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Decision Support Tools for Electricity Retailers, Wind Power and CHP Plants Using Probabilistic Forecasts

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ABSTRACT

This paper reviews a number of applications of optimization under uncertainty in energy markets resulting from the research project ENSYMORA. A general mathematical formulation applicable to problems of optimization under uncertainty in energy markets is presented. This formulation can be effortlessly adapted to describe different approaches: the deterministic one (usable within a rolling horizon scheme), stochastic programming and robust optimization. The different features of this mathematical formulation are duly interpreted with a view to the energy applications reviewed in this paper: trading for a price-maker wind power producer, management of heat and power systems, operation for retailers in a dynamic-price market. A selection of results shows the viability and appropriateness of the presented stochastic optimization approaches for managing energy systems under uncertainty.

Keywords:

stochastic programming;
robust optimization;
probabilistic forecasting;
energy market;
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1. Introduction

Energy systems worldwide have experienced a great degree of change due to the unprecedented deployment of renewables over the last years. In particular, the Danish energy system has almost become a symbol for this pursuit of sustainability as the penetration of wind power has been pushed beyond the record level of 30% in 2013 [5]. However, sustainability does not come free of challenges.

Renewable sources such as wind and solar are characterized by two features that distinguish them from conventional sources of electricity: they are *intermittent* and only *partly predictable*. At a power system level, the intermittency of these sources implies that other units in the system must be able to ramp up their production to meet the load at periods when wind and/or solar power are not available, or to ramp down when wind or solar irradiation pick up. Similarly, power

production must be planned in advance so that sufficient flexible units are on in order to be able to cope with the partly unpredictable future trajectory of wind and solar power production.

Among the solutions proposed to alleviate the challenges introduced by renewables in power systems, the integration of the latter with the heat system is a low hanging fruit that is believed to have great potential in Denmark [12]. As roughly 60% of the Danish houses is connected to urban district heating networks, the heat system can add a significant level of flexibility if managed smartly with the power system. For example, electricity-fueled units (heat pumps, electric immersion boilers) can turn wind power in excess of the demand of electricity into heat that eventually can be stored into hot-water tanks for future use. Similarly, the larger inertia of the heat system can be employed to counterbalance the fluctuations of renewable

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production, thereby providing balancing services to the electricity system.

To a large extent, the trade of heat and electricity takes place in short-term markets arranged in the time span between the day preceding the delivery of the commodity and real-time. For example, 70% of the electricity trade in the Nordic region takes place in the day-ahead market, Elspot, which closes at noon on the day before delivery [15]. Similarly, the bulk of heat production in the Copenhagen area, i.e., 34 500 TJ per year, corresponding to 20% of the total district heating load in Denmark, is settled on a day-ahead basis [17]. At the time of closure of these markets, producers must have made a decision on their trades, while market operators must run market clearing procedures to determine the dispatch of the units as well as the prices. Obviously, these decisions must be based on the information available at the day-ahead stage, which for uncertain parameters like wind power production or heat demand is a forecast of their future evolution.

The aforementioned trends in future energy systems, i.e., increasing uncertainty due to renewables and higher level of integration across systems and markets for different commodities, have implications for the decision-making problems that both utilities and market operators have to face. Firstly, decision-making models have to span *across* different commodities rather than consider them separately. Only in this way can the benefits of energy systems integration be reaped. Secondly, these optimization models should account for the stochasticity affecting the decisions to be made, since larger shares of renewables introduce greater degrees of uncertainty in the energy system.

The research project ENSYMORA produced a number of contributions to the state-of-the-art in decision support models for the energy industry, particularly tailored to the case of Denmark. This paper discusses and summarizes five of them. The first contribution is a model to optimize the trading strategy for a large wind power producer that, owing to its size, impacts the market as a price-maker [18]. The second contribution aims to support the day-ahead trading and dispatch processes of utilities managing Combined Heat & Power (CHP) plants [20]. Related to this topic is [14], where a day-ahead scheduling model is used for evaluating the economic value of heat pumps and electric boilers in the Danish energy system. Furthermore, [8] studies the possibility of operating CHP units and wind farms as a portfolio to reduce their

joint balancing cost. Finally, [19] considers the market strategy for a retailer operating in a real-time pricing environment.

The red thread behind these contributions is the increasing role of optimization under uncertainty due to the real-time uncertainty in forecasts of wind power generation, prices, load, etc., in the management of sustainable energy systems. In particular, three methods to deal with uncertainties in optimization are employed. In [8], we consider the use of a deterministic optimization model within a rolling-horizon framework. Furthermore, stochastic programming [3] is used in [18, 14, 19]. Finally, [20] employs robust optimization [1].

In Section 2 of this paper, we introduce the general formulation of a problem of optimization under uncertainty and describe how it can be tackled with a deterministic model in a rolling-horizon framework, using stochastic programming or robust optimization. The various types of forecasts needed as input for each of the aforementioned frameworks are described in Section 3. Then, Section 4 discusses the results obtained in some applications of these techniques to energy markets. Finally, conclusions are drawn in Section 5.

2. Mathematical Framework for Economic Problems in Energy

The classical model for optimization under uncertainty in energy markets can be written as

$$\text{Min.}_{\mathbf{x}, \mathbf{y}_\omega} \mathbf{c}^\top \mathbf{x} + \mathcal{M}_\omega \left\{ \mathbf{q}_\omega^\top \mathbf{y}_\omega \right\} \quad (1a)$$

$$\text{s.t. } \mathbf{A}\mathbf{x} \geq \mathbf{b}, \quad (1b)$$

$$\mathbf{T}\mathbf{x} + \mathbf{W}\mathbf{y}_\omega \geq \mathbf{h}_\omega, \quad \forall \omega \in \Omega. \quad (1c)$$

The subscript ω indicates that the linear cost coefficient \mathbf{q} in Eq. (1a) and the right-hand-side \mathbf{h} in Eq. (1c) are stochastic, i.e., functions of the realization of a random variable $\omega \in \Omega$. We assume in the following that an appropriate probability space (Ω, \mathcal{F}, P) is defined.

In optimization problems within the energy market domain, the objective is often aligned with the minimization of cost (possibly minus a term representing revenues) subject to the fulfillment of balance constraints that enforce that supply and demand for a commodity be equal. With this in mind, the

stochasticity in the cost coefficient q_ω reflects the uncertainty in the future realization of market prices. Furthermore, the uncertainty in the future demand for a commodity (heat or power) or in production (e.g., from a wind farm) results in stochastic right-hand side h_ω for the balance constraints.

The variable vector \mathbf{x} indicates the so-called *first-stage* variables. Because of the time structure of the decision-making problem, the decision on the value of these variables is to be made in advance and, thus, in the face of uncertainty. Indeed, only a statistical description (forecast) of the probability distribution of the stochastic parameters q_ω and h_ω is available at this stage, but not their true realization. Typically, in energy problems this type of variables include day-ahead offers and decisions on the on/off status of slow units, which cannot be changed in real-time, or nominal values (pre-dispatch) for the power and/or heat output of units. On the contrary, decisions y_ω can be adjusted when the uncertainty in the problem unfolds. These variables are referred to as *recourse* variables. In energy-related problems, they typically represent the real-time redispatch of flexible units or the purchase or sale of electricity in the balancing market.

Under the definitions above, the product $q_\omega^\top y_\omega$ indicates the cost of recourse decisions. As this cost is a function of the uncertainty, ω , it is stochastic itself. It is up to the modeler to decide which operator $M_\omega\{\cdot\}$ related to the random variable ω is to be included in the objective function. Typical choices are the expectation or an appropriate risk measure [13].

The vector \mathbf{x} can be a collection $[\mathbf{x}_1^\top \dots \mathbf{x}_M^\top]^\top$ of M first-stage decision variables. As energy markets often require producers, retailers and operators to make day-ahead decisions for each hour of the following day, optimization problems typically span multiple time periods. This implies that each decision variable \mathbf{x}_m is itself a vector including a decision for each time period $[\mathbf{x}_{m1} \mathbf{x}_{m2} \dots \mathbf{x}_{mT}]^\top$. Similarly, let us assume there are N types of recourse decisions to be made for each of the \mathbf{T} time periods. According to these definitions, model (1) is well defined if:

$$\mathbf{c}, \mathbf{x} \in \mathbb{R}^{MT}, \mathbf{q}_\omega, \mathbf{y}_\omega : \Omega \rightarrow \mathbb{R}^{NT}, \quad (2a)$$

$$\mathbf{A} \in \mathbb{R}^{L_1 \times MT}, \mathbf{b} \in \mathbb{R}^{L_1}, \quad (2b)$$

$$\mathbf{T} \in \mathbb{R}^{L_2 \times MT}, \mathbf{W} \in \mathbb{R}^{L_2 \times NT}, h_\omega : \Omega \rightarrow \mathbb{R}^{L_2}. \quad (2c)$$

Unless further assumptions on the cardinality of Ω are made, Eq. (1c) might involve an infinite number of constraints. Similarly, the applications of the operator $M_\omega\{\cdot\}$ on the recourse cost in Eq. (1a) might involve an infinite number of function evaluations. In the remainder of this section, we review strategies for approximating the solution to this (otherwise intractable) problem and describe some of their applications to energy market problems.

