

A combined spatial and technological model for the planning of district energy systems

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ABSTRACT

This paper describes a combined spatial and technological model for planning district energy systems. The model is formulated as a mixed integer linear program (MILP) and selects the optimal mix of technology types, sizes and fuels for local energy generation, combined with energy imports and exports. The model can also be used to select the locations for the energy sources, the distribution route, and optionally, to select the heat loads that will be connected to a district energy system. The optimisation model combines a map-based spatial framework, describing the potential distribution network structure, with a flexible Resource Technology Network (RTN) representation which incorporates multiple heat sources. Results for scenarios based on a test dataset are presented and show the impact of heat prices on the designed network length. The results illustrate the use of Combined Heat and Power (CHP) units to satisfy internal and external power demands, and also demonstrate their use in combination with heat pumps to satisfy emissions targets. A system value metric is introduced to quantify the incremental impact of investments in the heat network in areas of varying heat density. A procedure for screening potential supply locations to reduce computational requirements is proposed.

Keywords:

Heat-map;
District heating;
Optimisation;
MILP;

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1. Introduction

Heat-map based representations of energy systems show the locations of heat sources and sinks in a geographical domain, and can range in scope from district level to national level maps [1, 42]. In this paper detailed address level heat-maps for cities are used as the starting point for the development of an optimisation model for the planning of district energy systems. The heat-maps define the spatial framework for the model, identifying potential locations for the energy conversion processes together with the links for the heat distribution network. This is combined with a technological model, based on the Resource Technology Network (RTN) representation, which has been used in a range of applications for infrastructure planning [22-26]. The RTN for heat networks can incorporate supply technologies including

heat pumps, boilers and combined heat and power (CHP) units. The model can be used to select the type, size and location of each energy source and the connections for the distribution network to optimise an objective function that is the weighted sum of metrics for investment costs, operating costs/revenues and emissions. The value of this work lies in the integration with map-based tools, and the combination of features implemented. It is intended to bridge the gap between higher level map-based planning models and more detailed mechanistic models of the distribution network. A system value metric to quantify the incremental impact of investments in the heat network is also introduced.

The model combines features typically found in three categories of energy system models: spatial planning models for identifying areas where construction or

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Abbreviations:

CHP	Combined heat and power
HP	Heat pump
LiDAR	Light Detection and Ranging
MILP	Mixed Integer Linear Program
MINLP	Mixed Integer Non-linear Program
RTN	Resource Technology Network

expansion of heat networks may be feasible, models for the detailed optimisation of the routes and capacities of heat distribution networks, and models for the selection of the optimal mix of supply technologies (Figure 1). The first category of models often uses statistical data to estimate distribution costs for the area being studied [2, 11], but optimisation based methods employing a detailed spatial description of the distribution network have also been developed [3]. The second category employs non-linear or linearised models of the distribution network with varying levels of detail in computations of heat and mass flows, pressure drops and pump energy requirements [4, 14-16]. Formal optimisation methods [10], metaheuristics [8] or guidelines based on target pressure losses and flow velocities [15]

are used to select the route, pipe type and size for the heat network. The third category of models optimises the mix of technologies to meet a varying pattern of demands for heating, cooling and power [5]. A review of these three types of models is presented in the next section.

The combined model can be used for screening options in the early stages of planning a district energy system. The work described in this paper is intended to establish the feasibility and utility of the combined model for use within a map-based tool for the planning of district energy systems (Figure 2). To make it possible to embed within a map-based tool, the model incorporates a spatial framework to represent the layout of streets along which a heat network may be built, building locations, supply locations, and user choices as to whether buildings and network links are required or optional. Due to the complexity of the mapping application, the testing of the model has been done prior to the full development of the application. To facilitate testing, the model described in this paper has been implemented using standard optimisation languages and existing tools. An alternative implementation, written in Python, has also been developed and integrated within an early prototype of the map-driven application. Further

