Denne artikel er udgivet i det elektroniske tidsskrift
Artikler fra Trafikdage på Aalborg Universitet
(Proceedings from the Annual Transport Conference at Aalborg University)
ISSN 1603-9696
https://journals.aau.dk/index.php/td

Modelling choice of Time-of-Day in the Danish National Travel Model

Goran Vuk, travel demand modeller, MSc, PhD

1. Introduction

Modelling choice of Time-of-Day (ToD) is important when forecasting effects of congestion - both on roads and in Public Transport, and Road Pricing. That has been proven in COMPASS[1] and OTM models and now is included in the Danish National Travel Model, GMM4[2]. In this paper it is referred to the work related to the person demand modelling in GMM4, more specifically to the innovative work[3] related to the modelling of choice of ToD. The most important innovation is that, while we do not sample over modes and destinations, a sampling procedure is applied on the choice of ToD level.

GMM4 includes a Weekday (WD) model, an Overnight model and a model for International travel. In this paper we describe the WD model, which covers travel within the national border, for Monday to Friday, outside the summer holiday. The WD model is a tour-based model with segments for Home-Work (HW), Home-Education (HEdu), Home-Business (HBsn), Home-Leisure (HLsr), Home-Shopping (HShp) and Home-Escort (HEsc). Per definition, each tour starts and ends at home. We allow that a single stop is made on the tour, e.g. home-work-shopping-home. Two non-home based (NHB) trip model segments are included in the WD model, i.e. NHB leisure and NHB business models. The modes included are: Single Occupancy Vehicle (SOV), High Occupancy Vehicle (HOV) driver, Car Passenger (CP), Public Transport (PT), Bike and Walk. The SOV and HOV modes are further on split by vehicle type, i.e. conventional and electric vehicles.

Denmark is split in 3,670 zones (so-called 'Level 3 zonal system') in GMM4. All estimations in the WD model are based on the Danish National Travel Survey data, i.e. TU-data, for period 2006-2023. The model base year is 2023.

The main intention behind the completed estimations is to get a model that is:

- Robust: The model must be statistically sound – all the way from the overall model structure (i.e. relation of the ToD, Mode and Destination choices in a Hierarchical (Nested) structure) to the estimate of a single parameter. A number of tests (e.g. model elasticities) must ensure model's reliable

[1] Copenhagen Model for P erson Activity Scheduling and Simulations (COMPASS), an Activity-based model for the Greater Copenhagen.

[2] GMM stands for Green Mobility Model.

[3] All estimations were completed by Goran Vuk. The theoretical foundation of the ToD model was done by Andrew Daly (independent con sultant). James Fox (Jacobs UK) provided strong practical support throughout the project period

predictions by international standards.

- Policy Oriented: In model estimations the focus is also on the model application, e.g. impact of new infrastructure,
  Road Pricing, and new technologies (e.g. electric cars). The core of estimation efforts is on *time* and *cost* parameters via Cost Damping. GMM4 WD model's Cost Damping formulation includes VTTs that are unique for a) each tour, via tour's distance, and b) each person, via person's income.
- (Somewhat) Easy to understand: Beside the variables related to the LOS, the WD model includes a number of socio-economic variables that are found in the GMM4 Synthetic Population file. The model also includes the *Covid19* and *PFPT*[4] variables – both giving plausible estimates.

2. Cost Damping

In the GMM4 WD model, Cost Damping is implemented using Values of Travel Time, VTTs. Cost Damping refers to an estimation procedure that ensures that marginal importance of travel costs and time on person travel demand decreases with distance. In other Danish models, i.e. COMPASS and OTM, when Cost Damping implemented, we increased the model fit to the observed data. Importantly, the marginal utility of cost remains always negative in models with Cost Damping.

