impression that Percival might take it on, very reluctantly, but for the College's sake, so to speak.

Then was talking with Parton about arrangements for Ryle; about the Physics chair here; and about the Phil. chair at Otago. Physics chair: Jack Smart's friend has applied, & Parton is impressed by his application. And I was instrumental, through the course the conversation took, in drawing Parton's attention to the fact that the man won't have sent a parallel application to London, as Jim Logie had put it in the prospectus that this wouldn't be necessary for Australasian applicants. Parton thinks it

64 Editors' note: This letter has been edited by Andreas M. Müller, Peter Øhrstrøm and Martin Prior. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 38.

65 The letter is dated Tues 28/7/54. It is probably a mistake, as Tuesday was the 27/7/54.

important that the English committee should have something to say about this bloke, because he's an Englishman. {3}

He'd like to see him get English backing because there are people on the Council here who would be prejudiced by the fact that one of his referees is Oppenheimer. So he rang Jim Logie and asked him to write to the English people and get them to consider him - Re Otago Philosophy: - I lent him John Mackie's notes, & as a specimen showed him Mackie on Music & he was delighted, as the next Delphin is on 'Music & the Arts', & he's going to use Mackie's notes as a sort of magazine of provocative questions.

Have just turned off the wireless, with the announcer saying 'Hays present their feature. The Meredith Scandal', & as I turned it off, a loud squeak from Ann in the darkness - 'I want the Meredith Scandal!'

I took another Census with Stage I tonight, - asked them how many would assent to, deny, or be undecided about, the following {4} 3 props.

- (1) 'What is possibly possible is actually possible.' (S4).
- (2) 'What is possibly impossible is actually impossible.' (S5).
- (3) 'What is possibly false is possibly impossible.' (crazy).

15 definitely asserted to (1), 3 to (2) and 22 to (3); and the 3 who oriented to (2) asserted to (3) also, & were quite dismayed when I read (2) & (3) as a simple syllogism to infer that what is possibly false is actually impossible. - Perhaps Stuart M. Brown has somehow got to hear of this census-taking habit of mine, & that's why he sent me that article about Happiness Census for refereeing.

The von Freytag thing has also arrived for reviewing for the JSL⁶⁶. I might get van Rozen⁶⁷ along to English it for me. Will think about that.

There's no more news, but I'll be seeing you tomorrow[.]

- Love & love & love

Arthur

XOXOXOXOXOX

66 Arthur Prior reviewed Baron von Freytag Löringhoff's *Zur Logik als Lehre von Identität und Verschiedenheit* in the *Journal of Symbolic Logic*, March 1955.

67 It could alternatively be 'van Royen'.

2.16: Letter from Arthur to Mary Prior, August 2, 1954⁶⁸

CANTERBURY UNIVERSITY COLLEGE
CHRISTCHURCH, N.Z.
Mon. 2/8/54

Darling honey,

Another day gone. - Martin's cough's improving all the time, & tomorrow morning John H.⁶⁹ will be seeing him. - I've taken my spts. coat & cardigan to the cleaners: - And I reckon that's about all that's worth telling. - I'm very tired tonight, & have a pretty full week ahead, so won't make this very long. - Gave, I think, a very good lecture to Stage III⁷⁰ tonight, about medieval & modern views of time, with a fine proof that the logical conclusions of Thomism is Spinozism. If God sees the world spread out before Him in a timeless present, then the world is spread out in a timeless present, & is an eternal object & so can be no more be God's creation than the number 4 can. That was roughly it. - All this arose in the first place, of course, out of my time-calculus, & its {2} relation & contrast with the sort of time-calculus in which you translate

(A) 'It has been the case that Socrates is sitting down'
into

(B) 'For some time t , t is before t_0 and Socrates is sitting down at t ',

where t_0 is some impersonal dating of the time at which the statement happens to my mind. (B) is physicist's talk; & it is also, on the Thomist view, how God sees the fact of Socrates having sat down. And oddly, (A) is how the fact in question appears in medieval logic, though (B) is how it appears in medieval theology. I suspect that those boys were never

68 Editors' note: This letter has been edited by Andreas M. Müller and Martin Prior. It is part of the Martin Prior Collection, presently kept at Aalborg University folder E, item 44.

69 Editors' note: John Hansen, family doctor. He and his family had a flat in the same building as the Priors before the fire in 1949.

70 Stage III is the third year of a Bachelor degree in a particular subject, e.g. Philosophy.

really very far from the 2-truth theory. Look now, I'm keeping myself up!
& with dull professional stuff too. I love you, darling, & love you & love
- goodnight!

- Arthur

Xoxoxoxox

Chapter 3.

“The Syntax of Time-Distinctions”

by

A.N. Prior⁷¹

1. Truth and Time in Ancient and Modern Logic

Truth, on the face of it, is a property of propositions which is liable to alter with the time at which they are put forward. Thus 'Socrates is sitting down' is true at any time at which he is in fact sitting down, and false at all other times. Against this, it is not uncommonly argued that the sentence 'Socrates is sitting down' does not express a complete proposition, but rather a function of a date. It is short for 'Socrates is sitting down at -,' where the blank is understood as being filled by some unambiguous indication of the date at which the sentence is uttered ('4 P. M. on April 3, 325 B. C.,' for example). It therefore expresses different propositions when uttered at different times, and each one of the propositions it expresses is either true always or false always.