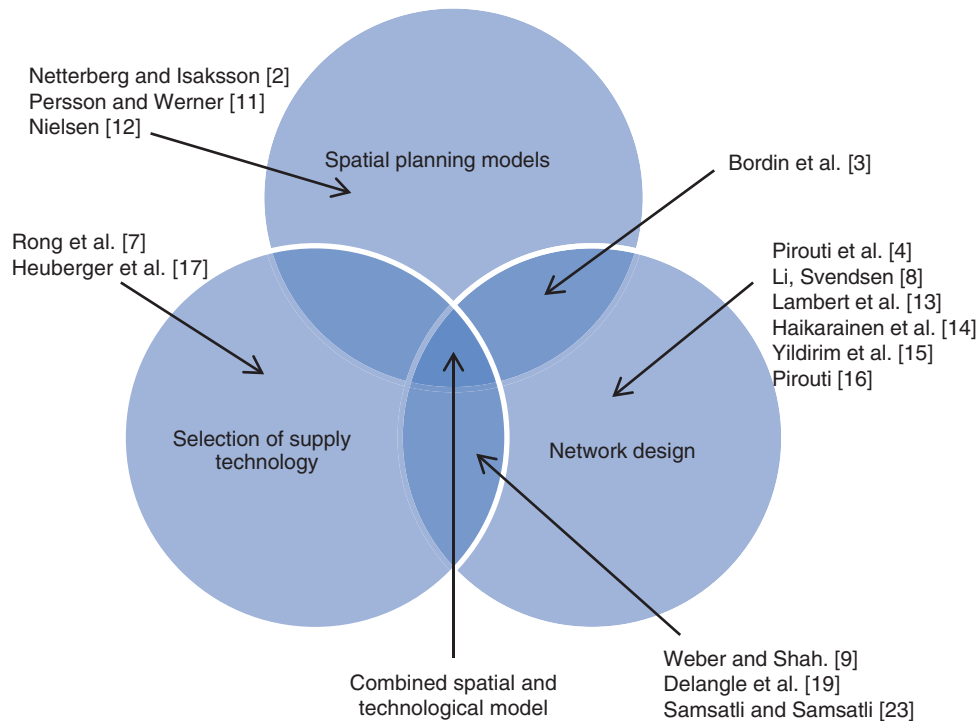


Figure 1: Classification of spatial and technological models for energy planning

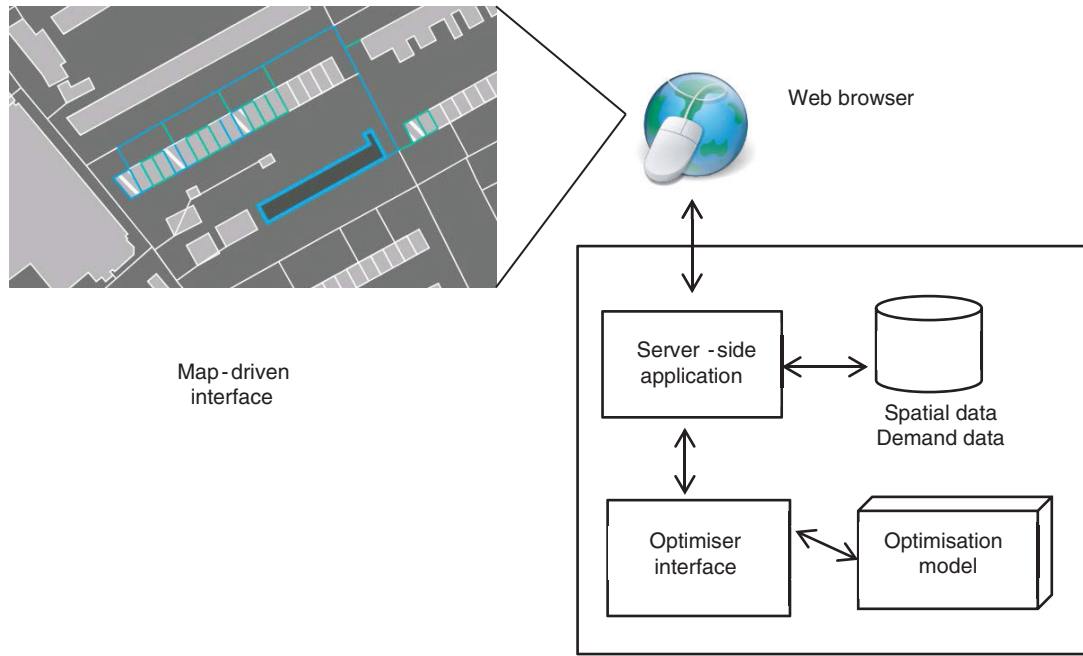


Figure 2: Map-driven modelling and optimisation of district energy systems

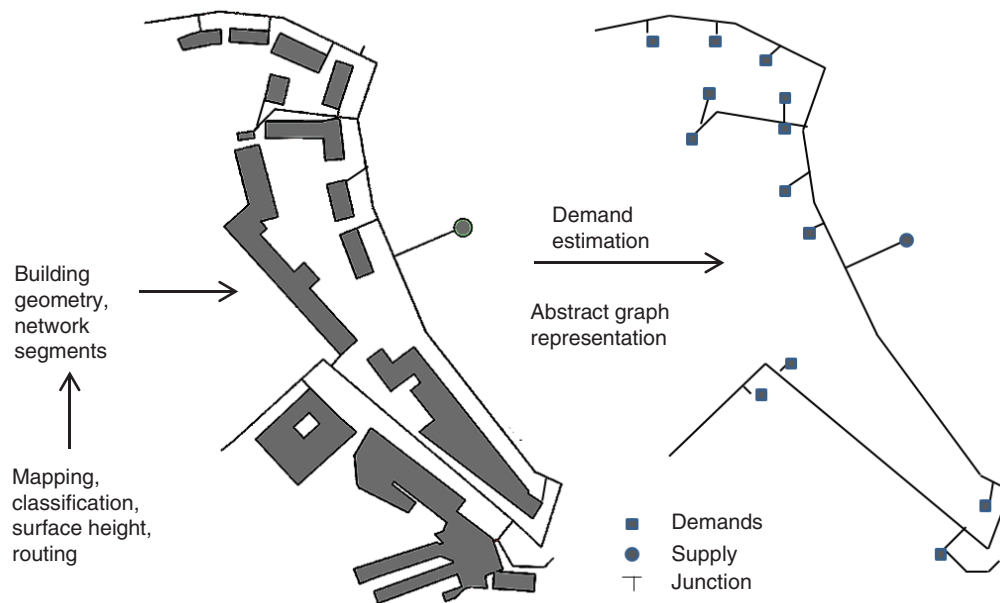


Figure 3: Map-driven construction of spatial framework for optimisation model

development of the integrated model and application is being undertaken by collaborators within the THERMOS (Thermal Energy Resource Modelling and Optimisation System) project [41], driven by feedback on the functionality and performance of the application from participating city partners.

The THERMOS application processes map data from sources such as OpenStreetMap into an abstract graph representation which serves as input for the optimisation model (Figure 3). Footprint polygons extracted from the map are augmented with height data obtained from LiDAR (Light Detection and Ranging, [38]). This

information is processed to obtain the building height, surface area, floor area, and volume. The paths for potential network links are derived from the roads in the map. Demands may be estimated by several different methods [20, 42]. One approach is to use benchmarks for annual heat demand per unit floor area for different types of buildings. An alternative approach is to estimate demands based on the heat loss through the external surface of the building, combined with typical values for internal and external temperatures. The spatial framework obtained from this information and demand values are used as inputs for the optimisation model described in section 3.

This paper describes the mathematical formulation of the combined optimisation model. An analysis of the incremental value of investing in a heat distribution network which can be obtained through repeated solution of this model is then developed. A test case is used to illustrate both the spatial and technological optimisation features of the combined model. To reduce computational requirements, a screening procedure is used to identify a limited set of potential supply locations prior to the optimisation of the supply technologies and distribution network structure.

2. Models of district energy systems

A broad review of energy system modelling tools may be found in [6]. Many papers focus on the integration of different technology types to supply the energy demands of a building or microgrid [7], whereas others emphasise the design of the distribution network [8]. A model that simultaneously optimises both the technology type and the distribution network routes is presented by Weber and Shah [9]. Models can also be categorised by the methodology used to formulate and solve the model. Models for the planning of district energy systems include MINLP (Mixed Integer Non-Linear Programming) models [10], MILP (Mixed Integer Linear Programming) models [3], stochastic programming models [13], multi-objective models for the optimisation of economic and environmental benefits [36], and models solved with metaheuristics [8].

Distribution costs for district heat networks may be estimated from aggregate characteristics of a district such as the population density, specific building space, specific heat demand and effective width [11]. More detailed estimates of transmission and distribution costs

can be obtained from a GIS system. An iterative procedure can then be used to identify expansion opportunities [12]. Models that can select the loads to be connected to a heat network using detailed spatial descriptions and optimisation models have also been developed. An MILP model for selecting loads to be connected to a heat network based on an economic criterion is described by Bordin et al. [3]. This model will be discussed in greater detail in the next section. Optimal staging of investments for expanding a heat network using stochastic programming is examined by Lambert et al. [13]. As part of the solution, these optimisation models also identify the routes and required network capacities for connecting the selected loads to the supply locations, and thus there is some overlap with the next category of models.

The models described above are used to identify potential expansion areas or select individual loads that may optionally be connected to a network. A second category of models optimises the distribution routes required to connect a fixed set of loads. Less detailed models with mass and energy balances can be formulated and solved as MILP models [14]. More detailed models may use non-linear expressions for pressure drops, pump energy requirements and heat losses. This leads to non-linear models with discrete decisions which can be solved using metaheuristics [8]. An alternative is to use piecewise linear approximations to characterise pump energy requirements [3]. These models emphasise the optimisation of the distribution network, but the technology selection and distribution network can also be optimised simultaneously [9]. More detailed models of the heat network may include detailed thermal and hydraulic calculations [15, 16]. Variations in heat demands characterised by load duration curves and corresponding adjustments to the supply flow rate and temperature may also be considered [16].

The optimisation of multiple generation technologies may be carried out in the context of selecting polygeneration technologies within microgrids [7], integration of large-scale heat pumps in district heating [37], and for the analysis of power generation with whole system models [17, 18]. The selection of technologies for the operation of a district heating centre is described by Delangle et al. [19]. The latter work also considers the details of sizing the heat network pipes based on projections of the required capacity, but this is decoupled from the subsequent optimisation of the

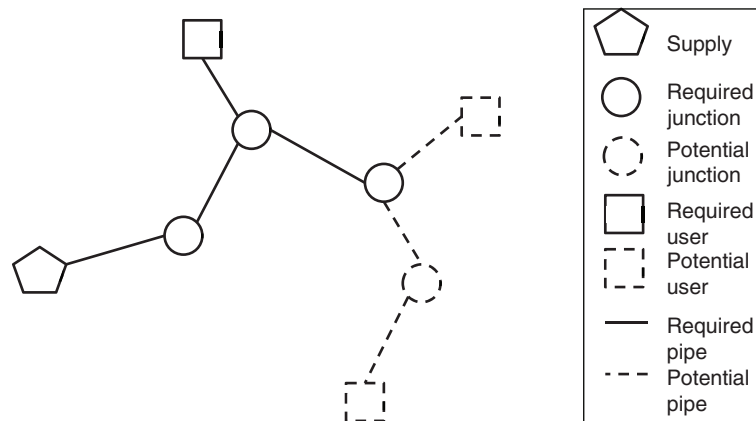


Figure 4: Spatial framework for optimisation model [3]

energy supply system. Optimisation methods are extensively used in this context, often with multi-objective formulations to address both cost and environmental concerns [36]. RTN based models, which are discussed in greater detail in the next section, have been used for technology selection in a wide variety of infrastructure planning applications [22-26]. A key feature of RTN models is that the technology mix can be easily restructured to include new technologies or combinations of technologies.

The work described in this paper combines an RTN based approach for technology selection, with the detailed spatial optimisation approach developed by Bordin et al. [3]. The model includes both environmental and economic metrics. The economic metric combines investment and operational costs for the supply technologies and the heat network. The model is coupled with a system analysis of the value of the distribution network which is similar to methods used in the analysis of storage and renewables in power systems [17, 18].

3. Planning model for district energy systems

Optimisation models for the planning of district energy systems may combine three frameworks that respectively represent the spatial, temporal and technological facets of the district energy system. The work described in this paper focuses primarily on the spatial and technological frameworks. A limited number of representative time periods, suitable for the early planning stages of a district energy system are used in the temporal framework. The spatial and technological modelling frameworks are outlined below, followed by a detailed description of a combined model.

3.1. Spatial framework for district heating network design

The spatial framework, which describes the location of energy demands, supply technologies and links for energy transport, is an abstract network representation which can be used for optimising the district energy system. The network includes nodes for required or potential users, supply points and junctions, and arcs for required or potential pipelines (Figure 4). Historical data for annual heat demand at each node may be available from local authorities or utilities, or may be estimated from building and consumer archetypes [20]. As described in the introduction the spatial framework may be constructed within an interactive map-driven application. The spatial framework and demand values can be used as inputs to an optimisation model which selects potential users to be connected to an existing network. An MILP formulation based on a cost objective which maximises revenues and minimises infrastructure and operational costs is described in [3]. This paper describes a model which can additionally select the supply technology type.