Omitting socio-economic variables and constants, a simple utility function for car travellers, for a given car segment and Origin-Destination pair, in GMM4 WD model can be written as:

$$\text{Util(Car)} = \beta_{GTCar} * (\text{Time} + \text{Cost} / \text{VTT(dist, inc)} + \text{CrossingCost} / \text{VTT(inc)}) \quad (1)$$

where: β_{GTCar} is the generalised time parameter for car, to be estimated
Time is the LOS car time in FF time units.
Cost is the LOS km-based driving costs
VTT(dist, inc) is the VTT as a function of both distance and income
CrossingCost is the bridge/ferry LOS costs
VTT(inc) is the VTT as a function of income, but not distance

Please note that the above formulation will include the part related to Road Pricing when using the model in forecasts.
For PT there is no specific bridge/ferry crossing cost and therefore the GMM4 PT utility is:

$$\text{Util(PT)} = \beta_{GTPT} * (\text{PT\_Time} + \text{PT\_fare} / \text{VTT(dist, inc)}) \quad (2)$$

where: β_{GTPT} is the generalised time parameter for PT, to be estimated
PT_Time is the LOS PT time, which includes both in-vehicle time and out-of-vehicle time components
PT_fare is the LOS PT travel costs
VTT(dist, inc) is the VTT as a function of both distance and income

In equations (1) and (2) VTTs are calculated, for each tour and person, as following:

$$\text{VTT} = \overline{\text{VTT}} \left(\frac{\text{Y}}{\bar{\text{Y}}}\right)^{\eta_{\text{Y}}} \left(\frac{\text{d}}{\bar{\text{d}}}\right)^{\eta_{\text{d}}} \quad (3)$$

where Y and d are the income of the traveller and the length of the trip (km) respectively
$\overline{VTT}$, $\bar{Y}$ and $\bar{d}$ are the mean values of value of travel time, income and distance
η_Y and η_d are the elasticities of VTT with respect to income and distance respectively

The average VTTs per mode are shown in Table 1, while the average Personal Income and Average Trip Distances, by TU data, are shown in Table 2. Income and Distance elasticities are presented in Table 3.

Table 1 – Average VTT values, 2023 values

Purpose	CD $\overline{VTT}$ (DKK/min)	CP $\overline{VTT}$ (DKK/min)	PT $\overline{VTT}$ (DKK/min)
Home-Work	1.40	1.36	1.36
Home-Education	1.29	1.29	1.29
Home-Business	4.13	3.80	3.80
Home-Leisure	1.29	1.29	1.29

[4] Primary Family Priority Time - PFPT

Home-Shopping	1.10	1.10	1.10
Home-Escort	1.23	1.23	1.23

Table 2 – Average Personal Income per mode and average Trip Distances per mode, by travel purpose, 2023 values

Purpose	Income (DKK) $\bar{Y}$, CD	Trip distance (km) $\bar{d}$
Home-Work	427.000 – CD 362.000 – CP 378.000 – PT	22.6 – CD 16.5 – CP 18.8 – PT
Home-Education	129.000 – CD 19.000 – CP 33.000 – PT	22.0 – CD 6.3 – CP 14.2 – PT
Home-Business	473.000 – CD 325.000 – CP 389.000 – PT	42.6 – CD 44.2 – CP 22.5 – PT
Home-Leisure	282.000 – CD 112.000 – CP 185.000 – PT	14.8 – CD 14.3 – CP 13.2 – PT
Home-Shopping	276.000 – CD 143.000 – CP 140.000 – PT	7.6 – CD 10.1 – CP 8.0 – PT
Home-Escort	311.000 – CD 144.000 – CP 189.000 – PT	8.7 – CD 14.2 – CP 10.0 – PT

Table 3 – VTT Income and Distance elasticity estimates per travel purposes

Purpose	Income elasticity η_Y	Distance elasticity η_d
HW and HEdu	0.181	0.157
HBsn	0.117	0.339
HLsr, HShp and HEsc	0.186	0.158

Splitting the cost components and associated VTT functions in equations (1) and (2) is deliberate. For per km-costs it ensures that the cost damping effect ensures that the marginal impact of a 1 kr. increase to a long tour is lower than the marginal impact of a 1 kr. increase to a short tour. However, the impact of the crossing costs for a given income group is constant, i.e. it does not vary according to the trip distance. For example, if a new crossing opens that significantly reduces trip distances for some movements, then the impact of the new crossing cost is not impacted by that reduction in trip distance. This arrangement deals with the problem of inappropriate utility variation for tours with high fixed costs.