Modern exact logicians commonly operate with 'propositions' in the second (timelessly true) sense, while ancient and medieval logicians had in mind 'propositions' of the first ('tensed') sort. It should be emphasised, however, that there are no grounds of a purely logical character for the current preference, and that 'propositions' in the ancient and medieval sense lend themselves as readily to the application of contemporary logical techniques and procedures as do 'propositions' in the modern sense. (At this point Strawson, who regards it as a limitation of modern methods that they cannot cope with 'propositions' in the ancient and medieval sense, and Quine,

⁷¹ Presidential Address given the 27th August, 1954, at the Second Philosophical Congress, Wellington 27th – 30th August, 1954, New Zealand section of the Australasian Association of Psychology and Philosophy. Published in *Franciscan Studies*, Vol. 18, No. 2 (JUNE 1958), pp. 105-120.

who objects to the use of such 'propositions' in logic because modern methods cannot handle them, would seem to be equally in error⁷²). Moreover, the actual application of these techniques and procedures to tensed propositions promises to yield results of considerable interest both logically and metalogically. This was dimly seen by C.S. Peirce, who 'never shared' the common opinion that time is an 'extra-logical' matter, though he thought, in 1903, that 'logic had not yet reached that state of development at which the introduction of temporal modifications of its forms would not result in great confusion.'⁷³ What the time was not ripe for in 1903, it may well be ripe for now, for in the intervening period we have acquired a vast fund of knowledge about the possible structures of modal systems, and (as the scholastic logicians knew⁷⁴) tense and mood are species of the same genus. We have also begun to learn how to handle a logic of three truth-values, and we shall find this to the point too.

Suppose we use the ordinary variables 'p', 'q', 'r', etc. for 'propositions' in the ancient and medieval rather than the modern sense, and employ the usual truth-operators in the following way (admitting in the meantime only two truth-values): -

- 'Np' [$\sim p$]⁷⁵ ('Not p') is true at any time at which 'p' is false, and false at all other times.
- 'Kpq' [$p \wedge q$] ('Both p and q') is true at any time at which both 'p' and 'q' are true, and false at all other times.
- 'Apq' [$p \vee q$] ('Either p or q') is false at any time at which both 'p' and 'q' are false, and true at all other times.
- 'Cpq' [$p \supset q$] ('If p then q') is false at any time at which 'p' is true and 'q' false, and true at all other times.
- 'Epq' [$p \equiv q$] ('If and only if p then q') is true at any time at which 'p' and 'q' have the same truth-value, and false at all other times.

The classical propositional calculus, with its symbols thus interpreted, will then hold in its entirety, unaltered. For in the formula 'Epp' [$p \equiv p$], for example, 'p' will be equivalent to 'p at x', where 'x' is the date of utterance, and the whole therefore to 'E (p at x)(p at x)' [(p at x) \equiv (p at x)],

72 For this dispute see W. V. Quine, 'Mr. Strawson on Logical Theory', *Mind*, October 1953, pp. 440-443.

73 C. S. Peirce, *Collected Papers*, 4. 523.

74 See E. A. Moody, *Truth and Consequence in Medieval Logic* (Amsterdam 1953), § 12.

75 Editors' note: Here and in the following we add the modern notation in [].

in which the arguments are propositions in the modern sense, substitutable for the variables in the 'Epp' [$p \equiv p$] of the propositional calculus as currently interpreted; and a similar proof will be available for all formulae of the calculus in its new interpretation.

When the new interpretation is employed, it becomes possible to enrich the calculus with a pair of non-truth-functional operators which cannot intelligibly be attached to the timelessly-true 'propositions' of the current interpretation. These are namely the tense-operators 'It has been the case that,' symbolised here by 'p', and 'It will be the case that,' symbolised here by 'F'. The functions formed by these operators are themselves propositions whose truth may vary with time. Thus 'Pp' ('It has been the case that p') will be false until it is the case that p for the first time (unless it has always been the case that p), and true thereafter; 'Fp' ('It will be the case that p') will be true until it has forever ceased to be the case p (unless it will never be the case), and false thereafter.

We may also introduce the following two abbreviations:

Df. H: $H = NPN [\sim P \sim]$

Df. G: $G = NFN [\sim F \sim]$

'Hp' may be read 'It has always been the case that p.' For 'It has been the case that p' means 'It has at some time been the case that p,' so that 'NPP' [$\sim Pp$] ('It has not been the case that p') means 'It has at no time been the case that p,' and 'NPNp' [$\sim P \sim p$] means 'It has at no time been the case that not p,' i.e. 'It has always been the case that p.' (A logic of only two truth-values, it may be recalled, is being used at this stage.) For similar reasons, 'Gp' may be read 'It will always be the case that p.'

With this symbolism, we shall begin by examining the formal features of what we may call the logic of futurity, i.e. the calculus obtainable by adding to the classical propositional calculus the operator 'F' and its derivative 'G' with suitable axioms and rules. We shall then discuss in succession the full P-F (past-future) calculus; the interpretation of this within a calculus employing 'propositions' in the modern sense, quantifiers binding 'date-variables' and the dyadic predicate 'later than'; and the modifications which would be imposed on these calculi by the admission of a third truth-value for certain future events and by certain results of modern physics.

