Extended Abstract
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Computational Circular Design of Planar Linkage Mechanisms Using Available Standard Parts

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Introduction

As a potential solution to increasing pressure on the environment and natural resources, Circular Economy has emerged as an approach that promises to close material flows, thus reducing resource use, pollution, and waste. To date, public effort has been focused on low value retention strategies like recycling, however, higher value retention solutions are still needed to better achieve a truly circular economy (Potting et al., 2017). Remanufacturing and repurposing are two high value retention strategies that focus on the reuse of existing parts of a product to create another product with the same function or a different function, respectively. These two strategies are already being explored in the building sector, where the creation of a database of available parts enables the reuse of building components (Brütting et al., 2019; Law et al., 2024).

However, for products with moving parts, like machines, current research lacks systematic circular design methods. This paper aims to address this gap by focusing on kinematics, specifically on linkage mechanisms, which are ubiquitous in modern machines across industries such as automotive, agriculture, and manufacturing. Though numerous works in kinematic analysis and synthesis have been conducted (Bächer et al., 2015; Nobari et al., 2022, 2024), these methods cannot be transferred directly into a sustainability context as they assume an unconstrained supply of parts, both in terms of the number of parts available and freedom in their geometry.

This paper presents a new computational method for synthesizing planar linkage mechanisms based on available standard parts, represented by LEGO® Technic beams. First, an adapted design representation is introduced, along with a generative design method to create mechanisms and their coupler curves based on a given inventory. Next, an optimization problem is formulated to address the inverse design task. The goal is to find combinations of reused and new parts that produce coupler curves as similar as possible to a given target curve. The resulting problem is an integer, non-linear, non-differentiable (black-box), multi-objective minimization problem.

Results

Figure 1 illustrates the two distinct parts of this work. Both the generation and optimization are implemented in Python and run on a single core of an i7-1195G7 processor.

Modeling and Generation

Mechanisms are represented with a graph with two types of nodes. Parts nodes, or LEGO® Technic beams, can connect to connector nodes, or LEGO® Technic pins. The edges in between them carry the location of the connection on the beam as an attribute. Designs are generated by randomly picking available parts from the inventory and adding them two by two to the start graph consisting of a fixed part and an actuator.

The kinematics analysis follows the arc intersection method (Bächer et al., 2015; Nobari et al., 2022). The position and orientation of the pairs of added parts is solved in the same order as they were added. Trajectories are extracted, normalized, and stored. While most designs are discarded because they are kinematically invalid, a large database of achievable mechanisms and their resulting curves is generated. The novelty of the mechanism synthesis method presented here is that it only considers the use of available parts.

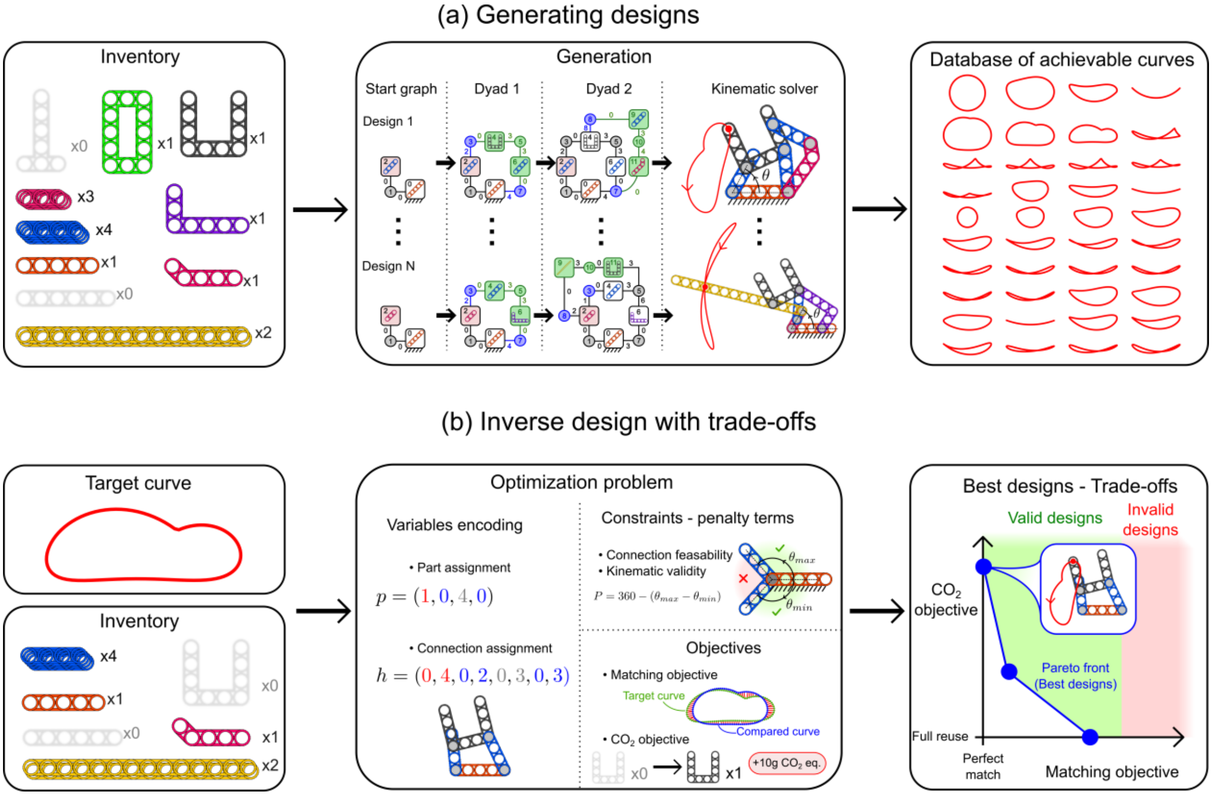

Figure 1: (a) Illustration of how designs are generated and saved to the database. (b) The inverse design method requires a target curve and a part inventory as an input. The optimization then finds a set of optimal designs based on a shape matching objective and a CO_2 objective.