2.1. Deterministic Optimization Within Rolling-Horizon Scheme

A deterministic solution to the problem of optimization under uncertainty (1) can be found by simply replacing the uncertain variables with a deterministic quantity related to the uncertainty, e.g., a point forecast. Finding the deterministic solution is perhaps the easiest, though roughest, approximation to a stochastic optimization problem. Typically, the conditional forecast expectation, see Section 3.1, is the chosen point prediction [4]:

$$\text{Min}_{\mathbf{x}, \hat{\mathbf{y}}} \mathbf{c}^\top \mathbf{x} + \mathbb{E}_\omega \left\{ \mathbf{q}_\omega^\top \right\} \hat{\mathbf{y}} \quad (3a)$$

$$\text{s.t. } \mathbf{A}\mathbf{x} \geq \mathbf{b}, \quad (3b)$$

$$\mathbf{T}\mathbf{x} + \mathbf{W}\hat{\mathbf{y}} \geq \mathbb{E}_\omega \left\{ \mathbf{h}_\omega \right\}. \quad (3c)$$

Note that the recourse variables lose their adaptive nature in this formulation, as y_ω is replaced by $\hat{\mathbf{y}} \in \mathbb{R}^{NT}$, which represents the response of the system when the realization of the uncertainty is equal to its deterministic counterpart (which in this case is the expectation).

As a trade-off for the simplicity of deterministic problem (3), there is in general no guaranteed bound on the degree of suboptimality introduced by using the deterministic solution instead of the “true” solution to (1). In fact, even the feasibility of the deterministic solution under all the realizations of the uncertainty with a non-negligible probability cannot be guaranteed. In energy markets, the risk of infeasibility may be not that problematic, as a deficit in electricity or heat production can be covered by a purchase in a real-time market, the start-up of an expensive backup unit or as, a last resort, the curtailment of load. However, these “emergency” decisions involve high costs (either monetary or in terms of corporate image) that quickly degrade the overall performance of the deterministic decision.

An approach to reduce the suboptimality of the deterministic decision consists in solving a sequence of deterministic problems (3) in a *rolling horizon* fashion. When the first problem in the sequence is solved, only the part of the solution corresponding to the first time period in the horizon, i.e., x_{m1} , \forall_m and \hat{y}_{n1} , \forall_n is implemented in practice. The horizon is then rolled one step forward by updating the variables x and y as well as the coefficients c , $\mathbb{E}_\omega \{q_\omega\}$ and $\mathbb{E}_\omega \{h_\omega\}$, before solving a new version of optimization problem (3). Note that rolling one step forward includes an update of the point forecasts used for the stochastic variables q_ω and h_ω .

The described rolling-horizon approach falls within the domain of deterministic Model Predictive Control [10]. It is better suited to problems of control of the output of a system in real-time than for market operation, as the former requires frequent updates (e.g., hourly) of the control strategy, while market offering or clearing problems are faced on a daily basis and do not allow for an update of the chosen strategy within the same trading floor. In [8], we determine the real-time production strategy for a portfolio consisting of a wind farm and a Combined Heat and Power (CHP) plant using the approach described in this section.

2.2. Stochastic Programming

The stochastic programming approach to (1) is based on a discretization of the uncertainty space Ω . By doing that, we approximate the probability distribution with a discrete number of scenarios $\omega_1, \dots, \omega_S$, see Section 3.3, with associated probability $p_{\omega_1}, \dots, p_{\omega_S}$. The optimization problem resulting from this discretization is:

$$\text{Min.}_{x,y_{\omega_s}} c^\top x + \sum_{s=1}^S p_{\omega_s} q_{\omega_s}^\top y_{\omega_s} \quad (4a)$$

$$\text{s.t. } \mathbf{A}x \geq \mathbf{b}, \quad (4b)$$

$$\mathbf{T}x + \mathbf{W}y_{\omega_s} \geq \mathbf{h}_{\omega_s}, \quad s = 1, \dots, S. \quad (4c)$$

Two important differences with respect to model (1) render its stochastic programming version (4) tractable. The first one is the fact that the recourse decision y_ω need to be determined only at a *finite* number (S) of points ω_s . The second difference is that constraint (4c)

need to hold for a *finite* number of realizations of the uncertainty ω . In contrast, observe that model (1) requires the determination of the whole recourse functions $y_\omega : \Omega \rightarrow \mathbb{R}^{NT}$ and includes an infinite number of instances of constraint (1c).

Moreover, observe that in (4) we implicitly made the assumption that the modeler wishes to include the expected value of the recourse cost in place of the operator $M_\omega\{\cdot\}$, which in the case of discrete uncertainty boils down to a sum weighted by the scenario probability. However, this is not the only description of the uncertainty allowed in stochastic programming. For example, the use of Conditional Value at Risk (CVaR) [16] would also result in tractable optimization problems, see [13].

While the deterministic formulation (3) has $MT + NT$ variables and $L_1 + L_2$ constraints, the size of stochastic programming model (4) is $(MT + SNT) \times (L_1 + SL_2)$. Despite being tractable, (4) can quickly grow too large if an excessive number of scenarios, S , is chosen.

2.3. Robust Optimization with Linear Decision Rules

In this section, we consider a special case of robust optimization, i.e., where the recourse decision y_ω is an affine function of the uncertainty. We make the following assumptions:

- A1** The uncertain parameters q_ω, h_ω depend linearly on the random variable, i.e., $q_\omega = \mathbf{Q}\omega, h_\omega = \mathbf{H}\omega$.
- A2** We require Eq. (1c) be valid $\forall \omega \in \mathcal{U}$, where \mathcal{U} is a bounded polyhedral set described by the set of \mathcal{R} linear inequalities $\mathbf{D}\omega \geq \mathbf{d}$. Note that \mathcal{U} is a subset of Ω that is chosen by the modeler depending on the desired level of conservativeness of the solution.
- A3** The recourse decision y_ω is restricted to be an affine function of the uncertainty, i.e., $y_\omega = \mathbf{Y}\omega$.

Note that A1 implies no loss of generality, as one could simply redefine the probability space on the new variables (q_ω, h_ω) . Then, uncertain parameters and random variables would coincide, hence the linear dependence between them would be trivial. Assumption A2 is a modeling assumption needed for tractability. Note that the modeler can freely choose the polyhedral set \mathcal{U} . Typically, the larger \mathcal{U} , the more conservative the solution, as feasibility must be ensured for a larger set of realizations of the uncertainty. Different types of closed convex sets, e.g., elliptical, can be chosen without destroying tractability; we refer the interested reader to [1]. Finally, A3 is also needed for tractability.

Under the assumptions above, model (1) can be reformulated in a robust optimization framework as follows:

$$\text{Min.}_{x,Y} c^\top x + \mathbb{E}_\omega \left\{ \omega^\top \mathbf{Q}^\top \mathbf{Y} \omega \right\} \quad (5a)$$

$$\text{s.t. } \mathbf{A}x \geq \mathbf{b}, \quad (5b)$$

$$\mathbf{T}x + \mathbf{W}Y\omega \geq \mathbf{H}\omega, \quad \forall \omega \in \mathcal{U}. \quad (5c)$$