2. The Logic of Futurity

In its general structure the logic of futurity bears an obvious resemblance to modal systems, the operator 'F' being analogous to 'M' ('It is possible that') and 'G' to 'L' ('It is necessary that'). To bring out this analogy, and also the precise point at which it fails, let us suppose that the 'Mp' of modal systems is interpreted as synonymous with our 'Fp', i. e. let us take it to mean 'It will be the case that p,' and let us see how the resulting system compares with the classical modal systems, e. g. the systems M, M' and M'' of von Wright.⁷⁶

The system M of von Wright adds to the classical assertoric propositional calculus the axioms

- B1. $CpMp$ $[p \supset Mp]$
 B2. $EMApqAMpMq$ $[M(p \vee q) \equiv (Mp \vee Mq)]$

the rules

- RB1. $E\alpha\beta \rightarrow E\alpha M\beta$ $[(\alpha \equiv \beta) \rightarrow (M\alpha \equiv M\beta)]$
 RB2. $\alpha \rightarrow L\alpha$

and the definition

- Df. L: $L = NMN$ $[L = \sim M\sim]$

If we interpret 'Mp' as 'It will be the case that p' (and 'Lp' in consequence as 'It will always be the case that p'), all these axioms and rules will be found to hold except B1. 'If it is the case that p then it will be the case that p' expresses no law, for it might be that p is now the case for the last time. What does still hold when 'Mp' is interpreted as suggested is the law $CLpMp$ $[Lp \supset Mp]$ (for if it will always be the case that p, then it will be the case that p), but whereas in the system M this is a theorem deducible from B1, in the present system it will require to be immediately laid down.

At this point our system closely resembles that which would be obtained if 'M' were interpreted as 'It is morally permissible that' (and 'L' in consequence as 'It is not morally permissible that not,' i.e. as 'It is obligatory that').⁷⁷ For just as 'It will be the case that p' does not follow from 'It is the case that p,' but does follow from 'It will always be the case that p,' so 'It is permissible that p be done' does not follow from 'p is done,' but does follow from 'It is obligatory that p be done.' There is, however, at least one significant formal difference between the futurity-system and

⁷⁶ G. H. von Wright, *An Essay in Modal Logic* (Amsterdam 1951), Appendix II.

⁷⁷ Ibid. Ch. V. See also A. N. Prior, *Formal Logic* (published by the Clarendon Press), III. i. 6.

the permissibility-system. It is not at all clear what the form 'It is permissible that it be permissible that p be done' would mean, but there is no such difficulty about 'It will be the case that it will be the case that p.' Moreover, it is clear enough that this form implies the simple 'It will be the case that p.' Hence if 'M' be interpreted as 'It will be the case that,' we may lay down the special axiom of von Wright's system M' (equivalent to Lewis's S4), namely

C1. CMMpMp [MMp \supset Mp]

Common notions on the subject of time suggest that we may also lay down the converse of this, CMpMMp [Mp \supset MMp]; for if it will be the case that p, then (at any time between now and the time when p is the case) it will be the case that it will be the case that p. This converse, however, cannot be deduced (as it may in ordinary modal systems) by substituting 'Mp' for 'p' in CpMp [p \supset Mp], since in the futurity system the latter does not occur.

The special axiom of von Wright's strongest system M'' (equivalent to Lewis's S5), namely

C2. CMNMpNMP, [M \sim Mp \supset \sim Mp]

cannot be affirmed in the futurity system, for if it will at some time be the case that it will not (i.e. will never) be the case that p, it does not follow that it is now already the case that it will never be the case that p. Its converse CNMpMNMp [\sim Mp \supset M \sim Mp] does hold, though once again it is not provable in the usual way by substitution in CpMp [p \supset Mp]. It need not be laid down as a special axiom, however, as it is provable from what we already have, as follows: -

CLpMp	[Lp \supset Mp]
CMMpMp	[MMp \supset Mp]
CCNpqCNqp	[(\sim p \supset q) \supset (\sim q \supset p)]
CCpqCNqNp	[(p \supset q) \supset (\sim q \supset \sim p)]
CCpqCCqrCpr.	[(p \supset q) \supset ((q \supset r) \supset (p \supset r))]
3 p/MNp, q/Mp X Df. L=C1-6.	
CNMpMNP	[\sim Mp \supset M \sim p]
4 p/MMp, q/Mp = C2-7.	
CNMpNMMp	[\sim Mp \supset \sim MMp]
5 p/NMp, q/NMMp, r/MNMP = C7-C6 p/Mp-8.	
CNMpMNMp.	[\sim Mp \supset M \sim Mp]

In sum, the system in which possibility is interpreted as futurity stands in between that in which it is interpreted as moral permissibility and von Wright's system M'. Von Wright's system M and the futurity system, while both are contained in M' and contain the permissibility system, are mutually independent. For CpMp [p ⊃ Mp] is provable in M but not in the futurity system, while CMMpMp [MMp ⊃ Mp] is provable in the futurity system but not in M. The above considerations suggest that the logic of futurity may be axiomatised by subjoining to the axioms and rules of the classical propositional calculus the following special axioms for 'F' and 'G':

- | | |
|----------------|------------------------|
| F1. CGpFp | [Gp ⊃ Fp] |
| F2. EFApqAFpFq | [F(p ∨ q) ≡ (Fp ∨ Fq)] |
| F3. CFFpFp | [FFp ⊃ Fp] |
| F4. CFpFFp, | [Fp ⊃ FFp] |

the rules

- | | |
|-----------------|-----------------------|
| RF: Eαβ → EFαFβ | [(α ≡ β) → (Fα ≡ Fβ)] |
| RG: α → G α | [α → Gα] |

and the definition

- | | |
|-----------------|-----------|
| Df. G.: G = NFN | [G = ~F~] |
|-----------------|-----------|

An equivalent axiomatisation would be that in which 'G' is taken as undefined, and we replace F2 above by