3.2. Technology selection framework for district heating

The Resource Technology Network (RTN) representation is similar to the State Task Network introduced by Kondili et al. [21] for planning the operation of batch chemical processes. This is a convenient representation for describing alternative pathways for producing intermediates and final products from different source materials. In the context of urban energy system models, resources may represent imported materials such as biomass, or natural gas, intermediates such as the

transport medium in district heating systems, or delivered energy for space heating. Technologies denote processes that consume and produce resources (e.g. a non-domestic gas boiler consumes natural gas and produces district heating). RTN-based infrastructure planning models have been applied to a wide range of applications including an analysis of the impact of urban energy governance policies [22], design of hydrogen networks [23], trade-offs in the design of urban energy systems [24], planning within the Water-Sanitation-Hygiene sector [25], and planning of the Energy-Food-Water nexus [26].

Past applications of the RTN model have typically employed an aggregated spatial framework where each zone in the model may represent a district within a city [22], or area within a region [23], and connections represent transport links between zones. As described in the previous section, the spatial model used here is more detailed, with the zones replaced by nodes representing individual buildings, supply points or junctions in the distribution network.

RTN model implementations may also differ in the level of temporal detail. A multi-level temporal framework which can capture seasonal and diurnal variability has been applied to model the use of hydrogen

generated from renewable energy for transportation [23]. This level of detail can result in large scale optimisation problems which are solved using a specialised algorithm. The overall optimisation problem is decomposed into technology selection and storage/transportation sub-problems that are solved iteratively. A low temporal resolution is used to identify an initial solution for the overall problem. In the present paper, a representative set of periods is used in the temporal framework. This is intended to reduce the computation times required within interactive applications used for the initial planning of district energy systems.

The RTN for this paper incorporates multiple “non-domestic” technologies for the production of district heat including CHP units, heat pumps and boilers (Figure 5). Multiple technology sizes are considered for the CHP units and non-domestic boilers. Natural gas and biomass can be used as fuels for boilers and CHP units. A dummy resource is defined to account for heat losses from all technologies. A generic technology for recovering heat from sources such as industrial plants can also be added to the model with user-specified capital and operating costs. Heat demands in buildings can be satisfied by heat exchangers connected to the district heating network. Renewable technologies such

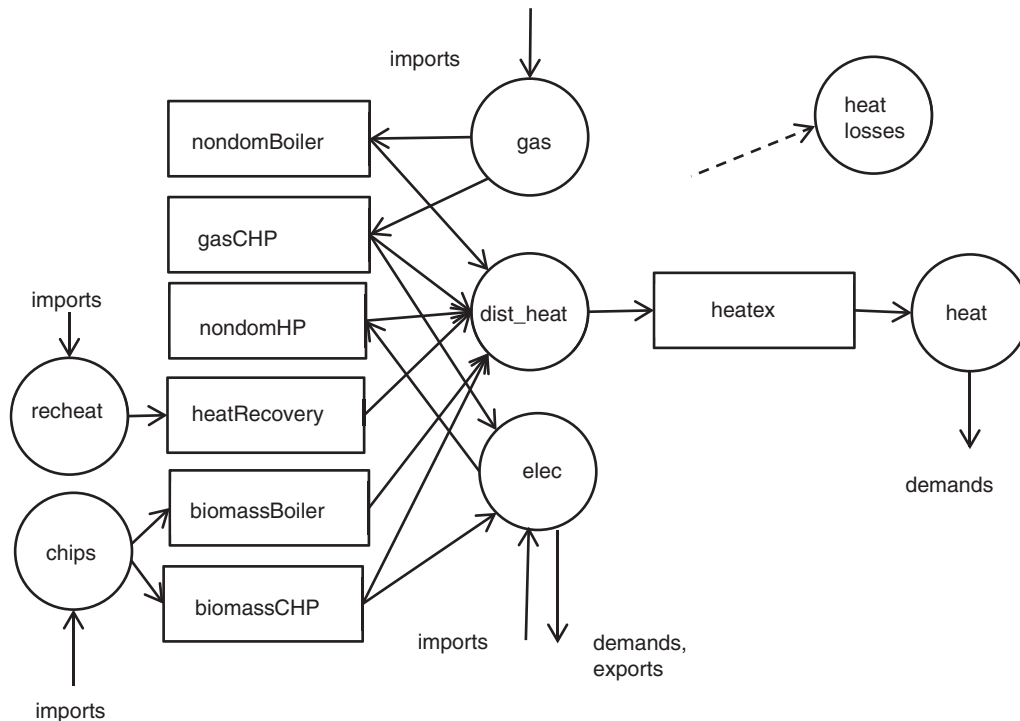


Figure 5: Resource Technology Network for district heating

Table 1 Short names and descriptions for resources and technologies (listed capacities are for technology sizes used in case study)

Short name	Description
biomassBoiler	boiler with biomass fuel
biomassCHP	CHP with biomass fuel
nondomBoiler	gas boiler for district heating supply (available in 0.5 MW and 1.0 MW sizes)
chips	fuel for biomass technologies
dist_heat	district heating resource produced by various technologies
elec	electricity resource imported, exported or produced by CHP
gas	natural gas fuel for boilers and CHP
gasCHP	CHP with natural gas fuel (available in the following sizes: 0.135 MW _e /0.22 MW _{th} small gasCHP, 0.5 MW _e /0.675 MW _{th} medium gasCHP, 1.0 MW _e /1.03 MW _{th} large gasCHP)
heatex	heat exchanger connected to district heating network
heatRecovery	user defined technology for heat recovery
nondomHP	non-domestic heat pump for district heating supply (0.5 MW)
reheat	user defined heat source for heat recovery technology
heat losses	dummy resource to account for heat losses

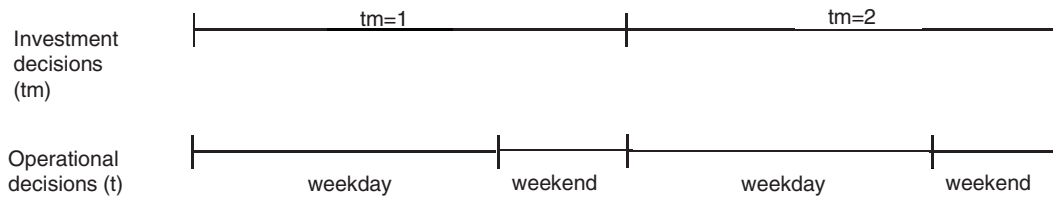


Figure 6: Example of temporal discretisation with two investment periods and minor periods for weekdays and weekends

as solar thermal have not been considered as these would require additional data on available installation area and solar irradiation. A higher temporal resolution would also be required in the optimisation model to capture the variability in these technologies, resulting in increased computation times for solving the model. Possible approaches towards managing the computational requirements for higher resolution models are discussed in the last section of this paper.

3.3. Combined spatial and technological model for district energy systems

The combined optimisation model uses an MILP formulation, similar to those used for other RTN-based infrastructure planning models [25]. The key constraints and objective function are described below. The model has been implemented within an existing interactive tool [24]. The tool, which is written in Java, generates scenarios for an MILP optimisation model in the GAMS modelling language which are then solved with the CPLEX solver.

The model incorporates a resource balance for each node in the spatial framework and each set of time intervals (t, tm) , where t are minor periods representing seasonal or diurnal demand variations, and tm are major periods for investment decisions (Figure 6). Major periods can be used to model staged investments, or to compare the energy system performance in a base period against a future period after investments to modify the system [27].