Note that constraint (5c) can be reinterpreted in the following equivalent reformulations:

$$\begin{aligned} (\mathbf{W}Y - \mathbf{H})\omega &\geq -\mathbf{T}x, \forall \omega \in \mathcal{U} \Leftrightarrow \min_{\omega \in \mathcal{U}} \\ \left\{ (\mathbf{W}Y - \mathbf{H})\omega \right\} &\geq -\mathbf{T}x, \end{aligned} \quad (6)$$

where the *min* operator in the latter inequality works row-wise. Replacing the inequality on the right side of Eq. (6) into (5), after some reformulations we can obtain:

$$\text{Min.}_{x,Y,\Lambda} c^\top x + \text{tr} \left\{ \mathbf{Q}^\top \mathbf{Y} \left(\sum_\omega \mathbb{E} \{ \omega \} \mathbb{E} \{ \omega \}^\top \right) \right\} \quad (7a)$$

$$\text{s.t. } \mathbf{A}x \geq \mathbf{b}, \quad (7b)$$

$$\Lambda^\top d \geq -\mathbf{T}x, \quad (7c)$$

$$\mathbf{D}^\top \Lambda = (\mathbf{W}Y - \mathbf{H})^\top, \quad (7d)$$

$$\Lambda \in \mathbb{R}_{\geq 0}^{R \times L_2}, \quad (7e)$$

where Σ_ω is the variance-covariance matrix of ω . In order to get Eq. (7a) from (5a), we performed the following substitutions:

$$\begin{aligned} E_\omega \left\{ \omega^\top \mathbf{Q}^\top \mathbf{Y} \omega \right\} &= \text{tr} \left\{ E_\omega \left\{ \omega^\top \mathbf{Q}^\top \mathbf{Y} \omega \right\} \right\} \\ &= \text{tr} \left\{ E_\omega \left\{ \mathbf{Q}^\top \mathbf{Y} \omega \omega^\top \right\} \right\} \\ &= \text{tr} \left\{ \mathbf{Q}^\top \mathbf{Y} E_\omega \left\{ \omega \omega^\top \right\} \right\} \\ &= \text{tr} \left\{ \mathbf{Q}^\top \mathbf{Y} \left(\sum_\omega + \mathbb{E} \{ \omega \} \mathbb{E} \{ \omega \}^\top \right) \right\}, \end{aligned} \quad (8)$$

where we exploited respectively: the fact that a 1×1 matrix is equal to its trace, the invariance of the trace operator to cyclic permutations of the arguments, the linearity of expectation and the definition of variance-covariance matrix. Constraints (7c)-(7e) are equivalent representations of the right-hand side of Eq. (6) based on linear duality [9]. We refer to [20] for further details on the latter transformation, whose derivation is rather lengthy.

2.4. Bilevel Programming

Bilevel programming can be employed to model different decision-making problems in energy markets [7]. In particular, we review here its applications to the offering problem of a price-maker wind power producer [18] and to model the Stackelberg game between retailers and residential consumers in a dynamic-price environment [19]. As both problems are subject to uncertainty, they can be cast in the framework (1).

In view of its large capacity, a price-maker agent can exercise a significant impact on the market price through its offering strategy. Hence, the market-clearing process conducted by the market operator has to be included within the optimization model to determine the optimal offer. Similarly, the real-time signal set by a retailer impacts the consumption plan from price-responsive consumers. The determination of the latter is an optimization problem in itself that has to be included within the retailer pricing problem.

In their most minimal formulation, the market-clearing problem for a market operator or the schedule determination for a consumer can be casted as linear programming problems

$$\text{Min.}_{\mathbf{u}} c_L^\top \mathbf{u} \quad (9a)$$

$$\text{s.t. } \mathbf{A}_L \mathbf{u} \geq \mathbf{b}_L \quad : \mu. \quad (9b)$$

In a market-clearing problem, \mathbf{u} represents the quantities to be dispatched, i.e., production and consumption for each market player, and c_L the marginal cost or benefit indicated in the offer or bid submitted by the corresponding agent. Constraints (9b) include a number of physical limits of the system (e.g., transmission capacity), market limits (e.g., dispatch limits specified in the offers), and balance between

supply and demand at each node. Constraints of the latter type are particularly important, as the associated dual variables, indicated in (11) with $\boldsymbol{\mu}$, can be interpreted as the electricity price at the corresponding location of the grid. We refer the reader to [13] for a more detailed description of the electricity market-clearing process.

In the consumer problem with dynamic pricing, \mathbf{u} represents the consumption along with some states of the system (e.g., temperature in the case of a heating system). The cost coefficient \mathbf{c}_L includes the price sequence sent by the retailer (multiplying consumption in the objective function) and possibly a penalty for states exceeding a comfort zone. Constraints (9b) include dynamic equations linking consumption and states as well as physical restrictions.

Since problem (9) is linear, the following Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality [9]:

$$0 \leq \boldsymbol{\mu} \perp \mathbf{A}_L \mathbf{u} - \mathbf{b}_L \geq 0, \quad (10a)$$

$$\mathbf{A}_L^\top \boldsymbol{\mu} = \mathbf{c}_L. \quad (10b)$$

Note that KKT conditions are non-linear, as the \perp operator implies that either the left or the right-hand-side are equal to zero. Hence, the \perp condition in Eq. (10a) could be replaced by $\boldsymbol{\mu} \circ (\mathbf{A}_L \mathbf{u} - \mathbf{b}_L) = 0$, where \circ is the pairwise product between corresponding vector elements. A reformulation that allows to linearize the \perp condition is proposed in [6]. The result is a reformulation of Eqs. (10) as a set of linear inequalities involving additional binary variables.

As a result of the observations above, we cast the trading problem of a price-maker wind power producer as a bilevel programming problem. In general terms, it can be formulated as follows:

$$\text{Min}_{\mathbf{x}, \mathbf{y}_\omega, \mathbf{u}_\omega, \mathbf{q}_\omega} \mathbf{c}^\top \mathbf{x} + \mathcal{M}_\omega \left\{ \mathbf{q}_\omega^\top \mathbf{y}_\omega \right\}. \quad (11a)$$

$$\text{s.t. } \mathbf{A} \mathbf{x} \geq \mathbf{b}, \quad (11b)$$

$$\mathbf{T} \mathbf{x} + \mathbf{W} \mathbf{y}_\omega \geq \mathbf{h}_\omega, \quad \forall \omega \in \Omega, \quad (11c)$$

$$0 \leq \mathbf{q}_\omega \perp \mathbf{A}_L \mathbf{u}_\omega - (\mathbf{b}_L - \mathbf{B}_L \mathbf{y}_\omega) \geq 0, \quad \forall \omega \in \Omega, \quad (11d)$$

$$\mathbf{A}_L^\top \mathbf{q}_\omega = \mathbf{c}_L, \quad \forall \omega \in \Omega. \quad (11e)$$

Note that the optimality conditions for the balancing-market clearing, Eqs. (11d)-(11e) control how the offer in the balancing market (included in the recourse vector \mathbf{y}_ω) affects the balancing market price \mathbf{q}_ω . Indeed, such constraints link \mathbf{y}_ω and \mathbf{q}_ω . The primal constraints for the balancing-market clearing, i.e., the conditions on the right of the \perp operator in Eq. (11d), are slightly different from the one in Eq. (10a). The presence of the \mathbf{y}_ω variable in Eq. (11d) captures the effect of the offer of the wind power producer at the balancing market stage on the clearing of the same market. Thus, (11) models the price-maker behavior of the wind power producer on the balancing market. Note that the balancing market offer of the wind power producer is in turn influenced by its day-ahead offer \mathbf{x} through Eq. (11c). Optimization model (11) is nonlinear, due to the multiplication between decision variables \mathbf{q}_ω and \mathbf{y}_ω in Eq. (11a) Some additional reformulations allow us to transform it into a Mixed-Integer Linear Problem (MILP). We refer to [18] for the details on this linearization.

Similarly, the problem of determining the optimal market and pricing strategy for a retailer in a dynamic pricing environment [19] can be cast as the following bilevel programming problem:

$$\text{Min}_{\mathbf{x}, \mathbf{y}_\omega, \boldsymbol{\pi}_\omega} \mathbf{c}^\top \mathbf{x} + \mathcal{M}_\omega \left\{ \mathbf{q}_\omega^\top \mathbf{y}_\omega - \boldsymbol{\pi}_\omega^\top \mathbf{u}_\omega \right\} \quad (12a)$$

$$\text{s.t. } \mathbf{A} \mathbf{x} \geq \mathbf{b}, \quad (12b)$$

$$\mathbf{T} \mathbf{x} + \mathbf{W} \mathbf{y}_\omega + \mathbf{V} \mathbf{u}_\omega \geq \mathbf{h}_\omega, \quad \forall \omega \in \Omega, \quad (12c)$$

$$0 \leq \mathbf{q}_\omega \perp \mathbf{A}_L \mathbf{u}_\omega - \mathbf{b}_L \geq 0, \quad \forall \omega \in \Omega, \quad (12d)$$

$$\mathbf{A}_L^\top \mathbf{q}_\omega = \boldsymbol{\pi}_\omega, \quad \forall \omega \in \Omega. \quad (12e)$$

The retailer (upper-level problem) influences the consumers (lower-level) by deciding the price signal $\boldsymbol{\pi}_\omega$.

In turn, the consumers decide their consumption plan u_ω , which enters the upper-level constraints (12c). Among other restrictions, Eq. (12c) include a balance condition between the electricity purchased in the day-ahead market, x , and in the balancing market, y_ω . Additional reformulations are needed, see [19], to get rid of the nonlinearity given by the revenue obtained for sales to consumers, $\pi_\omega^\top u_\omega$, in the objective function in Eq. (12a).

3. Forecasting

In this section, we review the different types of forecasts needed in the formulations of optimization under uncertainty described in Section 2.

3.1. Point Forecast

Point forecasts are the simplest type of prediction, as they aim at forecasting a single value describing a certain characteristic of the probability density function of a random variable.