- | | |
|-------------------|------------------------|
| F(2). CGCpqCGpGq, | [G(p ⊃ q) ⊃ (Gp ⊃ Gq)] |
|-------------------|------------------------|

delete RG, and replace Df. G by

- | | |
|-----------------|-----------|
| Df. F.: F = NGN | [F = ~G~] |
|-----------------|-----------|

The equivalence of these two systems may be proved in the same way as Sobocinski proves the equivalence of von Wright's system M and the system which he calls T, which is obtained by modifying M in the same way as we have modified our first system for F. What is required is to prove F(2) and the equivalence corresponding to Df. F from the first basis, and conversely to prove F2, RG and the equivalence corresponding to Df. G from the second basis. Sobocinski's proofs⁷⁸ of the analogous metatheorems do not employ the axiom Br, CpMp [p ⊃ Mp], or, when starting from the system T, the analogous axiom CLpp [Lp ⊃ p] (each of these is used only to prove the other); these proofs may therefore be taken

78 B. Sobocinski, 'Note on a Modal System of Feys-von Wright,' *Journal of Computing Systems*, July 1953, pp. 173-4.

over for our purpose without any alteration beyond the systematic replacement of 'M' by 'F' and 'L' by 'G'.

The form 'Mp' might be introduced into the F-calculus not simply as a synonym for 'Fp' but as an abbreviation for 'ApFp' [$p \vee Fp$] ('It either is or will be the case that p'). This is in fact how 'possibly' was defined by the Megaric logician Diodorus. If we adopt this definition, we obtain a system which is not merely contained in but equivalent to von Wright's middle system M' (or to Lewis's equivalent system S4), or which at all events is at least as strong as this system without being as strong as the system M'' (or Lewis's S5). This I have proved elsewhere,⁷⁹ using F2, F3, RF, RG and Df. G. (Von Wright's B2, C1, RB1, RB2 and Df. L are proved from their F-analogues and the new definition of M, while his B1. CpMp [$p \supset Mp$], is proved for the new definition of M and CpApq [$p \supset (p \vee q)$]; his C2 is disproved by proving its false F-analogue from it). The system M'' or a system at least as strong, is obtained if 'Mp' is introduced as an abbreviation for 'AApPpFp' [$p \vee Pp \vee Fp$] ('It either is or has been or will be the case that p'). This point cannot be enlarged upon, however, until we have passed from the F-calculus to the full tense-calculus in F and P.

3. The PF Calculus

A calculus of pure pastness would have exactly the same structure as the calculus of pure futurity; we could axiomatise it by simply taking the axioms, rules and definition of the latter calculus and replacing 'F' throughout by 'P' and 'G' throughout by 'H' ('It has always been the case that'). A complete tense-calculus in P and F would require more, however, than the simple adjunction of this P-calculus to the F-calculus. For it would need to contain also those laws which relate to the interaction of pastness and futurity, e. g. the pair

PF1, CpGPp [$p \supset GPp$]

('What is the case will always have been the case') and

PF2, CpHFp [$p \supset HFp$]

('When anything is the case, it has always been the case that it will be the case').

79 A. N. Prior, 'Diodoran Modalities' (forthcoming, in the *Philosophical Quarterly*), Section I.

PF1 and PF2, it may be noted, are obtainable from one another by the systematic replacement of 'P' by 'F' and 'F' by 'P' (written in full, PF1 is of course $CpNFNPp$ [$p \supset \sim F\sim Pp$], and PF2, $CpNPNFp$ [$p \supset \sim P\sim Fp$]). If we lay it down (as it seems we may) that if α is any law or rule of the PF calculus, we may obtain another law or rule by thus systematically interchanging 'P' and 'F' in α , we may cut down our other axioms and rules for this calculus by half.

A very wide range of laws is obtainable if we simply add PF1, and the rule just suggested (call it RA, the Rule of Analogy), to the basis already given for the F-calculus. Among the theorems provable is, for example,

$$CAApPpFpFPp, \quad [(p \vee Pp \vee Fp) \supset FPp]$$

'Whatever is or has been or will be the case, will have been the case.'⁸⁰ Its proof is as follows (using our *second* basis for the F-calculus): -

F1 ⁸¹ .	CGpFp	[Gp \supset Fp]
F(2).	CGCpqCGpGq	[G(p \supset q) \supset (Gp \supset Gq)]
F3.	CFFpFp	[FFp \supset Fp]
PF1.	CpGPp	[p \supset GPp]
1.	CCpqCNpNp	[(p \supset q) \supset (\sim q \supset \sim p)]
2.	CCpqCCqrCpr	[(p \supset q) \supset ((q \supset r) \supset (p \supset r))]
3.	CCpsCCqsCCrsCAApqrs	[(p \supset s) \supset ((q \supset s) \supset ((r \supset s) \supset ((p \vee q \vee r) \supset s)))]
	1 X RG = 4.	From 1 and RG:
4.	GCCpqCNqNp	[G((p \supset q) \supset (\sim q \supset \sim p))]
	F(2) p/Cpq, q/CNqNp = C4-5.	From 4 and F(2):
5.	CGCpqGCNqNp	[G(p \supset q) \supset G(\sim q \supset \sim p)]
	2 p/GCpq, q/GCNqNp, r/CGNqGNp = C5-CF(2) p/Nq, q/Np-6.	From 2, 5 and F(2):

80 This proposition is mentioned by Professor J. N. Findlay as one which a tense-calculus might contain, in a footnote to his 'Time: a Treatment of Some Puzzles,' in A. G. N. Flew's *Logic and Language* (first series, 1951), p. 52.