The balance equations for the model span all technologies and resources in the RTN (Figure 7). In the equation below RS represents the resource surplus at a node, P is the operating rate of technology j , $\mu_{j,r}$ is a coefficient that defines the production (or consumption) rate of resource r by technology j , IM and EXP are imports and exports, Q represents the flows and between nodes i and iI , and D represents the demands. Note that, although flow connections in both directions are permitted, due to the costs associated with flows, the optimisation will ensure that only one of $Q_{r,iI,i,t,tm}$ or $Q_{r,i,iI,t,tm}$ is non-zero. The flows Q can be modified by

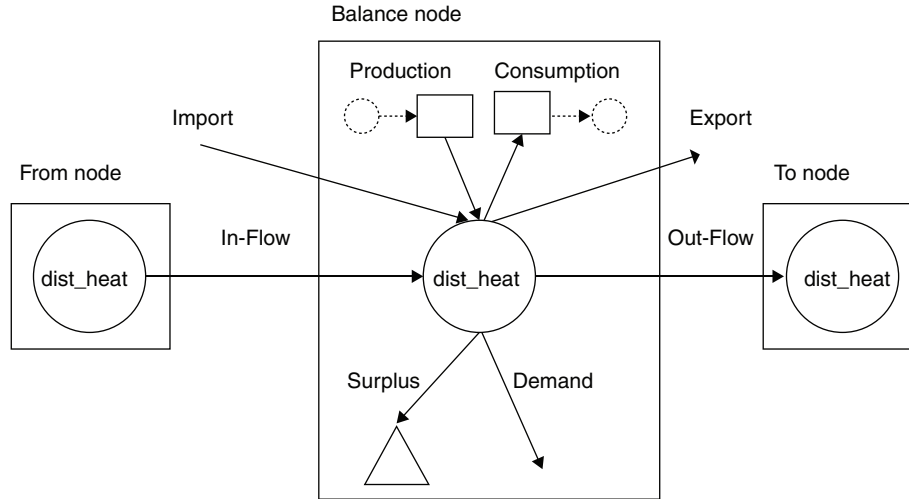


Figure 7: Resource balance for RTN model [27]

parameters reflecting heat losses or leaks [25]. The binary decision variable SAT in the balance equation selects nodes where the demands are satisfied (for required demand nodes the value of the decision variable is set to one). This is similar to the approach of Bordin et al. [3] where district heat connections are selected on the basis of an economic objective.

$$RS_{r,i,t,tm} = \sum_j \mu_{j,r} P_{j,i,t,tm} + IM_{r,i,t,tm} - EXP_{r,i,t,tm} + \sum_{i1} Q_{r,i1,i,t,tm} - \sum_{i1} Q_{r,i,i1,t,tm} - D_{r,i,t,tm} SAT_i \quad (1)$$

In general, the optimisation will tend to minimise resource surpluses due to the costs incurred in resource production. Non-zero surpluses may be permitted if storage is available or if a resource may be dissipated into the environment. For the case study in this paper, the resource surplus for all resources other than the dummy resource for heat losses was fixed to zero *a priori*.

The number of units N of technology j in cell i is determined by investment in INV new units in period tm . Investments in supply technologies are fixed at zero in all locations except the permitted supply locations.

$$N_{j,i,tm} = N_{j,i,tm-1} + INV_{j,i,tm} \quad (2)$$

Resources d represent the subset of resources r for which new networks must be built. The existence of a network link to transport resource d in period tm is

represented by the binary decision variable Y (for required links the decision variable is set to one). The following constraint ensures the existence of the link in periods following the one in which it is built.

$$Y_{d,i,i1,tm} \geq Y_{d,i,i1,tm-1} \quad (3)$$

Several types of infrastructure links may be defined with RTN models [23]: bidirectional links which can be used in either direction between a pair of nodes ($i, i1$); independent bidirectional links where a forward link allows transport from i to $i1$, and a reverse link allows transport from $i1$ to i ; and unidirectional links where only one of the two links may be built. Bidirectional links are used for the case studies described in this paper. These are convenient for use in the interactive planning application since they allow a user to indicate that a link should be built between two nodes without having to select a direction *a priori*. The following constraint indicates that a link in one direction implies a link in the opposite direction as well. A directional cost factor is then applied to the network costs so that the two links collectively are treated as a single bidirectional link.

$$Y_{d,i,i1,tm} = Y_{d,i1,i,tm} \quad (4)$$

Energy production in a node is constrained by the available capacity of the available units.

$$P_{j,i,t,tm} \leq CAP_j * N_{j,i,tm} \quad (5)$$

Flow between nodes is constrained by the capacity of the network links.

$$Q_{d,i,i1,t,m} \leq Q_{max,d} * Y_{d,i,i1,t,m} \quad (6)$$

The objective is to minimise the function *OBJFN* formed as the weighted sum of a value measure *VM* defined for metrics *m* representing operating costs, capital costs and emissions. The weights *OBJWT_m* for each metric are specified according to the desired objective. For the case study in this paper, an objective function which only considers the direct economic impacts is used, i.e. *OBJWT_{capex}* = 1, *OBJWT_{opex}* = 1, *OBJWT_{ghg}* = 0. A non-zero value of *OBJWT_{ghg}* may be used to incorporate a carbon cost in the objective.

$$OBJFN = \sum_{tm} \sum_m OBJWT_m VM_{m,tm} \quad (7)$$

The overall metric value *VM* is formed from the transportation cost *TC*, the production cost *PC*, import cost *IC*, export cost *EC*, tariffs *TR*, annualised equipment cost *EQ*, annualised network cost *NW*, and the annual maintenance cost *MC*.

$$VM_{m,tm} = TC_{m,tm} + PC_{m,tm} + IC_{m,tm} - EC_{m,tm} - TR_{m,tm} + EQ_{m,tm} + NW_{m,tm} + MC_{m,tm} \quad (8)$$

The model selects the mix of technology type and size, plant locations and distribution network links that minimises the objective function. Costs are represented as positive values and revenues as negative values.

Transport costs *TC* are proportional to the flows *Q*, while production costs *PC* are proportional to the production rates *P*. The network cost *NW* is calculated for all resources *d* requiring new networks, from the length *dist_{i,i1}* of each link and the annualized cost per unit distance *VY*. Alternatively, cost values *VYL* for individual links may be specified. The parameter β below is set to 0.5 for the bidirectional links used in the case study, so that only the cost of a single link is charged, even though links in both directions are created by equation (4). This is similar to the approach used in [23].

$$NW_{m,tm} = \sum_i \sum_{i1} \sum_d VY_{d,m} * \beta * dist_{i,i1} * Y_{d,i,i1,t,m} \quad (9)$$

$$NW_{m,tm} = \sum_i \sum_{i1} \sum_d VYL_{d,i,i1,m} * \beta * Y_{d,i,i1,t,m} \quad (10)$$

The equipment cost *EQ* is calculated from the annualised cost *VIJ* for each technology type *j*.

$$EQ_{m,tm} = \sum_i \sum_j VIJ_{j,m} * N_{j,i,tm} \quad (11)$$

The annualised costs are calculated by applying annuity factors *An* based on the interest rate *r* and lifetime *n*, to the equipment or network investment cost [9].

$$An = r(1+r)^n / ((1+r)^n - 1) \quad (12)$$

Import metrics *IC* are calculated from the unit cost (or emissions) *VI* for each imported resource *r*, weighted by the duration ϕ_t of period *t*. The value ϕ_t represents the number of hours for minor period *t* within a major period.

$$IC_{m,tm} = \sum_t \sum_i \sum_r VI_{r,m} * IM_{r,i,t,m} * \phi_t \quad (13)$$

Export metrics *EC* are calculated similarly. The parameters *Tariff* included in the metric *TR* may vary according to the technology type producing or consuming a resource. This permits the modelling of incentives that are targeted towards specific technology types, such as tax rebates on fuels and feed-in tariffs. The tariff metric used in the case study is based on the price of district heat delivered to each demand node.

$$TR_{m,tm} = \sum_t \sum_i \sum_j (\sum_r \mu_{j,r} Tariff_{j,r,t,m}) * P_{j,i,t,m} * \phi_t \quad (14)$$

3.4. System value of investment in heat distribution network

The system value of the heat distribution network can be calculated by placing an upper bound on the investment costs. The system value is measured by the change in the objective function produced by an increase in investment, which in turn results in an extension of the heat network. This is similar to a method used to evaluate the system impact of incremental investments in power generation and storage technologies [17], but here it is applied to investments in the distribution network. The optimisation model is solved repeatedly with an increasing value for an upper bound on the capital expenditure. The system value *SV_k*, at each iteration *k*, is calculated from the change in objective function per unit change in capital expenditure, as defined by Equations 7 and 8. The reference value of the objective function for the first iteration is equal to the investment and maintenance cost of the supply technologies, which is fixed for the remaining iterations. The change in the value of the objective function, $\Delta_k(OBJFN)$, at iteration *k* reflects the

value of incremental investments $\Delta_k(VM_{capex,tm})$ in the distribution network. The analysis here is restricted to the case where all investments are to be made within the first major time period ($tm=1$).

$$SV_k = \Delta_k(OBJFN) / \Delta_k(VM_{capex,tm}) \quad (15)$$

Since the system value is calculated from changes to the overall objective function value, it reflects the net impact of heat tariffs from newly connected loads, additional fuel costs and the annualised costs of extensions to the distribution network. An application of the system value calculation is provided in the next section.