Arguably, the most widely used point prediction is the *conditional forecast expectation*. Let us assume that, at time t , we are interested in the forecast of the expectation of random variable $h_{\omega, t+k}$, i.e., we are forecasting k -steps ahead or with lead-time k . We define this forecast as:

$$\hat{h}_{t+k|t} = \mathbb{E}_\omega \{ h_{\omega, t+k} \mid g, \gamma_t, \hat{\Theta} \} \quad (13)$$

Such a forecast is *conditional* on the information γ_t available at time t , which is in turn used to identify a suitable mathematical model g for the stochastic process h_ω , and to determine an estimate $\hat{\Theta}$ of the parameters of this model. Given the assumptions on model and parameters, $\hat{h}_{t+k|t}$ is a k -step ahead prediction (issued at time t) of the expected value of random variable $h_{\omega, t+k}$.

An example of conditional forecast expectation of the production of a wind farm with lead-times in the range between 1 and 24 hours is shown in Figure 1. Hourly forecast expectations are plotted along with the corresponding observations (i.e., the values measured in reality). As one can see, the forecast expectation overestimates wind power production during the first part of the day (roughly until hour 10), while it mostly underestimates it during the second part of the day.

Another type of point forecast is the *conditional quantile forecast*. Such a forecast aims at predicting a specific quantile of the distribution of a random

variable. Assuming the same issue-time and target of the expected-value forecast in Eq. (13), we can define the forecast α -quantile, $\hat{h}_{t+k|t}^\alpha$ by requiring that the following condition be met:

$$P_\omega \{ h_{\omega, t+k} \leq \hat{h}_{t+k|t}^\alpha \mid g, \gamma_t, \hat{\Theta} \} = \alpha. \quad (14)$$

According to this condition, the probability that $h_{\omega, t+k}$ is not larger than $\hat{h}_{t+k|t}^\alpha$, given model g and the estimated parameter set $\hat{\Theta}$, is equal to α . Note that this coincides with the definition of quantile for a continuous probability distribution. An important case of quantile forecast is the *conditional median forecast*, which is defined by setting $\alpha = 0.5$ in Eq. (14).

Despite providing a rather limited picture of the distribution of a random variable, point forecasts are widely used in decision-making as a result of their relative simplicity. For instance, the deterministic optimization framework introduced in Section 2.1 is based on the use of point forecasts such as the conditional forecast mean or median.

3.2. Probabilistic Forecast

Decision-makers may need more information on the distribution of a random variable than the single value provided by a point forecast. For example, they might be interested not only in knowing the expected value of an uncertain parameter at a point in time, but also on the uncertainty associated with such a point forecast. *Interval* and *density* forecasts provide this type of information.

An interval forecast with confidence β provides the decision-maker with a range where the random variable is forecast to take values in with probability β . Interval

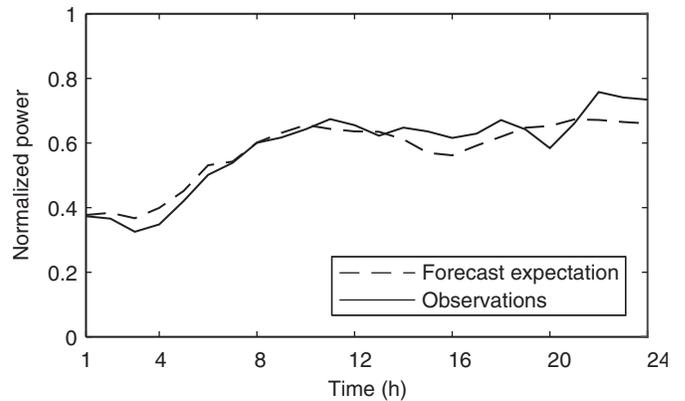


Figure 1: Example of day-ahead conditional forecast expectation along with the realized trajectory of wind power production.

forecasts can be obtained by pairing quantile forecasts, defined by Eq. (14), in the following manner:

$$\widehat{I}_{t+k|t}^\beta = \left[\widehat{h}_{t+k|t}^{(1-\beta)/2}, \widehat{h}_{t+k|t}^{(1+\beta)/2} \right]. \quad (15)$$

Note that there are multiple definitions for an interval forecast with a given confidence. For example, an interval forecast with confidence $\beta = 0.9$ could span both the quantile ranges 0–0.9 as well as 0.05–0.95. In the latter case, the interval is centered about the median, i.e., there is an equal probability of the random variable falling short or long of the median. The definition in Eq. (15) is for intervals centered about the median. Interval forecasts often find an application in robust optimization models, see Section 2.3. Indeed, the definition of the uncertainty set \mathcal{U} may include (among others) constraints enforcing that uncertain parameters be included within an interval with large confidence β .

Density forecasts give a full picture of the probability density function of a random variable. Essentially, they consist of a collection of interval forecasts issued with different confidence levels. Naturally, the finer the resolution in terms of confidence level, the more precise the information on the probability density function. In Figure 2, the example of forecast of wind power production in the previous section is enriched with the density forecast for the whole forecast horizon.

Probabilistic forecasts provide information on the uncertainty of a point forecast. They can be seen as a snapshot of a random process at a specific point in time in the future. Indeed, they model the probability density function of a random process at a given point in time, but they provide no information on the time-dependence structure of the forecast error. Scenarios fill in this last piece of information.

3.3. Scenarios

Many uncertain parameters in optimization problems are actually stochastic processes with non-negligible dynamic properties. For example, the forecast errors (i.e., the deviation between observation and the conditional forecast expectation) for wind power production at consecutive time periods have a significant positive correlation. This implies that, if production at time $t+k$ falls short of the forecast, there is a higher chance that it will also fall short of the forecast at time $t+k+1$. Scenarios provide a framework for modeling the dynamics of a random process.

Considering the random process $h_{\omega,t}$, we define a scenario as a plausible trajectory of this variable during

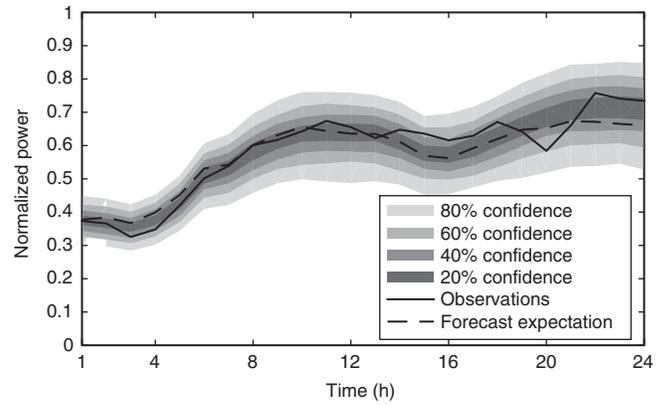


Figure 2: Example of day-ahead probabilistic forecast of wind power production at various confidence levels, along with its point forecast (expectation) and actual trajectory.

the time horizon of interest to the decision-maker. Considering a range of lead-times between 1 and K , we can define a set of S scenarios as:

$$\widehat{\mathbf{h}}^s = \left[\widehat{h}_{t+1|t}^s, \widehat{h}_{t+2|t}^s, \dots, \widehat{h}_{t+K|t}^s \right], \quad \forall_s = 1, 2, \dots, S. \quad (16)$$

If a sufficiently large number of scenarios is drawn, the (discrete) probability distribution of the S scenario values, $\widehat{h}_{t+k|t}^s$, for any given lead-time k can approximate reasonably well the (continuous) probability density function predicted for the same lead-time by the density forecast described in Section 3.2. Furthermore, the dynamics of the scenarios should comply with the estimated time-dependence structure (autocorrelation) of the random process $h_{\omega,t}$.

Figure 3 illustrates 10 scenarios simulating wind power production during the next 24 hours, along with the conditional forecast expectation and the observations already shown in Figure 1. Notably, the forecast errors for each scenario show positive autocorrelation, as scenarios that fall long of the forecast expectation at a given time tend to fall long also at neighboring time periods (and scenarios falling short tend to remain short).

Scenarios are extensively used within multi-stage stochastic programming models of the type introduced in Section 2.2. Typically, problems of this type are characterized by multiple sources of uncertainty, e.g., the cost q_ω and the right-hand side h_ω in model (4). Appropriate scenarios for these random variables should be issued so as to account not only for the autocorrelation for each random process, but also for their mutual correlation. We refer the interested reader to [13] for an introduction on the topic.