81 Editors' note: The original has 'CGppFp' which is obviously a typo.

6	CGCpqCGNqGNp 2 p/GCpq, q/CGNqGNp, r/CNGNpNGNq = C6--C1 p/GNq, r/GNp-7.	[G(p \supset q) \supset (G \sim q \supset G \sim p)] From 2, 6 and 1:
7.	CGCpqCNGNpNGNq 7 X Df. F = 8.	[G(p \supset q) \supset (\sim G \sim p \supset \sim G \sim q)] From 7 and Df. F:
8.	CGCpqCFpFq F3 X RA X RG = 9.	[G(p \supset q) \supset (Fp \supset Fq)] From F3, RA and RG:
9.	GCPPpPp 8 p/PPp, q/Pp = C9-10.	[G(PPp \supset Pp)] From 8 and 9:
10.	CFPPpFPP 2 q/GPp, r/FPP = CPF1-CF1 p/Pp-11.	[FPPp \supset FPP] From PF1 and F1:
11.	CpFPP 2 p/Pp, q/FPPp, r/FPP = C11 p/Pp-C10-12.	[p \supset FPP] From 10, 2, 11:
12.	CPpFPP 11 X RG = 13.	[Pp \supset FPP] From 11 and RG:
13.	GCpFPP 8 q/FPP = C13-14.	[G(p \supset FPP)] From 13 and 8:
14.	CFpFFPp 2 p/Fp, q/FFPp, r/FPP-C11-CF3, p/Pp-15.	[Fp \supset FFPp] From 14, 11 and F3:
15.	CFpFPP 3 q/Pp, r/Fp, s/FFPp = C11-C12-C15-16.	[Fp \supset FPP] From 3, 11, 12 and 15:
16.	CAApPpFpFPP	[(p \vee Pp \vee Fp) \supset FPP]

It cannot be claimed, however, that the F-calculus supplemented by PF1 and RA is sufficient to prove all laws about pastness and futurity. We shall shortly find reason for believing, for example, that although the thesis CMNMpNMp [M \sim Mp \supset \sim Mp] (the C2 of von Wright's system M") is clearly a law when 'Mp' is introduced into the PF calculus as an abbreviation for 'AApPpFp [p \vee Pp \vee Fp],' it is not provable from the basis suggested. But how our system may be best completed, and how completeness is to be tested in this field, is a matter that still awaits investigation.

4. The I-Calculus, and the Interpretation of the PF-Calculus within it.

The PF-calculus is interpretable within a calculus having as its elements

- a) the variables 'x', 'y', 'z', etc., standing for *dates*;
- b) the two quantifiers ' Πx ' and ' Σx ' (binding date-variables) and the usual truth-functions 'N', 'C', 'K', etc.;
- c) the variables 'p', 'q', 'r', etc., standing for 'propositions' in the ancient and medieval sense, but now considered as functions of dates, forming with the date-variables the timelessly true propositions 'px', 'qy', etc. (read 'p at x', 'q at y', and so on); and
- d) the dyadic function 'l', taking dates as arguments, and read 'is later than' (though no difference whatever would be made to the structure of the calculus if it were read 'is earlier than').

We may call this the 'l-calculus', after its distinctive operator.

In interpreting the PF-calculus within the l-calculus, we may use any arbitrarily chosen date-variable, say 'z', to represent the date at which the proposition under consideration is uttered. We then interpret

- 'p', 'q', 'r', etc., without tense-operators, as 'pz', 'qz', 'rz', etc.;
- 'Fp' ('It will be the case that p') as ' $\Sigma xKlxzpx$ ' [$\Sigma x(lxz \wedge px)$] ('For some x, x is later than z, and p at x,' or 'p at some time later than z');
- 'Pp' ('It has been the case that p') as ' $\Sigma xklzpx$ ' [$\Sigma x(lzx \wedge px)$] ('For some x, z is later than x, and p at x,' or 'p at some time earlier than z');
- 'Gp' as ' $\Pi xClzpx$ ' [$\Pi x(lxz \supset px)$] ('For all x, if x is later than z then p at x,' or 'p at all times later than z');
- 'Hp' as ' $\Pi xClzpx$ ' [$\Pi x(lzx \supset px)$] ('p at all times earlier than z').

And the laws of the PF-calculus will be not only interpretable but provable in the l-calculus if the latter contains

- (i) the usual laws and rules for truth-operators and quantifiers;
- and

- (ii) a set of special axioms expressing the properties of 'I', e. g.
 $ClxyClzlxz [lxy \supset (lyz \supset lxz)]$ (the law of transitivity for 'I'),
 $ClxyNlyx [lxy \supset \sim lyx]$ (the law of asymmetry for 'I'),
 $AAixylxylyx [ixy \vee lxy \vee lyx]$ ('Either the date x is identical with the date y or it is later than y or it is earlier' - the law of trichotomy for dates.