In equations (1) and (2) only the *GenTime* parameter for Car and PT is estimated. Thus, it is possible to make different transformations of the generalised cost, such as using both *log* and *linear terms*. This results in damping of both the time and cost components, while the use of a trip-length-dependent VTT function gives additional damping for the cost component.
To test for generalised time damping, generalised time was calculated using equations (1) and (2). For each linear generalised time term a second log generalised time term is tested. Taking Car mode as an example the utility function, in the context of Cost&Time damping, is:

$$U(car) = \beta_{GTCar} * GenTime(car) + \beta_{LogGTCar} * \log(GenTime(car)) \quad (4)$$

An equivalent Gamma model has the following formulation:

$$U(car) = \beta_{GmGTCar} * [\ \gamma * GenTime(car) + (1 - \gamma) * (\frac{E\_GenTm}{E\_LnGenTm}) * \log(GenTime(car))] \quad (5)$$

where: γ is a constant between 0 and 1,
E_GenTime is mean value for generalized time for given purpose, and

E_LnGenTime is mean value for log generalized time

Note that the $\frac{\text{E\_GenTm}}{\text{E\_LnGenTm}}$ ratio is a constant introduced for the scaling purposes when estimating $\beta_{GmGTCar}$. The constant ensures that, at their average values, linear and log generalized times have the same scale.

While we estimate freely GenTime parameters in Cost&Time damping models (as described by equation (4)), in the comparable Gamma model we determine the weights applied to the linear and Log-linear parts of equation (5), i.e. gamma is constant across modes, and this makes the Gamma model a reduced form of the log and linear cost model. When γ=1 we get a fully linear model while when γ=0 we get a fully logarithmic model. In the same manner, if we decide that, say, γ=0.8, then 80% is of the weight is given to the linear part and 20% to the log part of equation (5).

Note that the Gamma formulation model estimates only the βGmGTCar parameter in equation (5), while two GenTime parameters are estimated in equation (4), i.e. βGTCar and βLogGTCar parameters. When all else is equal, this means that the Gamma models converge faster than the Cost&Time damping models with free GenTime parameters, because of the correlations between the log and linear variables. Note also that the gamma formulation as applied here is a more restricted model form because γ is the same for each mode. That is in contrast with the Cost&Time damping specification where the relative contribution of linear and log terms varies between modes. The γ values, for each tour purpose, were decided based on detailed elasticity tests.

3. Theory behind the ToD modelling in GMM4 WD HW model

3.1. Introduction

To the rest of the paper we refer to the GMM4 Weekday (WD) model, for Home-Work (HW) purpose, that simultaneously estimate choice of Mode (M), Destination (D), and Time-of-Day (ToD) – in short HW ToD model.

In practical work to forecast travel demand over large areas, the models sometimes operate with large choice sets, perhaps several tens of thousands of alternatives. Such large choice sets raise difficulties in the run times necessary to estimate the models and potentially in the hardware and software requirements for the computer systems used. In a number of practical model systems, therefore, use has been made of sampling of alternatives to reduce the burden.

Sampling of alternatives for model estimation was introduced by McFadden [5] who set out conditions for unbiased estimation when sampling alternatives for multinomial logit (MNL) models. The fact that this theory applied only for MNL models was for several decades a severe limitation on the use of sampling in this context. However, the work of Guevara and Ben-Akiva[6] (subsequently referred to as GBA) removes the limitation in theory for the GEV family of models [7] and in other papers for mixed logit and random regret models. This theory is summarised in the latest authoritative overview by Bierlaire and Krueger[8], which indicates that no further theoretical advances have been made. The important advance given by GBA beyond McFadden's work is however obtained at the expense of introducing considerable theoretical complication. The issue of complexity was addressed by Daly, Hess and Dekker[9] (subsequently

[5] McFadden, D.L. (1978) Modelling the choice of residential location, in Karlqvist, A., Lundqvist, L., Snickars, F. and Weibull, J. (eds.) Spatial interaction theory and residential location, North-Holland, pp. 75-96

[6] GBA: Guevara, C.A. and Ben-Akiva, M. (2013) Sampling of alternatives in multivariate extreme value (MEV) models, Transportation Research Part B, 48, pp. 31–52

[7] GBA refer, perhaps more correctly, to 'MEV' models; however, the 'GEV' naming is more commonly used. This family includes MNL, tree-nested, cross-nested and other more exotic models.