4. Case study

The case study is based on a screening data set with 500 nodes. The data set is derived from the UK National Heat Map [42] for a location within one of the inner boroughs in London. The purpose of this heat map was to identify areas where heat networks were likely to be beneficial and to prioritise locations for more detailed investigation. Demand estimates are based on usage data collected at local authority level and address level characteristics obtained from public data sources. Point-to-point connections between nodes were used to identify potential network paths. The integrated application described in the introduction will use an improved methodology to estimate demands and identify potential routes from roads defined in the map data. The frequency distribution of demands across the nodes in the data set is shown in Figure 8. The majority of nodes represent building with demands less than or equal to 1.6 kW while there are a limited number of buildings with demands greater than 5 kW. All the nodes have heat demands, i.e. there are no nodes that function only as junctions. Representative values for the UK were used for network costs, fuel costs, and emissions factors (see Appendix 2 for sources). These are estimated values intended for use with this test case to demonstrate the key model features. It has been noted that network capital costs in the UK are high compared to other northern European countries [13, 28]. Annualised investment costs are calculated assuming a 3.5% discount rate and 30 year lifetime for the distribution network, and 15 year lifetime for supply technologies.

No potential supply locations were identified in the test data set. Preliminary testing showed that considering all 500 nodes as potential supply locations resulted in

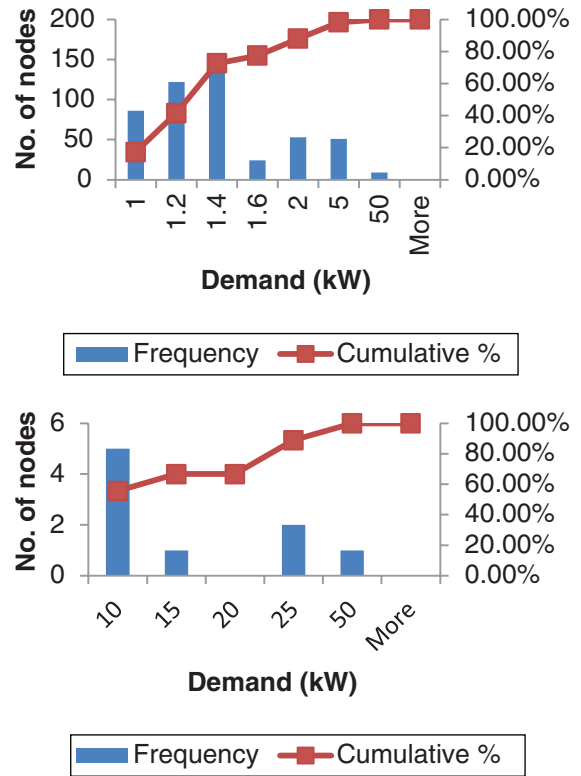


Figure 8: Frequency distribution of demand values: all nodes (top); nodes with demands > 5 kW (bottom)

very slow convergence towards an optimal solution, with estimates of the relative gap (defined as the percent difference between the best solution and the estimated optimum) ranging from 20-90% after 12000 seconds of computation for the test cases considered in section 4.1. The large gaps are in some part due to weak estimates of the optimal solution, but this still creates a difficulty in specifying a suitable convergence criterion to achieve reasonable run times. These initial runs with a full set of possible supply locations were therefore treated as screening runs, and the supply locations identified within the best solution were used as potential supply locations for the scenarios in the next section. With these limited supply locations the solution times for the scenarios in section 4.1 were considerably reduced, with solution times less than 900 seconds in almost all cases, and often less than 60 seconds, with relative gaps in the range 1-5%. The main steps in the construction and solution of the combined spatial and technological model are shown in Figure 9. Sections 4.1 to 4.3 describe scenarios that illustrate the main features of the model.

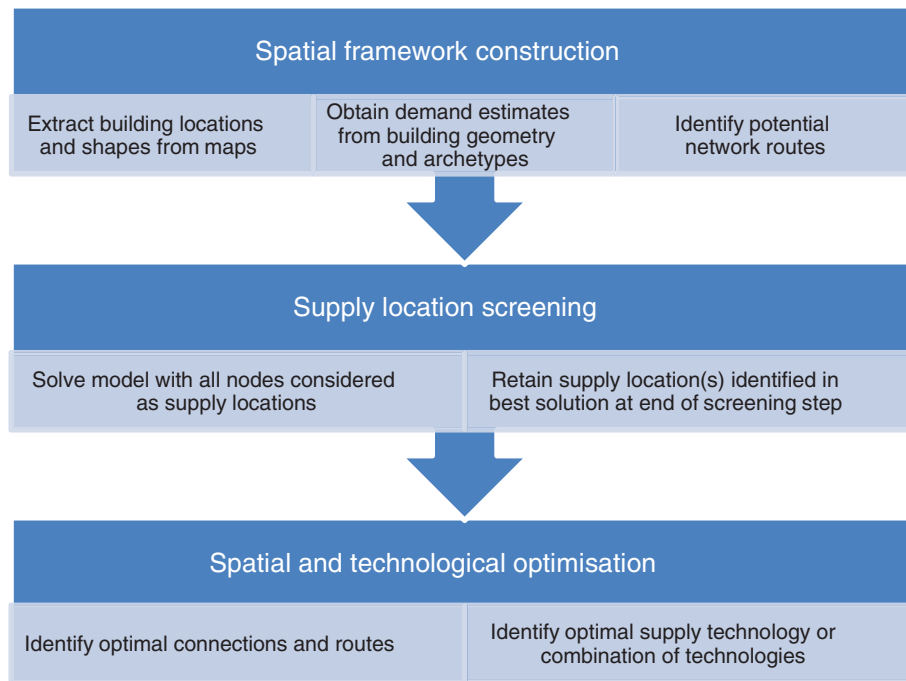


Figure 9: Construction and solution of optimisation model with screening of supply locations

Table 2 Cost breakdown for connection selection scenarios (k€) (Imp.=Import, Maint.=Maintenance, Ntwk.=Network, Equip.=Equipment, Obj.= Objective function).

Tariff level	Boiler type	Imp.	Maint.	Tariffs	Ntwk.	Equip.	Obj.	Length	GJ/m
2.0x	1 MW	34	6	-67	8	17	-3	277 m	16.02
2.5x	1 MW	112	21	-281	52	58	-37	1814 m	8.03
3.0x	1 MW	165	35	-496	92	90	-113	3191 m	6.84
2.0x	1x0.5 MW	33	3	-66	8	11	-10	277 m	16.02
2.5x	2x0.5 MW	102	18	-254	40	49	-46	1367 m	9.82
3.0x	2x0.5 MW	151	31	-455	77	79	-116	2680 m	7.49

4.1. Impact of district heat tariff levels and supply locations on network design

The overall annual heat demand for the area is 24,894 GJ. A heat network connecting all nodes would potentially be 4153 m in length; with a linear heat density of 6 GJ/m. Linear heat density is often used to screen potential district heating areas. As an example, UP-RES [29] suggests that linear heat density should be greater than 7.2 GJ/m for a heat network to be economically viable. This indicates that it may not be viable to connect all 500 nodes to a heat network, and the model is used to select connections based on minimising the objective function. Results for three scenarios with heat supplied

by a single 1 MW natural gas boiler are shown in Figure 10. The numbers in the lower right corner of each scenario in the figure indicate the district heat tariff, the length of the designed network, and the linear heat density of the selected loads. Three scenarios with district heat tariff levels at multiples of 2.0, 2.5 and 3.0 times the natural gas price are considered. The number of connected nodes and length of the designed network increase with higher district heating network tariffs as it becomes economically viable to supply areas with lower heat densities.