When we consider those laws of the PF-calculus which have so far come to our notice, the formulae by which they are interpreted in the I-calculus fall into two distinct groups. Those of the first group are provable by means of the ordinary logic of truth-operators and quantifiers alone. This group includes F2, F(2), PF1 and PF2, and also RF and RG. For example, PF1 ($CpGPp [p \supset GPp]$) becomes on interpretation

$$Cpz \Pi x Clxz \Sigma y Klxypy \quad [pz \supset \Pi x (lxz \supset \Sigma y (lxy \wedge py))],$$

('If p at z, then if x any time later than z, there is a time than which x is later, at which p'; e. g. if Socrates is sitting down at z, then if x is any time later than z, there is a time than which x is later, at which Socrates is sitting down'). This is provable as follows (using Łukasiewicz's rules for quantifiers, and 'a', 'b' and 'c' as variables standing for propositions in the timelessly-true sense): -

1. $CKabKba \quad [(a \wedge b) \supset (b \wedge a)]$
2. $CCKabcCaCbc \quad [((a \wedge b) \supset c) \supset (a \supset (b \supset c))]$
 $1 \ a/py, b/lxy \ X \Sigma 2y = 3.$
3. $CKpylxy \Sigma y Klxypy \quad [(py \wedge lxy) \supset \Sigma y (lxy \wedge py)]$
 $2 \ a/pz, b/lxz, c/\Sigma y Klxypy$
 $= C3 \ y/z-4.$
4. $Cpz Clxz \Sigma y Klxypy \quad [pz \supset (lxz \supset \Sigma y (lxy \wedge py))]$
 $4 \ X \ \Pi zx = 5.$
5. $Cpz \Pi x Clxz \Sigma y Klxypy. \quad [pz \supset \Pi x (lxz \supset \Sigma y (lxy \wedge py))]$

PF2 ($CpHFp [p \supset HFp]$) becomes on interpretation

$$Cpz \Pi x Clzx \Sigma y Klyxpy [pz \supset \Pi x (lxz \supset \Sigma y (lyx \wedge py))],$$

which is of course provable in exactly the same way. Both alike hold in virtue of the principle that guarantees, for example, that if Susan is a hairdresser, then whoever loves Susan loves someone that is a hairdresser.

The other group require for their proof one or more of the special axioms for 'I'. This group includes even F1, CGpFp [Gp ⊃ Fp], which becomes on interpretation

$$\text{C}\Pi\text{xClxzpx } \Sigma\text{xKlxzpx}^{82} \quad [\Pi\text{x}(lxz \supset px) \supset \Sigma\text{x}(lxz \wedge px)],$$

though all that this requires for its proof, over and above the laws of truth-functions and quantification, is the law Σxlxz^{83} , asserting that there is a date later than any given date. The dependence of F1 upon the infinite extent of the future is not perhaps immediately evident; but the dependence of its analogue CHpPp [Hp ⊃ Pp], i.e. CNPNpPp [$\sim P \sim p \supset Pp$], upon the infinite extent of the past, is evident enough. For if time had a beginning, then if at the beginning of time it has not been the case that not p, it has not yet been the case that p either. And similarly if time has an end, if at the end of time it will not be the case that not p, it will not be the case that p either. Of our other axioms F3, CFFpFp [FFp ⊃ Fp], requires for the proof of its l-interpretation the law of transitivity ClxyClyzlxz; and F4, CFpFFp [Fp ⊃ FFp], requires the law

$$\text{Clxz}\Sigma\text{yKlxlyz} \quad [lxz \supset \Sigma\text{y}(lxy \wedge lyz)],$$

asserting that between any two dates there is another date. The law that if it is or has been or will be the case that it neither is, nor has been nor will be the case that p, then it neither is, has been nor will be the case that p (CMNMpNMp [M \sim Mp ⊃ \sim Mp], where Mp = AApPpFp [p ∨ Pp ∨ Fp]), would seem to require for its proof the law of trichotomy. To see that this is so, let us consider our reason for assenting to the l-interpretation of one part of this law, namely that if it ever *has been* the case that p neither is, has been, nor will be the case, then it *is now* the case that p neither is, has been, nor will be the case. In terms of the l-calculus this means

'If for any time x, earlier than z, p at no time earlier than or later than or identical with x, then p at no time earlier than or later than or identical with z.'

And we believe this because we believe that 'p at no time earlier than or later than or identical with x', no matter what time x may be, amounts to 'p at no time at all'; and this is just the negative side of the law of trichotomy. Since the law of trichotomy is independent of the

82 Editors' note: The original paper has CΠxCΠlxzpx ΣxKlxzpx, which must be a typo.

83 Editors' note: The original paper has xlxz, which must be a typo.

laws which we have seen here to be required for F1, (F2), F3, F4, PF1 and RG, and is not derivable from them by any 1-equivalent of the Rule of Analogy, we seem to have here a proof that CMNMpNMP $[M \sim Mp \supset \sim Mp]$ is independent of F1, F(2), etc. This is in any case one *kind* of method by which independence in the PF-calculus might be established.

The interpretation of the PF-calculus within the l-calculus is clearly a device of considerable metalogical utility. But is it more than that? Can we turn it into an 'interpretation' in the sense of a metaphysical explanation of what we mean by 'is', 'has been' and 'will be'? If so, we would have to regard our symbols 'P' and 'F' as not being genuine propositional operators but as artificially constructed quasi-propositional operators, very much as the class-symbols of the Boole-Schröder algebra of classes are treated in *Principia Mathematica* not as genuine names of objects but as artificially constructed quasi-names. Moreover, if the l-calculus is in this sense metaphysically fundamental, we would, I think, have to agree substantially with those logicians of whom Peirce wrote, who considered time to be an 'extra-logical' matter. For the l-calculus, as we have seen, consists essentially of the ordinary logic of truth-operators and quantifiers, with special axioms concerned with the relation 'I' - which could be, so far as logic is concerned, any 'material' relation ordering objects in an infinite and continuous linear series - superimposed upon it.