8 Bierlaire, M. and Krueger, R. (2024) Sampling and discrete choice, in Hess, S. and Daly, A. (eds.) Handbook of Choice Modelling (second edition, June 2024), Edward Elgar, pp. 693-719.

[9] DHD, Daly, A., Hess, S. & Dekker, T. (2014) Practical solutions for sampling alternatives in large scale models, Transportation Research Record 2429

DHD) who specified simpler but still not straightforward procedures for the two-level tree-nested logit models often used in practical transport applications.

In this series of papers (McFadden, GBA and DHD), sampling is considered on the basis of the 'importance' of each of the alternatives. By giving more weight to alternatives that are more likely to be chosen, without excluding entirely the less-relevant alternatives, a very large improvement in estimation accuracy can be obtained relative to uniform sampling, where each alternative has the same sampling probability. It is notable that the key research papers on sampling alternatives, McFadden and GBA, use the example of residential choice, where the differences in choice probability are less than in travel demand forecasting.

3.2. Overview of theoretical basis

The two key equations on which the estimation of GEV models with sampled alternatives are based are, first, the result from Ben-Akiva and Lerman[10] that *any* GEV model can be written as a 'corrected' MNL, where the utility function of alternative i is adjusted by adding the term:

$$\log G_i(C) \tag{6}$$

where G_i is the derivative of the GEV function underlying the model with respect to its ith argument

C is the full choice set

The second is the result from McFadden that an MNL based on a random importance-based sample of alternatives D can be used to make unbiased estimates of the model over the full set of alternatives, provided the utility function of alternative i is adjusted by adding the term:

$$\log \pi(D|i) \tag{7}$$

where π gives the sampling probability for the set D, given that i is the chosen alternative. The sampling probability π varies systematically over the alternatives to reflect the importance of each alternative. Note that the chosen alternative *must* be included in D.

The breakthrough given by GBA is that these two older results can be combined to estimate any GEV model with sampled alternatives by adding *both* corrections to the utility functions of an MNL and then that the first correction can be calculated as $\log G_i(D^*)$, where D^* is a reduced choice set, which may be the same as D or sampled on some other basis. The GBA paper gives proof of this assertion and states the conditions that must apply to D^*. GBA goes on to give further properties and illustrations, including that the 'robust' measures of error must be used for the estimates.

However, for practical application a number of further steps need to be taken. In particular, how should we (a) specify the sets $D^*$and D, (b) calculate G_i for a given model and (c) determine the sampling procedure and the sampling probability π. These are the topics addressed by DHD in the context of a two-level tree-nested logit model; for the GMM application the G_i calculation must be extended to three or more levels, as it is necessary to test the nesting for time period choice and nesting among the modes, as well as the standard nesting of modes and destinations.

First, another issue has to be raised. In all cases we are aware of, alternative sampling has applied to zones, for destination choice or residential choice. However, in the GMM4 work, it seems that sampling time period combinations may be preferable. This issue is discussed in the following section

3.3. Sampling destinations and time periods

In GMM4 we sample time period combinations (TPC), rather than destinations. This proposal is based on two considerations.

[10] Ben-Akiva, M., Lerman, S. (1985) Discrete Choice Analysis, Theory and Application to Travel Demand, MIT Press, Cambridge, MA.

First, sampling by time periods is simpler than sampling by destination. It seems reasonable to apply constant sample probabilities across all the origins, i.e. assuming that differences by origin in aggregate time period choice for each purpose are negligible. These sample probabilities can be obtained by using the aggregate shares of time-period combinations. In contrast, when sampling by destination, the sampling must be based on attraction variables and separation, for which functions must be created for each purpose. Sampling rates must be calculated separately for each origin.