Inverse Design and Trade-Offs

For the optimization problem, designs are encoded by two vectors of integers. The part assignment vector determines which part from the inventory is selected for each part node in the graph. The hole assignment vector lists the indices of the locations of all connections on their respective parts.

Constraints are modeled as penalty functions for simplified optimization. Two penalty functions ensure that the locations of all connections exist on the parts and are not redundant, in order to avoid kinematic singularities. A final penalty function penalizes designs where the actuator is not free to complete a full rotation.

The optimization model includes two objectives to minimize. The first objective, called the matching objective, calculates the degree to which the resulting curves match the target curve. The second objective, or CO_2 objective, calculates the greenhouse gas emissions caused by the fabrication of a new part, which is not available in the inventory.

An analysis of the resulting design space reveals a complex structure with a multitude of sparse local minima, most of which have significantly higher objective values than the known global minima. A genetic algorithm and a pattern search using NOMAD 4 (Audet et al., 2022) are employed to solve the defined problem.

Compared to a naive random search, both optimizers demonstrate little to no advantage in terms of convergence to designs with good matches to the target curve.

The feasible solution space and pareto front confirm the expected trade-offs between kinematic performance and environmental impact. When specific parts are missing from the inventory, designs benefit more from the addition of new parts, albeit at the expense of a CO_2 impact.

Conclusion

This work introduces a computational method to explore and select feasible designs with a given set of available standard parts represented by LEGO® Technic parts. A design representation is defined and used to generate achievable mechanisms. Black-box optimizers are used to solve the inverse design problem. Finally, this paper explores the tradeoffs between the kinematic performance and the embodied CO2.

Developing a circular design method for mechanism synthesis remains challenging due to the computational cost, the combinatorial nature of the problem, the non-linearity of the relationships between components, and the complexity of enforcing the validity of designs. New representations and solution approaches could potentially offer a substantial gain in performance and enable scenarios of reuse for more complex and industry-motivated case studies. Additionally, a more thorough life cycle assessment and the embedding of circularity metrics would improve the relevance of the designs discovered.
In the broader context, this work represents an initial step in the future development of a more holistic computational circular design method that can assist design practitioners in creating new mechanical products with reused parts.

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References

Audet, C., Le Digabel, S., Montplaisir, V. R., & Tribes, C. (2022). Algorithm 1027: NOMAD version 4: Nonlinear optimization with the MADS algorithm. *ACM Transactions on Mathematical Software (TOMS)*, *48*(3), 1–22. https://doi.org/10.48550/arXiv.2104.11627

Bächer, M., Coros, S., & Thomaszewski, B. (2015). LinkEdit: Interactive linkage editing using symbolic kinematics. *ACM Transactions on Graphics*, *34*(4), 99:1-99:8. https://doi.org/10.1145/2766985

Brütting, J., Senatore, G., & Fivet, C. (2019). Form Follows Availability – Designing Structures Through Reuse. *Journal of the International Association for Shell and Spatial Structures*, *60*(4), 257–265. https://doi.org/10.20898/j.iass.2019.202.033

Law, I., Chang, K. C., Önalan, B., Reisach, D., Griffioen, S., de Saussure, A., Dillenburger, B., & De Wolf, C. (2024). Web-Based Material Database for Circular Design. In P. Eversmann, C. Gengnagel, J. Lienhard, M. Ramsgaard Thomsen, & J. Wurm (Eds.), *Scalable Disruptors* (pp. 116–127). Springer Nature Switzerland. https://doi.org/10.1007/978-3-031-68275-9_10

Nobari, A. H., Srivastava, A., Gutfreund, D., & Ahmed, F. (2022). LINKS: A Dataset of a Hundred Million Planar Linkage Mechanisms for Data-Driven Kinematic Design. *Volume 3A: 48th Design Automation Conference (DAC)*, V03AT03A013. https://doi.org/10.1115/DETC2022-89798

Nobari, A. H., Srivastava, A., Gutfreund, D., Xu, K., & Ahmed, F. (2024). *LInK: Learning Joint Representations of Design and Performance Spaces through Contrastive Learning for Mechanism Synthesis* (No. arXiv:2405.20592). arXiv. https://doi.org/10.48550/arXiv.2405.20592

Potting, J., Hekkert, M. P., Worrell, E., & Hanemaaijer, A. (2017). Circular Economy: Measuring Innovation in the Product Chain. *Planbureau Voor de Leefomgeving*, *2544*. https://dspace.library.uu.nl/handle/1874/358310