There are strong reasons, however, for refusing to attach this metaphysical significance to the interpretability of the PF-calculus in the l-calculus. As an 'interpretation' in the metaphysical sense of the 'now' which is understood in all the 'propositions' with which the PF-calculus is designed to deal ('Socrates is sitting down' means 'Socrates is sitting down *now*,' 'Socrates will be sitting down' means 'It is *now* the case that it will be the case that Socrates is sitting down,' and so on), the 'z' which we have used in the l-calculus is surely a complete sham. For 'now' is not the name of a date (it has the same meaning whenever it is used, but does not refer to the same date whenever it is used). In fact the whole movement of events from the future through the present into the past is inexpressible in the l-calculus. If there is to be any 'interpretation' of our calculi in the metaphysical sense, it will probably need to be the other way round; that is, the l-calculus should be exhibited as a logical construction out of the PF-calculus

rather than *vice versa*. How this could be achieved in detail has yet to be investigated, but as a first step we may point out that 'The date of p's occurrence is later than the date of q's occurrence' seems to be equivalent to 'It either is or has been or will be the case that it both is the case that p and is not but has been the case that q'

$$(AAKpKNqPqPKpKNqPqFKpKNqPq \\ [(p \wedge \sim q \wedge Pq) \vee P(p \wedge \sim q \wedge Pq) \vee F(p \wedge \sim q \wedge Pq)]).$$

5. The l-calculus and the Three-valued PF-Calculus

To what has just been said we may add that even on the purely formal side the interpretability of the PF-calculus within the l-calculus can only be asserted with a qualification. The PF-calculus which was sketched in Sections 2 and 3 can only be asserted in its entirety if only two truth-values are admitted. If we assign a 'neuter' truth-value to propositions in the future tense about matters whose outcome is undetermined at the time of utterance, the PF calculus will need to undergo radical revision; and it is by no means certain that the calculus when thus amended will be interpretable even in an amended l-calculus. (We shall certainly not, if it is, be able to employ exactly the same interpretations as before.)

The most striking difference between the two-valued and the three-valued PF calculi is that in the latter the Rule of Analogy fails. For example, while it continues to be a law that what is the case will always have been the case, it is no longer (on the three-valued hypothesis) a law that what is the case has always been going to be the case; that is, we now have PF1, CpGPp [p \supset GPp], but not its analogue PF2, CpHFp [p \supset HFp]. (We are assuming throughout that expressions beginning with 'P' or 'H' cannot take the third truth-value, that 'Pp' is true if 'p' has at some time been true, and false if 'p' has at no time been anything but false or indeterminate, and that 'Hp' is true if 'p' has always been true, and false if 'p' has at any time been false or indeterminate.)

But it is quite impossible to discriminate in this way between the l-interpretations of these two laws. For these l-interpretations have exactly the same structure, and are proved in exactly the same way; their proof, moreover, involves no special assumptions about the character of the relation 'I', but only the rules for quantifiers and two laws of the propositional calculus, CKabKba [(a \wedge b) \supset (b \wedge a)] and

CCKabcCaCbc $[(a \wedge b) \supset c) \supset (a \supset (b \supset c))]$. Nor does their proof even depend on the fact that the I-calculus, as presented in the last section, uses the propositional calculus in its two-valued form. For CKabKba and CCKabcCaCbc⁸⁴ are laws in Łukasiewicz's three-valued calculus also (though the converse of the second is not. This is one of a number of points at which three-valued logic and the calculus of strict implication stand as it were on opposite sides of the two-valued assertoric calculus.⁸⁵ In this last we have both a 'law of exportation' CCKabcCaCbc and a law of importation CCaCbcCKabc $[(a \supset (b \supset c)) \supset ((a \wedge b) \supset c)]$; but where 'C' is interpreted as strict implication we have the latter but not the former, and where it is the 'if' of Łukasiewicz's three-valued calculus we have the former but not the latter). It seems obvious, indeed, that the I-calculus is deterministic in its whole conception - time is represented in it as spread out once for all, with no ever-moving 'now' but only a series of 'dates' timelessly characterised in various ways (Socrates eternally sitting down at x but not sitting down at y, and so on). There can really be no 'neuter' truth-value for the only kind of 'proposition' which this calculus will admit. Time, one might say, figures in the I-calculus not as it does in medieval logic (which, as we have pointed out earlier, took tenses far more seriously than our own common logic does, and which already had such laws as our PF1,⁸⁶ but rather as it does in medieval theology, in which God is said to behold all events in an unchanging present.

The introduction of the third truth-value into tense-logic also destroys the equivalence which makes it reasonable to introduce the form 'It has always been the case that p' as an abbreviation for 'It has not been the case that not p.' Formally it is of course still possible to do this, but in its natural acceptance the form 'It has always been the case that p' is not equivalent to 'It has not been the case that not p' if we admit the third truth-value; for if 'p' has always been neuter the latter is true but the former is not. It may be noted that it is only in its natural acceptance that CpHFp $[p \supset HFp]$ is not a law on the three-valued assumption, if it is merely short for CpNPNFp