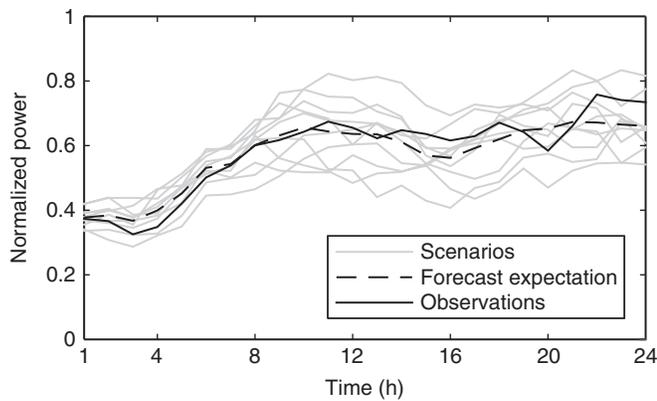


Figure 3: Example of scenarios modeling wind power production for the next day.

4. Applications

In this section, we review some applications of the methods described above. Section 4.1 deals with the determination of the optimal trading strategy for a price-maker wind power producer. Then, the management of heat and power systems is considered in Section 4.2. Finally, Section 4.3 focuses on market strategies for an electricity retailer participating in a real-time pricing environment.

4.1. Trading Wind Power as a Price-Maker

The problem of trading wind power is addressed in [18]. That work considers a producer whose size is sufficiently large to impact the balancing market prices as a result of its trading strategy. The problem is particularly relevant in Denmark, where wind power penetration has already surpassed 30% [5] and few large producers dominate the market. However, its relevance extends to other markets as the installed production capacity from wind (or solar, which could be addressed in a similar fashion) is constantly growing.

The stochastic programming approach described in Section 2.2 is employed in [18]. Realistic scenarios of day-ahead price, wind power production and system demand are used as input to the optimization problem. In turn, the latter outputs an offering curve specifying a given number of volume-price pairs. Note that the latter is the specific form in which producers are required to submit their day-ahead market offer to Nord Pool [15].

As described in Section 2.4, the price-maker nature of a market participant can be accounted for by casting the trading problem as a bilevel programming model. In this case, the lower-level problem represents the clearing process of the balancing market. Equilibrium conditions of the type of Eqs. (11d)–(11e) model how the offer submitted at the day-ahead market, the actual wind power production and the deviation of the other market players affect the balancing market price. Since the lower-level problem involves stochastic parameters (wind power production and deviation from other market participants), there is an instance of such equilibrium conditions per scenario.

Table 1 summarizes the structure of the optimization problem. For each stage, it lists the variables representing the decisions to be made and the uncertainty that is revealed after making those decisions.

Figure 4 illustrates the optimization model with a diagram. Forecast scenarios of day-ahead prices, wind power production and system deviation are inputs. The output of the model are the quantity offers to be submitted at the day-ahead and balancing markets. The feedback line from the balancing market represents that the determination of the clearing prices for this market is endogenous in the optimization model, so that the price-maker behavior of the producer can be taken into account.

Financial results obtained with the strategic offering model described above are reported and compared to the

Table 1: Stages, decisions and uncertainty in the optimization problem modeling the trading problem for a price-maker wind power producer [18].

Stage	Decision variables	Uncertainty revealed after stage
1) Day-ahead	price-offer pairs	<ul style="list-style-type: none"> • day-ahead price • wind power production • aggregate deviation from other players
2) Balancing	<ul style="list-style-type: none"> • volume purchased/sold (upper level problem) • balancing market dispatch (lower level problem) • balancing market price (lower level problem) 	

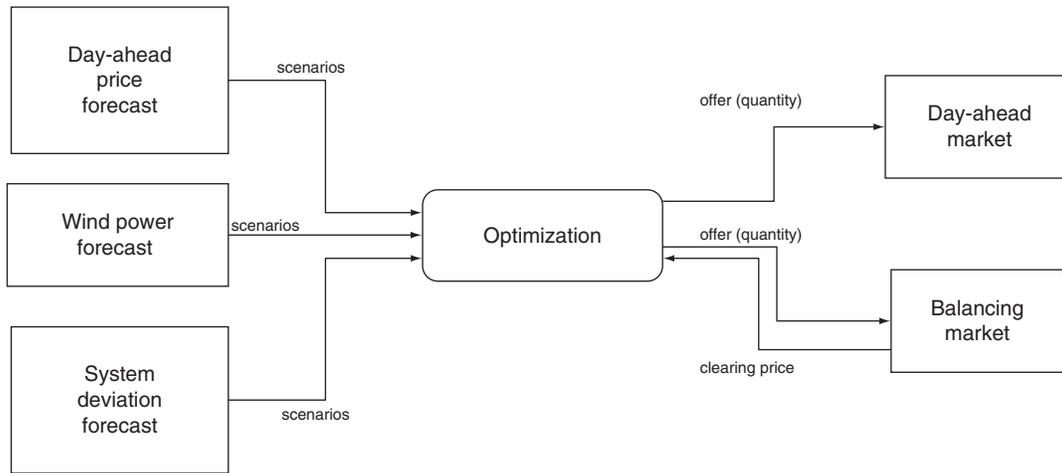


Figure 4: Diagram of the optimization model to determine the optimal trading strategy for a price-maker wind power producer in [18].

ones obtained with simpler deterministic offers in [18]. Three price-inelastic strategies, where a certain quantity is offered at any price level, are chosen as benchmarks. In the first one, the day-ahead offer is the conditional *mean* forecast of wind power production. This implies that the difference between the actual production from the wind farm and the day-ahead conditional mean forecast is to be sold (or purchased, if the production is smaller than the forecast) in the balancing market. Note that balancing market prices are in general different from day-ahead prices, so this strategy is not necessarily optimal. The second strategy consists in offering the conditional *median* forecast in the day-ahead market, and then settling the difference from the actual production in the balancing market. The last strategy consists in selling *all* the production at the balancing stage (i.e., submitting a zero offer in the day-ahead market).

A base case is considered first where the wind power producer's penetration in the balancing market is 20% and there is a small positive correlation between wind power production and the deviation from the other wind power producers. Results show that the strategic offering model outperforms the benchmarks by roughly 1.5% (zero offer) and 3% (mean and median offers).

Furthermore, the degree of improvement provided by the strategic offer is analyzed at different levels of penetration of the wind power producer. Figure 5 is constructed from simulation results published in [18]. It illustrates the percentage improvement in profits obtained when switching from the simpler trading strategies described in the fourth paragraph of this section to the proposed price-maker trading strategy. It

shows that offering no electricity at the day-ahead market is nearly optimal as long as the producer is small. However, the performance of this strategy deteriorates as the size of the producer increases. On the contrary, the degree of suboptimality of the strategies where forecast mean and median production are offered at the day-ahead market tends to drop as the size of the producer gets larger.

The impact of correlation between wind power production and the aggregate net system deviation is also assessed in [18]. Figure 6 illustrates results from the same paper. It emphasizes that the suboptimality of the zero-offer is a decreasing function of this correlation. On the contrary, the mean and median offers perform comparatively better when this correlation is negative.

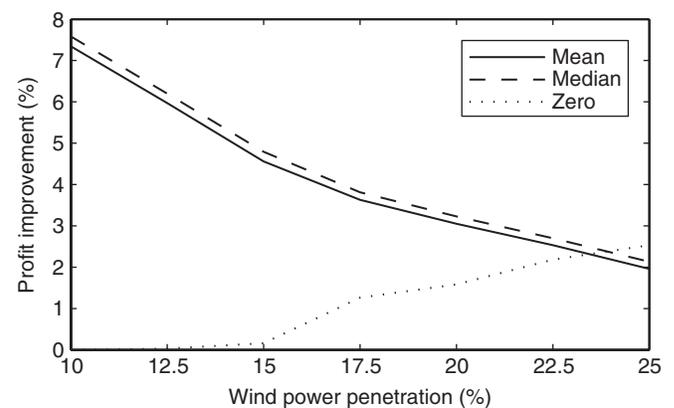


Figure 5: Profit improvement with respect to basic trading strategies at different levels of wind power penetration.

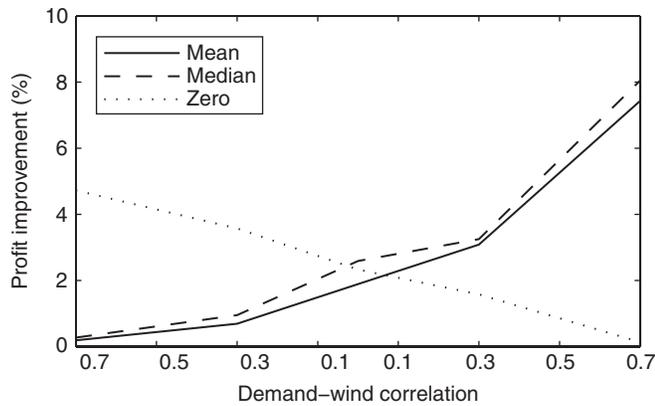


Figure 6: Profit improvement with respect to basic trading strategies at different levels of correlation between wind power and real-time deviation of power demand.