Table 2 shows the costs and revenues (shown as negative values since the model is formulated as a cost

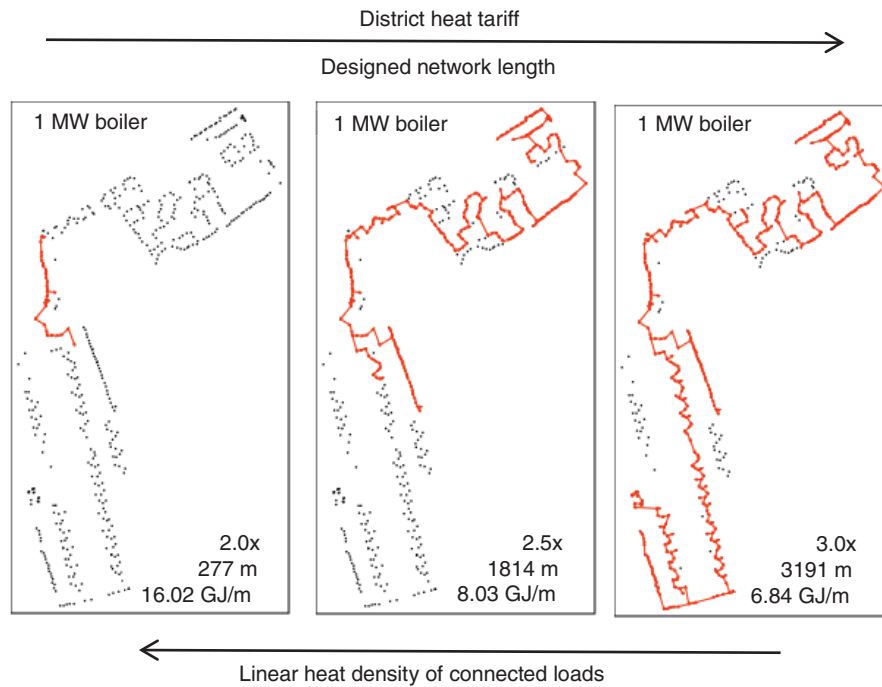


Figure 10: Impact of district heat tariff on connections with single supply location (numbers in lower right of each scenario indicate tariff level, network length and heat density)

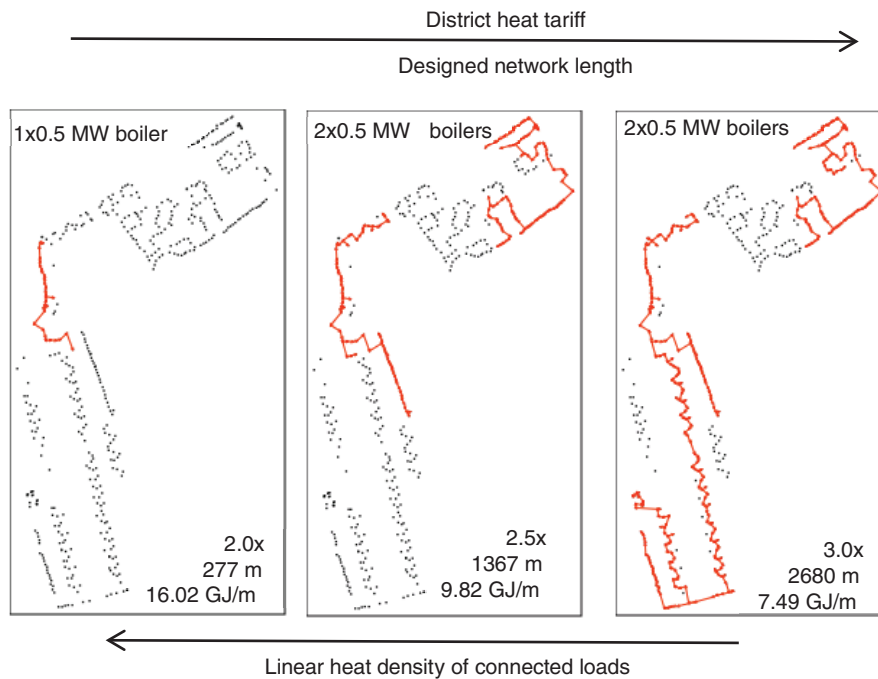


Figure 11: Impact of district heat tariff on connections with two supply locations (numbers in lower right of each scenario indicate tariff level, network length and heat density)

minimisation problem) for the scenarios in Figure 10, followed by those in Figure 11. The distribution network and equipment costs are annualised costs. Equipment

costs include both the non-domestic boiler costs and the costs of heat exchangers and other required equipment within the buildings connected to the network. The

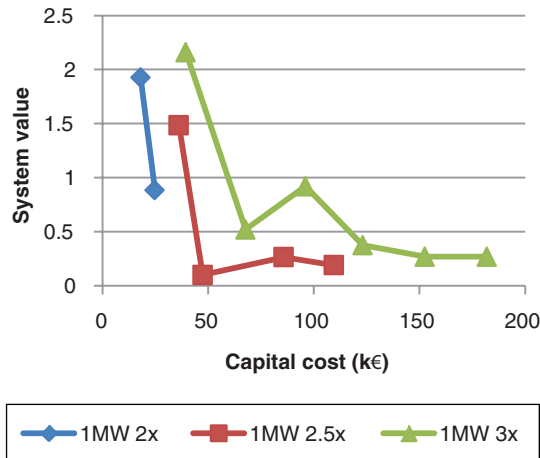


Figure 12: System value of distribution network with single supply location

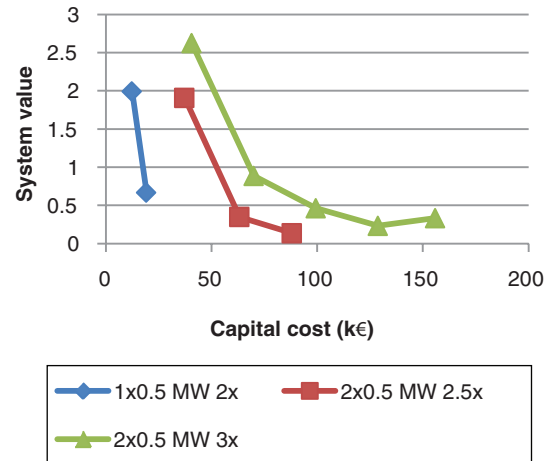


Figure 13: System value of distribution network with two supply locations

objective function is the sum of the operating and investment components listed in the previous five columns.

The second set of scenarios in Figure 11 shows the results with 0.5 MW boilers deployed in up to two locations. The second location makes it possible to supply a second cluster of loads without connecting through an intermediate area with a lower heat density, improving the economic performance of the heat network.

Further insight can be obtained from an analysis of the system value from incremental investments in the distribution network. The system value SV^k at each iteration k , calculated using the procedure described in section 3.4, is plotted against the corresponding capital expenditure VM_{capex}^k . Figure 12 shows the system values for the scenarios with a single supply location, while Figure 13 shows the system values with two supply locations. The plots for the higher tariffs in Figure 12 have a local maximum in the middle of the plot. At investment levels below this point, there is insufficient capital to construct a heat network from the supply location in the central area to the top right corner in the heat map. The intervening area contains low value connections where the heat revenues are insufficient to recover the added investment costs. These are included in the solution only at higher investment levels where the revenues from higher value connections from the top right corner can be used to offset the additional costs of building a network through this area. With two supply locations there is no need to bridge these low value locations and the system value plots show a more regular

shape. The high initial system values in both figures show that the screening procedure described at the beginning of section 4 identifies supply locations in areas with higher value connections.

4.2. Technology selection with combined heat and power generation

This section illustrates the use of the technology selection features of the model to optimise scenarios with combined heat and power generation and consumption. A base “heat only” scenario is defined in which the heat demand at all 500 nodes must be supplied by a heat network, with the total heat demand being approximately 0.8 MW. This is compared with two scenarios which include power generation. The second “heat and electricity” scenario has electricity demands at each node in addition to the heat demands specified in the “heat only” scenario. The electricity demands are specified as 65% of the heat demands, for a total of approximately 0.5 MW, and can be satisfied either by electricity imports from the grid or local power generation. The third “electricity exports” scenario has the same heat demands as the “heat only” scenario, no internal electricity demands, but electricity can be exported to the grid. All three scenarios are optimised with a 1 MW natural gas boiler, 0.135 MW_e/0.22 MW_{th} small CHP, 0.5 MW_e/0.675 MW_{th} medium CHP, and a 1.0 MW_e/1.03 MW_{th} large CHP available as potential supply choices. Single representative values of the electricity import and export prices were used here (see Appendix 2 for sources). This is due to the low temporal resolution of the combined

model used in this paper. A more detailed approach to estimating electricity market prices, reflecting seasonal and diurnal variations, may be used in models with a higher temporal resolution [30].

Figure 14 shows the technology types selected to supply the heat demands and electricity demands in the three scenarios. In the base “heat only” scenario a 1 MW natural gas boiler operating at 80% of its capacity is used to supply all the heat demands. In the second “heat and electricity” scenario, a medium CHP unit is selected to supply the bulk of the electricity demands (0.5 MW)

together with a small amount of imports, while heat is supplied to the heat network by the CHP and a non-domestic boiler. The medium CHP is selected as the supply technology as its capacity (0.5 MW_e) provides the closest match to the level of internal electricity demands, substituting for more expensive electricity imports while also supplying much of the heat demand. In the third “electricity exports” scenario a large CHP unit is selected to supply all of the heat demands, while the generated electricity is exported. In this case the operating level of the large CHP is curtailed below the 1 MW_{th} maximum because the internal heat demands only amount to 0.8 MW. The revenues from electricity exports reduce the overall operating costs for the heat network. These scenarios illustrate the capability of the model to select the technology type depending on the specific requirements and economic criteria.