Second and more important, GMM4 must have a sharp focus on trip length, as kilometrage elasticity and water crossings receive close attention. The relevant variables in the model, i.e. *times* and *costs*, vary much more across destinations than across time periods, and the accurate estimation of parameters for these variables is helped by retaining as much variation across destinations as possible. In contrast, there is only one key parameter for time period choice, the structural parameter defining the scale of time period choices, and we expect this parameter can be estimated adequately from a sample of time period combinations.

For GMM4 we developed tree-nested models for three interacting choices made by travellers: mode M, destination D and out-and-return time period combination T. Considering the possibility that models can be developed assuming equal variance for two or three of the choices, there are 13 potential structures that need to be considered.[11] Given that sampling is done only in the T dimension, these 13 structures can be classified into three groups, as follows.

- 'Easy', 6 structures: T>D>M, T>M>D, T>M=D, T=M>D, T=D>M, T=M=D. The characteristic of these structures is that T is at the highest level, where the model is effectively MNL. In consequence, the logsums are not affected by sampling and no correction G_i is needed.
- 'Intermediate', 5 structures: M=D>T, M>D=T, D>M=T, M>T>D, D>T>M. Here T is one level below the highest and the approaches of GBA and DHD (who apply 2-level tree-nested logit) could be used as examples.
- 'Difficult', 2 structures: M>D>T, D>M>T. T is two levels below the root of the model, so that the approximation of sampling for T propagates through the intermediate level. While we would not expect to be able to identify three separate levels in the model, we might like to test these to indicate which of the simpler structures we should be using.

The question is then what the correction G_i ought to be, in the intermediate and difficult cases. GBA indicate a procedure (or rather a series of possible procedures) for 2-level tree-nested logit, but there are serious complications. DHD simplify the procedure somewhat and convert it to the RU1 parametrisation of tree-nested logit. which is required for working with ALOGIT. However, although the DHD approach could be used in principle, in practice they offer two alternatives, one iterative and one involving an additional level of nesting, neither of which are really practical in the GMM4 context.

A further issue is that it is necessary to perform the model estimation using ALOGIT, which makes the use of a different choice set for the logsum calculation difficult (though not impossible). In the context of GMM4 estimation, we need to avoid these complications and use the same set of sampled TPC throughout the model.

In conclusion, it is not recommended that the GBA correction term G_i should be used. To test models not in the 'easy' category (where the correction is not needed), tests should be made with different sample sizes to explore the extent of any bias. In practice, by experience it is suggested that T>M>D and its simplifications T=M>D, T>M=D and T=M=D are the most likely structures. Testing T>M>D should indicate what the appropriate structures should be.

3.4. GMM4 sampling approach

It was decided to implement alternative sampling in the GMM4 estimations to avoid very long run times and potential memory issues. The latest theory (GBA) indicates that two corrections are needed to achieve an unbiased estimation. One is the McFadden correction $\pi(D|i)$ which corrects for the sampling itself, the other Gi adjusts for the fact that the model

[11] For this note it is assumed that further nesting, e.g. within the modes, can be ignored.

is not MNL. However, calculating Gi without using all of the destinations (which would defeat the purpose of sampling) is challenging.