4.2. Managing Heat and Power Systems

Owners of heat and power producing units typically have to come up with production plans with a certain advance in time to the actual delivery of these commodities into the respective grids. This is partly caused by the fact that the preferred floor in which electricity is traded is the day-ahead market. In this section, we review some applications of stochastic optimization to the management of heat and power systems.

4.2.1. Unit Commitment and Dispatch for Heat and Power Systems

Because of the time structure of electricity and heat markets, heat and power production units have to be pre-dispatched on a day-ahead basis. Furthermore, these units may need some time to turn on and off. At the time of making the dispatch decision, important parameters like the actual heat demand or power prices are unknown. Hence, this optimization problem calls for a stochastic approach.

A robust optimization approach with the use of linear decision rules along the lines of Section 2.3 is proposed in

[20]. In this approach, redispatch decisions are made affine functions of the uncertain heat demand. A suitable budget uncertainty set [2] specifies intervals for the maximum deviation of heat demand at each time period, as well as a limit for the total deviation over the optimization horizon. The conditional expectation of the power price is also given as input to the optimization model, along with its correlation with heat demand. The optimization model outputs the plan for the on/off status of the units as well as for heat and power production. Table 2 sketches the structure of the optimization problem.

Figure 7 illustrates the optimization model with a diagram. Point forecast of day-ahead power prices, heat demand as well as uncertainty set for the latter and their correlation are inputs to the model. The output of the model are the unit-commitment and heat dispatch for day-ahead scheduling of the heat network, as well as offers to the day-ahead and balancing power markets.

The work in [20] establishes the viability of the robust optimization approach with linear decision rules for this type of problems by showing tractability in a representative instance of the problem, including two CHP units, an expensive heat-only unit as backup and a heat storage. The presence of storage renders the approach especially interesting, as it allows to consider a large number of stages (24 hourly periods in the case of [20]) without having to give up on the non-anticipativity of the solution, on the contrary of stochastic programming.

In the illustrative example in [20], the storage appears to be the unit that is used the most to guarantee the instantaneous heat balance. Since this is not a production unit, the CHP plants contribute by filling up the storage after deviations have taken place. Furthermore, the example shows that extraction CHP units may increase heat imbalance when the correlation between heat demand and power price is positive. This occurs because when a unit of this type is running at its

Table 2: Stages, decisions and uncertainty in the model optimizing the unit commitment and dispatch for a heat and power system [20].

Stage	Decision variables	Uncertainty revealed after stage
1) Day-ahead	<ul style="list-style-type: none"> • unit commitment • heat production pre-dispatch • power production pre-dispatch 	heat consumption
2) Balancing	<ul style="list-style-type: none"> • heat production redispatch • power production redispatch 	electricity price

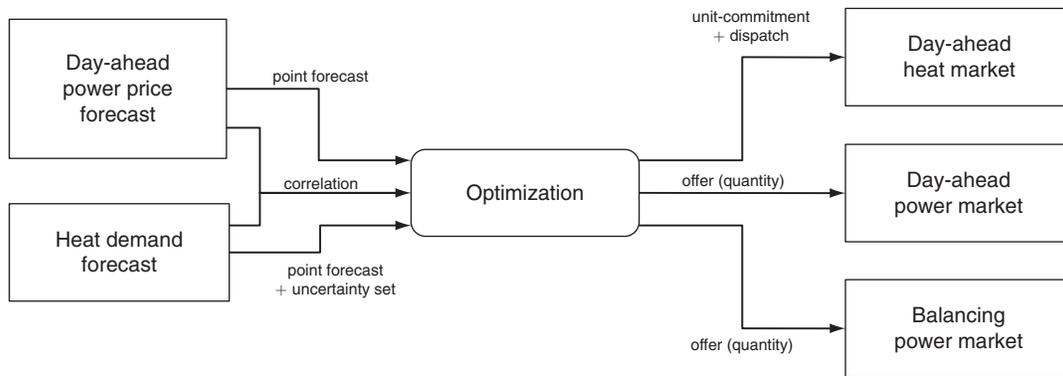


Figure 7: Diagram of the model optimizing the unit commitment and dispatch for a heat and power system [20].

maximum total production level, an increase in the power output can only be obtained by a proportional drop in heat production. Hence, this unit may decrease its heat production when power price and heat demand increase simultaneously. Figure 8 shows the ratio between heat output and heat-demand increase for the extraction CHP unit according to the linear decision rules in the example in [20]. The negative values indicate decreasing heat output when heat demand increases to allow for a larger power output. As a result, other production or storage units in the system must ramp-up to guarantee heat balance in these cases.

4.2.2 Assessment of the Economic Value of Heat Pumps and Electrical Boilers

A setup similar to the one in the previous section is considered in [14]. The focus on that paper is to assess the potential for the instalment of heat pumps and electric immersion boilers into the heating system serving the Greater Copenhagen area. In order to do that, realistic technical data for the units included in the CHP plant Amagerværket are employed along with actual realizations of heat and electricity prices.

The economic value of heat pumps and electric boilers is assessed by simulating the day-to-day market operation of a heat and power system. This includes decisions on unit commitment, pre-dispatch of heat and day-ahead trade of electricity, and the heat redispatch in real-time. This operational model is built along the principles of stochastic programming described in Section 2.2. Table 3 sketches the structure of the optimization model. Scenarios modeling the stochastic parameters (power prices, heat demand) are generated using time series models [11]. The simulation of the

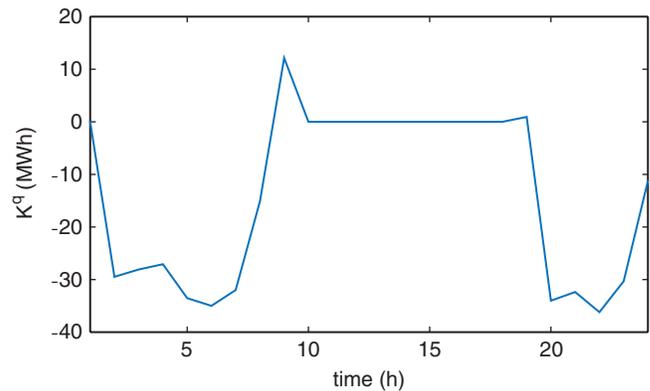


Figure 8: Ratio between heat output and heat-demand increase for an extraction CHP unit dictated by the linear decision rules in the example in [20].

system operation spans four representative weeks, from which yearly financial results are extrapolated.

Figure 9 illustrates the optimization model with a diagram. Scenarios of day-ahead power prices and heat demand are inputs to the model. The output of the model are the unit commitment and heat dispatch for day-ahead scheduling of the heat network, offers to the day-ahead power markets, and the updated unit-commitment and redispatch to cope with the real-time need for heat.

The simulations performed in [14] show that the financial improvement obtained by the use of a stochastic model instead of a deterministic one, i.e., the value of the stochastic solution, varies between around 0.5% up to above 17%, depending on time of year. The highest improvement is obtained during summer, where the dispatch of the system is less flexible as the heat pump and electrical boiler are both turned off. Fall and spring trail with an improvement of about 1.5%, while the smallest figure is obtained during winter. Another interesting result is that the value of the stochastic

Table 3: Stages, decisions and uncertainty in the model to assess the value of heat pumps and electric boilers in a heat and power system [14].

Stage	Decision variables	Uncertainty revealed after stage
1) Day-ahead	<ul style="list-style-type: none"> • preliminary unit status • heat production pre-dispatch • day-ahead power offer 	<ul style="list-style-type: none"> • day-ahead price • heat demand
2) Balancing	<ul style="list-style-type: none"> • final unit status • heat production redispatch • power production redispatch 	

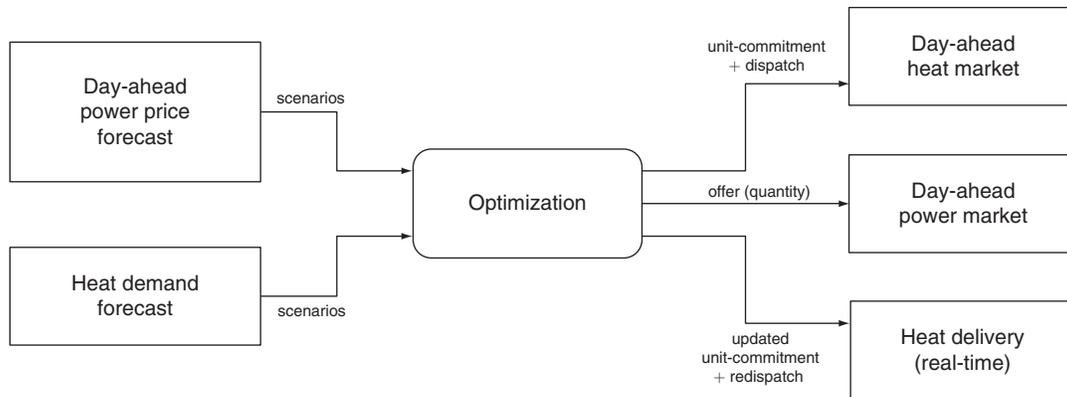


Figure 9: Diagram of the optimization model used in the assessment of the economic value of heat pumps and electrical boilers in [14].

solution is highly influenced by the installed capacity of these units. Indeed, as these units provide flexibility, they render the deterministic solution less and less suboptimal. Hence, the authors argue for the importance of stochastic models when making investment decisions.