84 Editors' note: The original has 'CCKabcCbc' here which is probably a typo.

85 Cf. A. N. Prior, 'Three-valued Logic and Future Contingents, *Philosophical Quarterly*, Oct. 1953, p. 321.

86 See, e. g. William of Ockham, *Tractatus de Praedestinatione*, Franciscan Institute edition (1945), p. 4.

$[p \supset \sim P \sim Fp]$, 'If it is the case that p , it has not been the case that it will not be the case that p , 'it holds even in the three valued system. On the other hand, the law $CHpPp [Hp \supset Pp]$, 'If it has always been the case that p , then it has been the case that p , 'holds in the three-valued system if its natural interpretation is assumed, but not if it is taken to be short for $CNPNpPp [\sim P \sim p \supset Pp]$, 'If it has not been the case that not p , it has been the case that p .' But $F1, CGpFp [Gp \supset Fp]$, holds on both interpretations, and so does $PF1, CpGPp [p \supset GPp]$. In its I -interpretation, the equivalence of H and $NPN [\sim P \sim]$ amounts to the equivalence in quantification theory of Πx and $N\Sigma xN [\sim \Sigma x \sim]$, and so does that of G and $NFN [\sim F \sim]$. This equivalence - of Πx and $N\Sigma xN [\sim \Sigma x \sim]$ - is one which still holds if the ordinary rules for Π and Σ are superimposed on Łukasiewicz's three-valued propositional calculus, though it fails if they are superimposed on Heyting's intuitionist calculus.⁸⁷ But even if we operate in the I -calculus with intuitionist quantification, we cannot obtain in it (at least by the interpretations suggested in the last section) a model of the three-valued PF calculus; for while we could then destroy (as we want to) the equivalence of the interpretations of H and $NPN[\sim P \sim]$, we would not preserve (as we also want to) the equivalence of the interpretations of G and $NFN [\sim F \sim]$. This confirms our contention in the last paragraph that the whole conception of time underlying the I -calculus is different from that underlying the three-valued PF calculus.

Another difference between two-valued and three-valued tense-logic is that whereas $F(2), CGCpqCGpGq [G(p \supset q) \supset (Gp \supset Gq)]$, is a law in both, $F2, EFApqAFpFq [F(p \vee q) \equiv (Fp \vee Fq)]$, is a law in the former but not in the latter (though the P -analogues of both hold in both systems). For suppose we have a situation of *limited* indeterminacy (the sort of situation which seems often to occur in atomic physics), in which what will happen must be either p or q , but which of them it will be is not fixed. We will then have $FApq [F(p \vee q)]$ but not $AFpFq [Fp \vee Fq]$. It is true that - as we have mentioned earlier - Sobocinski has proved the M -analogue of $F2$ from that of $F(2)$; but this proof assumes the law of importation $CCpCqrCKpqr$

87 Cf. A. Heyting, 'On Weakened Quantification,' *Journal of Symbolic Logic*, Vol. XI (1946), p. 119 ff.

$[(p \supset (q \supset r)) \supset ((p \wedge q) \supset r)]$, which does not hold in the three-valued calculus.

So far as I can see, the admission of the third truth-value does not affect the axioms F3 and F4, the rules RF and RG, or the P-analogues of these (CPPpPp [PPp \supset Pp], etc.). A query that might be raised about F3, CFFpFp [FFp \supset Hp], is this: - A proposition of the form 'It will be the case that p' is liable (on the three-valued assumption) to be indeterminate at first and then to become true (as the element of freedom is eliminated by later choices and the outcome of the matter becomes inevitable); and might we not describe this position by saying that 'It will be the case that it will be the case that p' is now true though 'It will be the case that p' is as yet only neuter? This will not do, however. If it is really undecided whether it will be the case that p, then it is undecided whether it will be the case that it will be the case that p. (It may be decided *that* it will be decided, but it cannot be decided *how* it will be decided.)

Modern quantum mechanics and relativity theory would no doubt suggest yet other modifications of our ordinary logic of time-distinctions.⁸⁸ For example, according to some versions of quantum mechanics time is discontinuous, and if this is so we must drop our F4, CFpFFp [Fp \supset FFp]; for it may be that it will be the case that p after the minimum time-interval, so that there is no time future to now but past to the being-the case of p at which it 'will be the case that it will be the case' that p. The formal consequences of abandoning F4 are in case worth studying. (For ordinary purposes they turn out to be very slight; none of the theorems which seemed worth establishing in Sections 2 and 3 required F4 for their proof). The most obvious effect of the theory of relativity is on the laws of asymmetry and trichotomy in the I-calculus. Relativity theory distinguishes between an absolute and a relative sense of 'later'; and if 'lx y' means 'x is absolutely later than y,' the law of asymmetry holds (no time is at once absolutely later and absolutely earlier than the same time) but the law of trichotomy does not (time x may be neither absolutely earlier nor absolutely later than time y without being identical with

⁸⁸ In what little I have been able to say on this point, I am indebted to conversations with Mr. J. Gabriel of the University of Otago, and Mr. W. W. Sawyer of Canterbury College.

time y); whereas if ' lxy ' only means ' x is later than y from *some* point of view,' the reverse is the case.

The theory of relativity also raises a more profound and vexing question. The three-valued PF-calculus gives formal expression to the deep-seated ancient feeling that what is past is beyond our control (and so 'necessary') in a way that what is future is not; but it may well be doubted whether relativity theory is compatible with so radical a distinction between the past and the future as this. At least in many of its presentations, relativity theory seems to be as closely bound up with the 'spread-out-eternally' view of time underlying the l -calculus as medieval theology was. On this, one possible comment is that this may be simply a philosophical defect of which the theory of relativity will eventually have to rid itself, in the same way as the differential calculus eventually had to rid itself of the incoherences pointed out by Berkeley. Further, within relativity theory itself it has often been pointed out that the only events which occur in one time-order from one point of view and in the opposite time-order from another, are events which from both points of view are 'outside the future' in the sense of being incapable of being affected by what happens 'here and now' and 'outside the past' in the sense of being incapable of affecting what happens here and now; and this very way of putting the matter reflects something of that ancient feeling about past and future to which we have referred. The solution of such questions as these, however, must depend on future collaboration between mathematical logicians and mathematical physicists, or on the work of those who have become familiar with both fields.

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