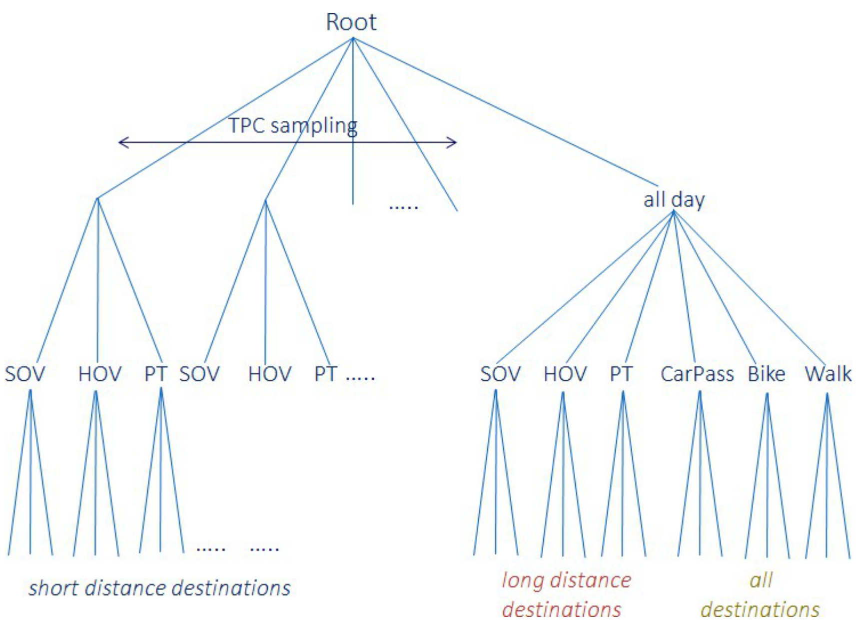
The sampling is done on the basis of time period combinations, rather than on destinations as in several previous studies. Sampling TPC is simpler than sampling destinations and focusses the estimation effort more accurately on the issues of interest in the model.

The calculation methods proposed by GBA and DHD for the correction Gi are complicated and not practical for the GMM4 context. The use of ALOGIT means that the same set must be used for calculating logsums and model probabilities (without further complication) and GBA indicate that this implies an iterative procedure, which is also not practical for GMM4. However, a number of model structures can be estimated without any such issues. Tests should be made with larger sample sizes to determine whether bias is present if other model structures are used.

It was decided to use replacement sampling for TPC which allows a simple calculation of the correction factor $\pi(D|i)$.

A tree structure can be set up to represent the T>M>D structure, which can also be used for simplifications of that structure – see Figure 1. This structure contains an all-day TPC which deals with the issue that TPC choice will not be modelled for modes and destination associated with long-distance tours. An ASC for the all-day TPC should be included as part of the model estimation.

Figure 1 – GMM4 WD model Tree Structure



Three modes are included in the ToD modelling, i.e. SOV, HOV driver, and PT. ToD is modelled for the tours of the maximum distance of 200 km. The ToD modelling is based on 11 Time Period (TP) choices in the TU data: 03:00-04:59 (UM_a), 05:00-05:59 (UM2), 06:00-06:59 (MM3), 07:00-07:59 (MM), 08:00-08:59 (MM2), 09:00-14:59 (UM3), 15:00-15:59 (EM), 16:00-16:59 (EM2), 17:00-17:59 (EM3), 18:00-20:59 (UM4) and 21:00-02:59 (UM_b). In the LOS files the TP 03:00-04:59 and 21:00-02:59 are presented in one file.

4. Model estimations and base elasticities

4.1. Introduction

Due to the sampling procedure applied in GMM4 we succeeded to reduce the number of alternatives to about 90.000. In running time, that allowed us to make, roughly, one estimation per 8 hours. For the HW tour purpose, we observed the TP choices[12] as shown in Table 4.

Table 4 – TP share for the HW tours

	UM_a	UM2	MM3	MM	MM2	UM3	EM	EM2	EM3	UM4	UM_b	total
UM_a	0.0098	0.0131	0.0131	0.0427	0.0525	0.9780	0.2658	0.1936	0.1313	0.1608	0.0394	1.90
UM2	0	0.0033	0.0230	0.0263	0.0558	3.2457	1.4637	0.8894	0.4660	0.5448	0.1542	6.87
MM3	0	0	0.0263	0.0459	0.0689	5.0966	8.9528	4.9129	2.0675	1.6048	0.3873	**23.16**
MM	0	0	0	0.0558	0.1050	6.2518	7.8763	12.3987	5.4970	3.3803	0.8040	**36.37**
MM2	0	0	0	0	0.0328	2.4614	1.9986	3.9250	3.5214	2.2743	0.5382	**14.75**
UM3	0	0	0	0	0	2.2973	1.2077	1.7590	1.8280	2.7863	3.4722	13.35
EM	0	0	0	0	0	0	0.0295	0.0853	0.0820	0.2986	0.7122	1.21
EM2	0	0	0	0	0	0	0	0.0558	0.1017	0.2888	0.4037	0.85
EM3	0	0	0	0	0	0	0	0	0.0328	0.1707	0.3577	0.56
UM4	0	0	0	0	0	0	0	0	0	0.2264	0.5251	0.75
UM_b	0	0	0	0	0	0	0	0	0	0	0.2199	0.22
total	0.01	0.02	0.07	0.17	0.32	**20.33**	**21.79**	**24.22**	13.73	11.74	7.61	100