The results in Figure 14 are for scenarios where all the heat loads must be satisfied. An analysis with optional connections is shown in Figure 15. The supply technology is a medium CHP unit and the heat tariff is 2.5 times the gas price. The length and heat density of the designed network are similar to those for a single 1 MW boiler with a 3x heat tariff (Figure 10, right). CHP operation is infeasible with a 2x heat tariff since the connected heat loads are below the minimum part load operating level for the CHP. No additional connections are found with a 3x heat tariff as the CHP is already operating at its maximum capacity.

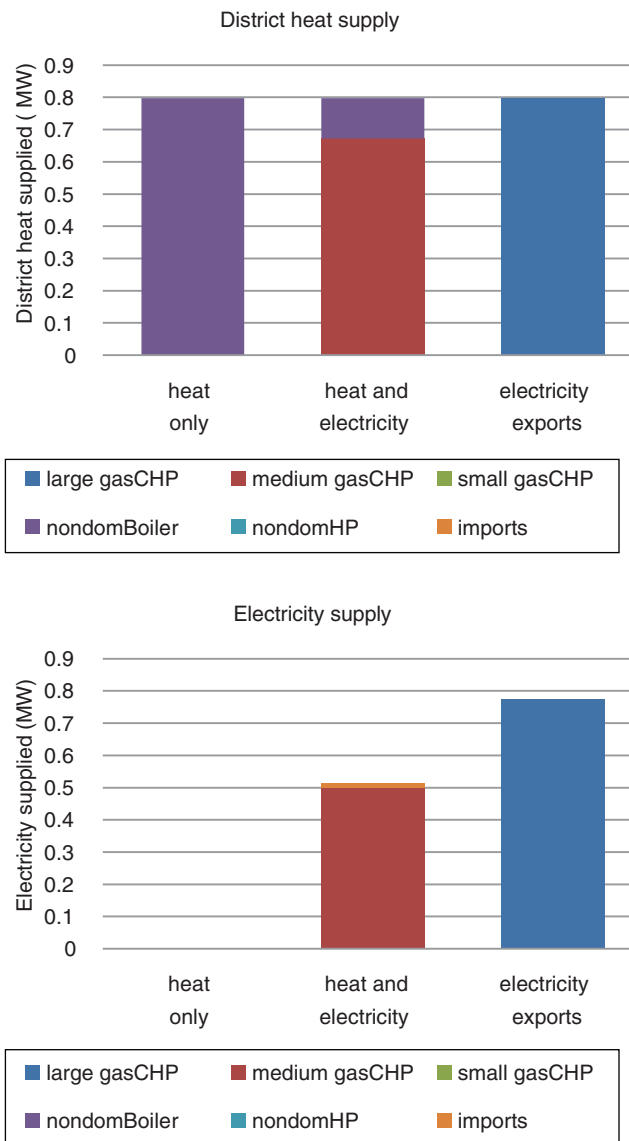


Figure 14: Technology selection for combined heat and power scenarios: Heat supply (top), electricity supply (bottom)



Figure 15: Selection of connections with medium CHP and electricity exports

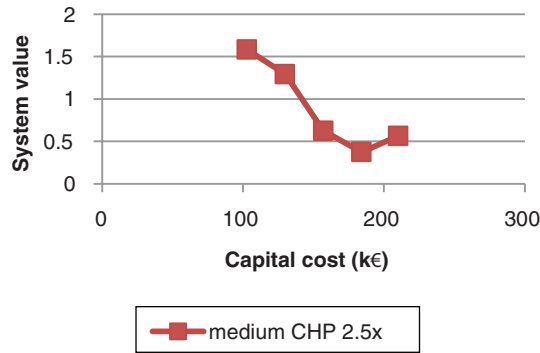


Figure 16: System value of distribution network with medium CHP

Figure 16 shows the system values for the distribution network with a single medium CHP. The range of the plot is bounded by the operational limits of the CHP and consequently does not show the same pattern as the system value plot for a single boiler (Figure 12). Due to the lower bound on part load CHP operation, the distribution network must be large enough to cover both the central and top right areas in the heat map. At investment levels below the values shown in Figure 16, the operation of the CHP would be infeasible due to insufficient demand.

4.3. Technology selection with emissions reduction target

The base scenario with 500 nodes supplied by a 1 MW boiler considered in the previous section produces greenhouse gas emissions of 1.428 kt per year. Figure 17 shows the results compared to scenarios with targets of 30% and 40% reduction in emissions. These scenarios include a 0.5 MW heat pump in the technology selection. The heat pump is selected in both emissions reduction scenarios, with a small CHP in the 30% reduction scenario, and a medium CHP in the 40% reduction scenario. The district heating is supplied by a combination of CHP, heat pump and boiler. The capital costs, operating costs and GHG emissions are shown in the table below. The heat pump COP was taken to be 2.897 and the emissions factor for natural gas was taken as 0.18416 kg/kWh (see Appendix 2 for sources). The heat pump COP value is for an ammonia based ground source heat pump with source temperature of 12 °C and sink temperature of 90 °C as reported in [35], based on the methods and tools described in [20]. A more detailed approach to modelling and optimisation of large-scale

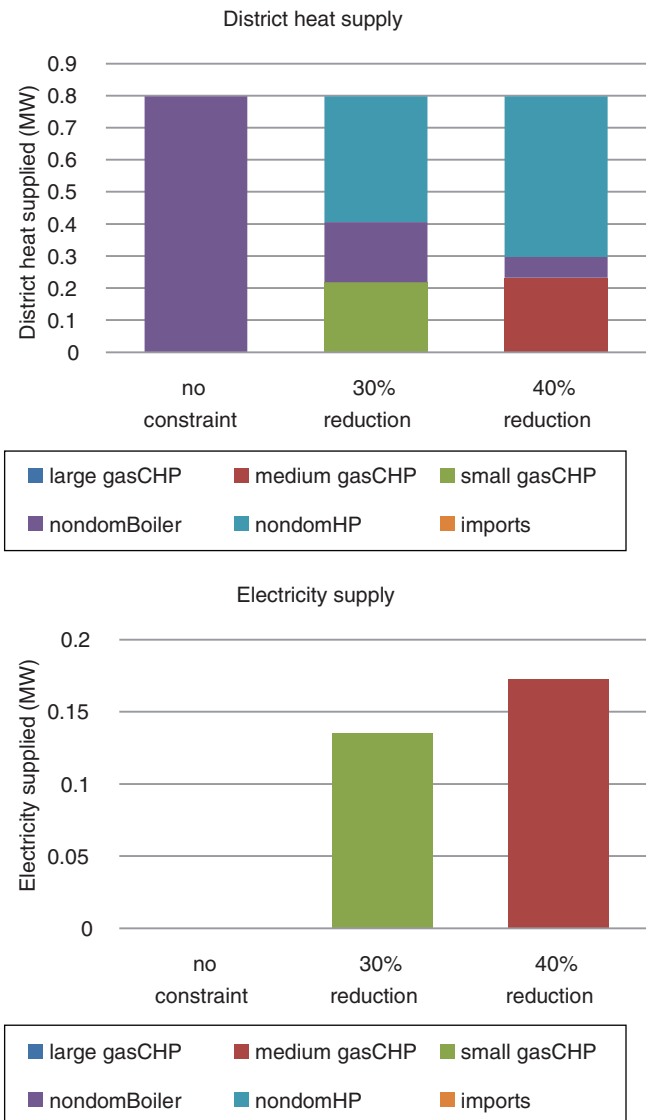


Figure 17: Technology selection with emissions reduction targets: Heat supply (top), electricity supply (bottom)

heat pumps in district heating considering variations in COP with temperature is given in [37].

4.4. Summary of results

To summarise, the scenarios in section 4.1 illustrates the interaction between the heat price and the economic viability of the heat network. Higher heat prices make it economical to expand the network to additional locations. With a single supply location the network has to be built through an area with unprofitable connections, whereas this area may be bypassed with two supply locations.

Table 3: Costs (k€) and emissions (kt/year).

Scenario	Capital cost	Operating cost	Total	Emissions
No constraint	229	232	462	1.428
30% reduction	313	202	515	0.975
40% reduction	345	218	564	0.850

Section 4.2 examines different scenarios involving combined heat and power generation. These show it is possible to obtain an economic benefit either by substituting local power generation for electricity imports, or by exporting to the grid. The scenarios in section 4.3 show that significant emissions reductions can be achieved by using combined heat and power generation and heat pumps. Overall, the results illustrate how the model can be used for both spatial planning and technology selection.