Moreover, [14] shows that additional benefits between €3 m and €4.5 m can be obtained by installing a heat pump and electric boiler of reasonable size. However, the yearly benefits from these units could increase by as much as €7.3 m in a future scenario with lower electricity prices (with an average decrease of €6.7/MWh with respect to the current price level), which would imply cheaper operation for these units.

4.2.3 Portfolio Strategies for Jointly Balancing Wind Power and CHP Plants

A portfolio consisting of a wind farm and a heat-and-power system is considered in [8]. In that paper, the operation of the portfolio in the balancing market is optimized so as to minimize the cost of its total imbalance, i.e., the deviation between actual production and the day-ahead offer for these units. The problem is of particular relevance to Northern Europe and specifically to Denmark, where cogeneration is believed

to have large flexibility potential to support the integration of wind power [12].

The model in [8] represents the operation of the portfolio in the balancing market only. Hence, the results (dispatch) of the day-ahead electricity market is considered as an input. The optimization problem is built on the deterministic equivalent and is simulated with a rolling-horizon strategy as described in Section 2.1. Point forecasts (expected conditional mean values) are used for the uncertain parameters, which include heat demand, wind power production and balancing penalties (i.e., the differences between up-/down-regulation prices and the day-ahead price). Forecasts are issued with a horizon spanning from 1 to 23 hours ahead, since the time horizon for the optimization model includes 24 hourly time periods. The realization of heat demand and wind power production during the hour of operation is assumed to be known.

Figure 10 illustrates the optimization model with a diagram. Point forecasts of heat demand, balancing market penalties and wind power are inputs to the model along with the dispatch resulting from the day-ahead power market. The output of the model are the updated unit-commitment, the redispatch to cope with the real-

time need for heat and the corresponding balancing market electricity offer.

The model described above is run over a period spanning 10 months in 2012 within a fully operational framework, i.e., with state-of-the-art forecasts of uncertain parameters and actual Nord Pool market data [15]. The joint management of the heat and power system and the wind farm is compared to the independent operation of these assets. From a financial perspective, operating the system as a portfolio provides an average revenue increase of 0.55% over the simulation period. Furthermore, it brings about a reduction of 16.32% in the total volume of imbalances, i.e., involuntary deviations from the day-ahead schedule that are to be settled in the balancing market. Note that the latter is an important figure, as it signals that producers prefer to balance their portfolios internally, rather than through the market. Those results refer to the case where the operational objective is the maximization of revenues with no account for imbalances. By further imposing that the total imbalance of the portfolio be no larger than the one of the wind farm alone, financial improvement drops to 0.19%. However, imbalances in this case are reduced by 41.31% compared to the independent operation.

4.3 Market Strategy for a Retailer under Dynamic Pricing

The case of a retailer operating in a demand-response environment with dynamic pricing is considered in [19]. In that paper, it is assumed that a retailer purchases all the

electricity necessary to supply a group of residential consumers in the wholesale markets. In turn, the consumers purchase electricity from the retailer paying a real-time price chosen by the latter. Consumers are assumed to be flexible in their load for heating purposes (e.g., if they are equipped with a heat pump) as long as the temperature in the dwelling is within a given comfort band.

The model developed in [19] is a three-stage stochastic programming model with two levels. The upper-level problem consists in the profit maximization for a retailer, while the lower-level ones aims at maximizing the benefit (minus the costs) for the residential consumers. Table 4 summarizes the decision variables and the uncertainty unfolding at each stage. The model outputs the market strategy for the retailer in terms of purchase of electricity in the different market floors and of dynamic price signal to be sent to the consumers.

Figure 11 illustrates the optimization model with a diagram. Scenarios of day-ahead and balancing market prices, temperature and inflexible consumption are inputs to the model. The output of the model are the power purchase at the day-ahead market, the purchase/sale at the balancing power market and the price for the flexible consumers. The feedback arrow from the flexible consumers indicates that the power consumption is modeled endogenously through the lower level problem in the optimization model.

The illustrative example in [19], among other results, is used to compare the financial performance of the real-time pricing model with deterministic approaches. The

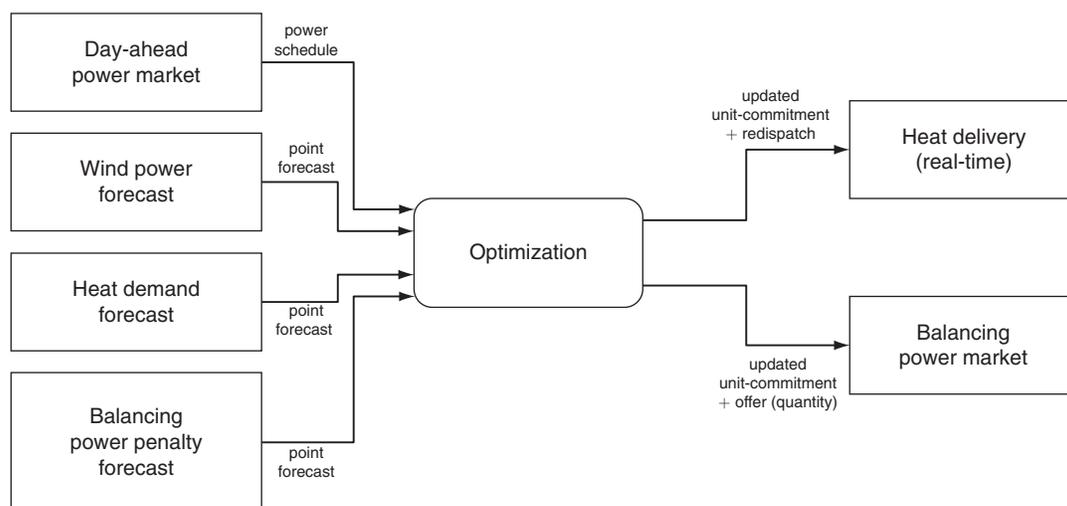


Figure 10: Diagram of the optimization model to jointly balance wind power plants and CHP units in [8].

Table 4: Stages, decisions and uncertainty in the model to determine the optimal market strategy for a retailer in a dynamic price environment [19].

Stage	Decision variables	Uncertainty revealed after stage
1) Day-ahead	<ul style="list-style-type: none"> day-ahead electricity purchase 	<ul style="list-style-type: none"> day-ahead electricity price weather-related uncertainty
2) Balancing	<ul style="list-style-type: none"> real-time price charged to end-consumers (upper level problem) energy purchased by the consumer (lower level problem) temperature of consumer building (lower level problem) 	<ul style="list-style-type: none"> up-/down-regulation price consumption from inflexible load
3) Ex-post	<ul style="list-style-type: none"> purchase/sale at balancing market 	

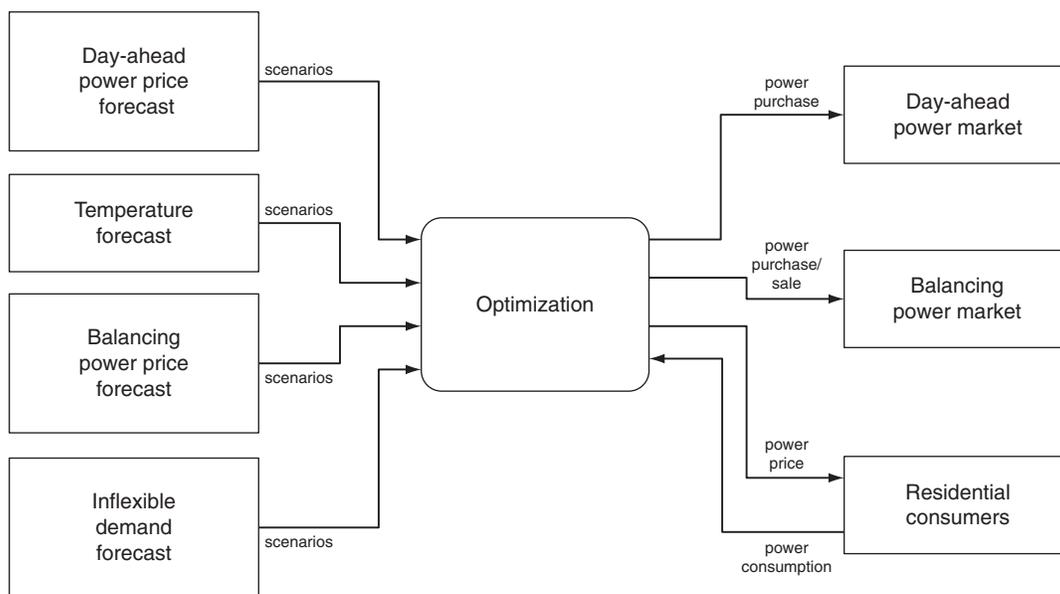


Figure 11: Diagram of the optimization model to determine the market strategy for a retailer under dynamic pricing [19].

first benchmark is the case where the consumer price for electricity is flat, while the second one is a *time-of-use* pricing scheme where the price is higher when consumption peaks and lower at valley periods. The reported profit improvement ranges from 4.96% against the fixed pricing scheme to 8.93% against the time-of-use one. Such an improvement is boosted by an increase in revenues from consumers (2.47% and 5.68%, respectively) and a reduction in market cost for electricity procurement (-2.75% and -0.97%, respectively). In particular, balancing costs for deviations of total consumption from the electricity purchase in the day-ahead market drop by 13.54% and 5.68%, respectively.