Most of the 2006-2023 TU data HW tours have the midpoint of the outbound trip in MM3, MM and MM2 (i.e. between 06:00 and 08:59), while most of the homebound trips happen in EM and EM2 (i.e. between 15:00 and 16:59). There are also many homeward HW trips happening between the two peaks, most likely close to 16:00.
As many as 12.4% of the TUs HW tours start in MM (i.e. 07:00-07:59) and end in EM2 (i.e. 16:00-16:59).

4.2. Model estimations

In the TU data, for the HW segment, we have potentially 66 shares of 11x11 time periods (TP). In Model v14 (see table 5) we have estimated 63 TPC constants. They are applied to all three modes involved in the ToD choice, i.e. SOV, HOV and PT. The base constant is *S_5_42*, defining the TP choice of Outward trip of the HW tour being in between 8:00 and 9:00 AM, while the Return trip of the HW tours is in between 4:00 and 5:00 PM. Those 2 TPC constants that are related to unchosen combinations, or combinations that are hard to estimate, are set to -20. In this model we also estimate structural parameters for the T>M>D structure. Note that the structure clearly indicates that TPC needs to be at the highest level of the structure, supporting the decision to sample alternatives on the basis of TPC.

In the GMM4 WD Technical Report we stated that: *In the COMPASS model there is a sub-model, i.e. Primary Family Priority Time model (PFPT)[13], that explains how Danes put priority on Family Quality Time, which, in return, impacts their activities/travel during the day – both in scheduling, duration, use of car, etc. In Phase 2 we will look at interactions of time-period choices with what we know about the family. For example, young parents with children might come home earlier from work.* For many good reasons we couldn't test fully the concept of PFPT in a tour-based model, such as the GMM model. The GMM4 version of the *PFPT variable* includes:

1. HHsize>=2 and AdultsSize>=2 and ChildrenSize>=1 and PersonAge between 20 and 40.
2. Home-Work Trip of the HW Tour is in TP4 (07:00-08:00) or TP5 (08:00-09:00),
   while the Work-Home trip of the HW Tour is in TP7 (15:00-16:00) or TP8 (16:00-17:00).

[12] It is the midpoint of the TU tour that decided to which of the 11 TPC a Home-Work or Work-Home trip belongs to.
[13] Goran Vuk, John L. Bowman, Andrew Daly and Stephane Hess. "Impact of family in-home quality time on person travel demand". Transportation. Volume 43. Issue 4. Page 705-724. Print ISSN 0049-4488. Online ISSN 1572-9435. DOI 10.1007s11116-015-9613-2. July 2016.

In Model v14, the estimated parameter *GMMpfpt* is positive (as expected), and significant in t-value, i.e. +0.3934, t==+8.8.
The parameters' description is also given in the table below.