5. Discussion and conclusions

A combined spatial and technological model of district energy systems formulated as a mixed integer linear program (MILP) has been described in this paper. The model implements a unique combination of map-driven modelling, detailed optimisation of the distribution network, and selection of supply technologies (Figure 1). In contrast with empirical methods that are based on aggregate measures such as linear heat density [2,11], the decisions are based on a detailed optimisation of the capital, operating and environmental costs of supply technologies and individual connections within the heat network. The spatial framework for the model, which is similar to the graphical representation proposed by Bordin et al. [3], makes it possible to integrate the model within a map-driven application, and to identify subsets of buildings within a neighbourhood where it is economically viable to construct a network, and conversely to exclude locations where the heat revenues would be insufficient to recover the investment in the heat network. This paper further analyses the impact of supply locations and heat prices on the selected structure of the distribution network. The RTN representation, which has been applied in diverse infrastructure planning applications [22-26], makes it possible to evaluate multiple supply technology types including heat pumps, CHP and boilers, and to construct scenarios with combined heat and power generation. The model can be used with environmental objectives and constraints.

A series of test cases based on a screening dataset with 500 nodes have been presented to illustrate the main features of the model. Preliminary testing showed that considering all 500 nodes as potential supply locations resulted in very slow convergence towards an optimal solution and so a screening procedure was used to identify a limited set of supply locations for the test cases. The results for the test cases show that a mix of technology types, such as heat pumps and combined heat and power units, may be required to achieve emissions reduction targets, and that it is important to consider the interactions between heat and power supply on both environmental and economic indicators. The system value measure, which has been proposed as a method for analysing the impact of storage and renewable technologies in power systems [17, 18], has been adapted to quantify the impact of incremental investments in the heat network. This measure provides a means for visualising the overall effect of heat prices, supply technology type and location, and increasing investment levels on the economics of the heat distribution network.

An alternative implementation of the model described in this paper, written in Python using the Pyomo modelling language [40], has been integrated within a browser based application which is being tested by city partners within the Thermos project [41]. The prototype application includes spatial datasets compiled in collaboration with the city partners which can be used to construct the spatial framework required by the model. Further development of the prototype application and model is being undertaken in response to feedback from the city partners on the features and performance of the integrated application.

This paper outlines a broad conceptual framework for modelling district energy systems. Directions for future development include improving estimates of infrastructure and operational costs, and developing solution methods for larger problems. Currently, the cost and capacity of potential network links must be estimated beforehand and provided as inputs to the model. One alternative is to select from a range of discrete pipe sizes [19], but this could be computationally demanding if it is directly integrated within the overall system optimisation. Another alternative, which would be less computationally intensive, is to use cost estimates that include both a fixed component and a linearised variable component [37]. Similar functions, or piecewise linear functions, could also be used in place of discrete values

Table 4: List of scenarios for connection selection, combined heat and power generation, emissions reduction targets.

4.1	Scenarios with 1 MW boiler and varying heat tariffs (2.0x, 2.5x, 3.0x) Scenarios with 0.5 MW boilers in two locations and varying heat tariffs (2.0x, 2.5x, 3.0x) Scenarios for 1 MW and 0.5 MW boilers with increasing investment levels
4.2	Heat demands for all nodes (“heat only”) Heat demands and electricity demands for all nodes (“heat and electricity”) Heat demands for all nodes, electricity exports permitted (“electricity exports”) Medium CHP with heat tariffs (2.5x), electricity exports permitted
4.3	Heat demands for all nodes, no constraint on emissions Heat demands for all nodes, 30% reduction in emissions Heat demands for all nodes, 40% reduction in emissions

for technology sizes and costs [9]. The use of explicit diversity functions for connected heat loads within the model can lead to bi-linear terms involving the number of loads and heat flows in expressions for pipe capacities. Iterative methods for solving models with these expressions are being investigated.

Incorporating technologies such as solar thermal heating would require the use of a higher temporal resolution in the model to accurately represent the seasonal and diurnal variability in the heating supply. Time series aggregation methods based on clustering algorithms can be used to reduce the number of minor periods required to model the operation of the energy system. The granularity of the clustering can be adjusted within the optimisation algorithm so that the error introduced by this procedure is bounded [39]. The solution of larger problems for combined spatial and technological optimisation will require the use of specialised algorithms or approximate solution methods. One possibility is to decompose the overall problem into sub-problems for selecting the energy source and designing the distribution network, which can then be solved iteratively [23]. The spatial sub-model could be reformulated to facilitate the use of parallelised algorithms. Preliminary work has been carried out on developing an iterative procedure, which is inspired by genetic algorithms, for optimising large distribution networks. An initial solution is found by partitioning the original problem. This solution is improved with alternating expansion and refinement steps. The optimisation model described in this paper is used for each step, with different sets of required, optional or excluded nodes. Switching optimisation strategies from step to step accelerates the process of finding improved solutions.

Acknowledgments

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Appendix 1: Nomenclature

Sets and indices:

d	Subset of resources that require network construction
i	Cell index
j	Technology type
m	Metric (opex, capex, ghg, custom)
r	Resource
t	Minor period
tm	Major period

Parameters:

$\mu_{j,r}$	Conversion fraction for resource r by technology type j
Φ_t	Weighting (hours per major period) for minor period t
CAP_j	Maximum operating level for technology type j (MW)
$D_{r,i,t,tm}$	Demand for resource r in node i in period (t,tm) (MW)
$dist_{i,il}$	Distance between cell i and cell il (m)
$OBJWT_{m,tm}$	Objective function weight for metric m in major period tm
$Q_{max,r}$	Maximum transportation rate for resource r (MW)
$Tariff_{j,r,t,m}$	Tariff for resource r produced by technology j in period t (kGBP/MWh)
$VE_{r,m}$	Export metric for resource r (kGBP/MWh, t CO _{2e} /MWh)
$VI_{r,m}$	Import metric for resource r (kGBP/MWh, t CO _{2e} /MWh)
$VIJ_{j,m}$	Investment metric for technology type j (kGBP)
$VY_{d,m}$	Investment metric for network to transport resource d (kGBP/m)
$VYL_{d,i,il,m}$	Investment metric for link to transport resource d from i to il (kGBP)

Variables:

$EXP_{r,i,t,tm}$	Export of resource r from node i in period (t,tm)
$IM_{r,i,t,tm}$	Import of resource r into node i in period (t,tm)
$INV_{j,i,tm}$	Number of units of technology type j added in node i in major period tm
$N_{j,i,tm}$	Number of units of technology type j available in node i in major period tm
$P_{j,i,t,tm}$	Production rate of technology type j in node i in period (t,tm)
$Q_{r,i,il,t,tm}$	Flow of resource r from node i to node il in period (t,tm)
$RS_{r,i,t,tm}$	Surplus of resource r in node i in period (t,tm)
SAT_i	Demands in node i are satisfied
$Y_{d,i,il,tm}$	Network link to transport resource d from node i to node il exists in major period tm

Objective function expressions:

$TC_{m,tm}$	Transportation cost
$PC_{m,tm}$	Production cost
$IC_{m,tm}$	Import cost
$EC_{m,tm}$	Export cost
$TR_{m,tm}$	Tariffs
$EQ_{m,tm}$	Equipment cost
$NW_{m,tm}$	Network cost
$MC_{m,tm}$	Maintenance cost
$VM_{m,tm}$	Overall cost

Appendix 2: Sources for economic and environmental parameters

Table 5 Sources of parameter values

Parameter	Source
Network cost per unit length	Capital cost typical benchmarks normalised to non MWh metrics [31]
Import price of gas	Prices of fuels purchased by non-domestic consumers in the United Kingdom (including the Climate Change Levy), medium consumer, 2017 [32]
Import price of electricity	Industrial electricity prices in the EU for medium consumers (including environmental taxes and levies) [32]
Export price of electricity	Wholesale electricity prices [33]
Emissions factor for electricity	Methodology paper for emission factors, Base electricity generation emission factors [34]
Emissions factor for natural gas	Conversion factors 2017 - Condensed set, Gaseous fuels [34]
Investment cost for non-domestic boilers	Hypothesis used to model non domestic boilers [35]
Investment cost and operational parameters for CHPs	Hypothesis used to model CHPs [35]
Investment cost and operational parameters for heat pumps	Hypothesis used to model heat pumps [35]