5. Conclusion

This paper reviews a number of contributions to decision-making under uncertainty in energy markets resulting from the project ENSYMORA. From a methodological point of view, the red thread unifying these studies is the use of techniques of optimization under uncertainty and of probabilistic forecasting within decision-making and optimization. The common focus is on problems of interest to future energy systems, including the large-scale deployment of renewables, integration across different energy systems and smart grids.

We first give a general formulation that is directly applicable to problems of decision-making under

uncertainty in energy markets. From this general formulation, we show how to derive a deterministic version of a problem of optimization under uncertainty (which can be easily implemented within a rolling-horizon framework in control problems) as well as how to apply stochastic programming and robust optimization. In parallel, we show how various elements in these formulations can be interpreted in different energy-market applications and we introduce the types of forecasts needed to account for uncertainty within these models.

The applications reviewed in this paper span the perspectives of a broad range of actors involved in energy markets. The case of a wind power producer trading in two electricity market floors (day-ahead and balancing) is considered in [18]. Furthermore, we review decision-making problems on different time-scales for owners of Combined Heat and Power (CHP) plants. The considered applications include investment analysis [14], optimal day-ahead unit-commitment and dispatch [20] as well as operation in the balancing market [8] as a portfolio with a wind farm. Finally, the perspective of an electricity retailer operating in a dynamic-price environment is considered in [19].

A selection of results from the case-studies included in the reviewed papers is presented. These results confirm the viability of different techniques of optimization under uncertainty for decision-making in energy markets with a large fraction of stochastic, and hence partly unpredictable, renewable power production. Comparisons with deterministic solutions for these problems show that stochastic methods can bring average financial improvement of a few percentage points in the considered problems.

Besides being a review-paper, this article can be considered as an introduction to the topics of optimization under uncertainty as well as of modeling and forecasting of stochastic processes. Indeed, it includes the basic formulation of an economic optimization problem, and guides the reader through the main solutions to account for uncertainty in the parameters, namely deterministic optimization with rolling-horizon, stochastic programming and robust optimization. The main forecasting concepts and products to be used within these optimization models are also briefly but rigorously reviewed. Finally, the reader is presented example results that show the potential of the optimization strategies within decision-making in

energy markets. The reader interested in a more complete treatment of these topics is referred to state-of-the-art textbooks throughout this paper.

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References

- [1] Aharon Ben-Tal, Laurent El Ghaoui, and Arkadi Nemirovski. *Robust Optimization*. Princeton University Press, 2009. URL <http://press.princeton.edu/titles/9099.html>.
- [2] Dimitris Bertsimas, David B. Brown, and Constantine Caramanis. Theory and applications of robust optimization. *SIAM Review*, 53(3):464–501, 2011. URL <http://epubs.siam.org/doi/abs/10.1137/080734510>.
- [3] John R. Birge and Francois Louveaux. *Introduction to Stochastic Programming*. International Series in Operations Research and Financial Engineering. Springer, New York, NY, USA, 2nd edition, 2011. URL <http://link.springer.com/book/10.1007%2F978-1-4614-0237-4>.
- [4] Antonio J. Conejo, Miguel Carrion, and Juan Miguel Morales. *Decision Making Under Uncertainty in Electricity Markets*, volume 153 of *International Series in Operations Research and Management Science*, chapter 2, pages 27–62. Springer, New York, NY, USA, 2010. URL <http://link.springer.com/book/10.1007%2F978-1-4419-7421-1>.
- [5] Energinet.dk. 2013 was a record-setting year for Danish windpower, December 2014. URL <http://energinet.dk/EN/El/Nyheder/Sider/2013-var-et-rekordaar-for-dansk-vindkraft.aspx>.
- [6] Jose Fortuny-Amat and Bruce McCarl. A representation and economic interpretation of a two-level programming problem. *Journal of the Operational Research Society*, 32(9):783–792, 1981. URL <http://www.jstor.org/discover/10.2307/2581394?sid=21105660360363&uid=3737880&uid=2&uid=62&uid=5910136&uid=67&uid=22998&uid=3&uid=18666672>.
- [7] Steven A. Gabriel, Antonio J. Conejo, J. David Fuller, Benjamin F. Hobbs, and Carlos Ruiz. *Complementarity Modeling in Energy Markets*, volume 180 of *International Series in Operations Research & Management Science*. Springer, New York, NY, USA, 2012. URL <http://link.springer.com/book/10.1007%2F978-1-4419-6123-5>.
- [8] Anna Hellmers, Marco Zugno, Anders Skajaa, and Juan Miguel Morales. Operational strategies for a portfolio of wind

- farms and CHP plants in a two-price balancing market. *IEEE Transactions on Power Systems*, 2015. doi: 10.1109/TPWRS.2015.2439060.
- [9] David G. Luenberger and Yinyu Ye. *Linear and Nonlinear Programming*, volume 116 of *International Series in Operations Research & Management Science*. Springer, New York, NY, USA, 3rd edition, 2008. URL <http://link.springer.com/book/10.1007%2F978-0-387-74503-9>.
- [10] Jan Marian Maciejowski. *Predictive Control with Constraints*. Pearson Education, Harlow, UK, 2002. URL <http://www.pearsonhighered.com/educator/product/Predictive-Control-with-Constraints/9780201398236.page>.
- [11] Henrik Madsen. *Time Series Analysis*. Chapman & Hall/CRC, Boca Raton, FL, USA, 2008. URL <http://link.springer.com/book/10.1007%2F978-0-387-74503-9>.
- [12] Peter Meibom, Klaus B. Hilger, Henrik Madsen, and Dorthe Vinther. Energy comes together in Denmark: The key to a future fossil-free Danish power system. *IEEE Power Energy Magazine*, 11(5):46-55, 2013. URL <http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6582012>.
- [13] Juan Miguel Morales, Antonio J. Conejo, Henrik Madsen, Pierre Pinson, and Marco Zugno. *Integrating Renewables in Electricity Markets: Operational Problems*, volume 205 of *International Series in Operations Research and Management Science*. Springer, New York, NY, USA, 2014. URL <http://link.springer.com/book/10.1007%2F978-1-4614-9411-9>.
- [14] Maria G. Nielsen, Juan Miguel Morales, Marco Zugno, Thomas E. Pedersen, and Henrik Madsen. Economic valuation of heat pumps and electric boilers in the Danish energy system. *Applied Energy*, 2015. doi: 10.1016/j.apenergy. 2015.08.115.
- [15] Nord Pool Spot website. <http://www.nordpoolspot.com/#/nordic/table>, December 2014.
- [16] Ralph Tyrrell Rockafellar and Stan Uryasev. Optimization of conditional value-at-risk. *Journal of Risk*, 2(3):21–41, 2000. URL <http://www.risk.net/journal-of-risk/technical-paper/2161159/optimization-conditional-value-risk>.
- [17] Varmelast.dk. Varmeplaner, December 2014. URL <http://varmelast.dk/da/varmeplaner/varmeplaner>. In Danish.
- [18] M. Zugno, J. M. Morales, P. Pinson, and H. Madsen. Pool strategy of a price-maker wind power producer. *IEEE Transactions on Power Systems*, 28(3):3440–3450, 2013. URL http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=6494365.
- [19] M. Zugno, J. M. Morales, P. Pinson, and H. Madsen. A bilevel model for electricity retailers' participation in a demand response market environment. *Energy Economics*, 36:182–197, 2013. URL <http://www.sciencedirect.com/science/article/pii/S0140988312003477>.
- [20] M. Zugno, J. M. Morales, and H. Madsen. Robust management of combined heat and power systems via linear decision rules. In *2014 IEEE International Energy Conference (EnergyCon)*, pages 479–486, Dubrovnik, Croatia, 2014. URL http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=6850470.