Table 5 – HW ToD model, v14

```
File                  P3_HWtod_v14_GV.F12
Title              HW TPC-M-D choice model
Converged                            True
Observations                        40392
Final log (L)                   -309034.5
D.O.F.                                 93
Rho²(0)                             0.273
Rho²(c)                             0.116
Estimated                       23 Oct 25
________________________________________

GmGenTmSOV          -0.06145 (-205.8  - Generalised time parameter for SOV
GmGenTmHOV          -0.06695 (-121.3  - Generalised time parameter for HOV driver
GmGenTmCP           -0.09323 (-81.7)  - Generalised time parameter for CP
GmGenTmPT           -0.07377 (-74.6)  - Generalised time parameter for PT

tmWalk              -0.05443 (-25.2) - Travel time for Walk
tmBike              -0.05303 (-80.4) - Travel time for Bike

intraB                0.3170  ( 7.3) - Intrazonal travel for Bike
intraW                0.5702  ( 6.7) - Intrazonal travel for Walk
TwoCarsS               1.993  (43.3)
TwoCarsH               1.591  (29.5)
MLcar                 0.4690  (12.0) - Male using car
MLwalk                0.1595  ( 1.8) - Male walking
MLbike                0.1184  ( 2.5) - Male using bike
HHchd                -0.2058  (-2.3) - Presence of children in the HH, impact on CP mode
HHwkids                1.376  (34.1) - Presence of children in the HH, impact on CD mode
CPcrown               0.9352  ( 6.8) - CP is more likely when HH owns a car
PTWdist             -0.01063 (-21.4) - Part time workers are less likely to travel long
OnePrsHH               1.417  (24.4) - One person HH, impact on CP mode
TrainDist            -0.1176  (-7.7) - Distance to the nearest Train station
GMMpfpt               0.3934  ( 8.8) - Impact of Primary Family Priority Time on ToD choice for young families
covidPT              -0.3326  (-2.2) - negative impact of COVID on using PT

Allday                 3.697  (39.6) - an All-day constant (see section on Theory)
ascSOVel             -0.1887  (-1.7) - ASC
ascHOVel              -1.910 (-12.8) - ASC
ascHOVcn              -1.807 (-29.6) - ASC
ascCPas               0.4299  ( 2.7) - ASC
ascPT                  1.737  (15.3) - ASC
ascW                 -0.1723  (-1.2) - ASC
ascB                  0.5078  ( 4.4) - ASC

S_1_1 throughout S_11_66   - TPC constants, i.e. 66 time period constants that replicate the share presented in Table 4

TotEmp                 1.000     (*) - attraction variable for working places
TR_M_D                 1.000     (*) - a structural parameter proving that M>D
TR_TPC_M              0.6732  (69.6) - a structural parameter proving that T>M
```

4.3. Base elasticities for Car mode

Based on Model v14, we have calculated Travel costs and Travel time, Tour and Km elasticities, for Car (i.e. the sum of SOV and HOV, over short and long tours).

Those elasticities, where appropriate, are compared to the expected ranges of elasticities, as recommended by Significance[14]. The recommended elasticities are not purpose specific values, but rather aggregate values across travel purposes. Importantly, the recommended ranges cover 85% of the reported model runs, i.e. there might be reasons why GMM4 elasticities could get different elasticities.

Car km-cost

The Car LOS include two types of costs, i.e. driving costs and ferry/bridge costs. To calculate Km-cost elasticities, the driving costs for SOVshort, HOVshort, SOVlong, HOVlong and Car Passenger modes are increased by 10%.

The model predicted Tour elasticity is -0.125, a value in the lower part of the recommended interval of -0.1 to -0.3. The impact of choice of time-of-day part of the model is that the overall elasticity decreases relative to the comparable Mode-Destination model. That is because when driving costs increase over all TP, then it is expected that some car tours shift from one TP to another, while the mode and destination remain.

The model predicted Km elasticity is -0.338, a value that fits nicely into the recommended interval of -0.25 to -0.6.

The PT tour elasticity is +0.267 while the PT km elasticity is +0.349.

Table 6 – Km-cost Tour and Kms elasticities

	HW ToD	*Recommended values, all purposes*
Tour	-0.1253	*-0.1 to -0.3*
Kms	-0.3385	*-0.25 to -0.6*

Table 7 shows Km-cost Tours and Kms elasticities for two comparable models, i.e. models with and without choice of ToD. As expected, the elasticities in the model without choice of ToD are higher, i.e. when driving costs increase then some car drivers would shift ToD before shifting mode. Second, the effect of choice of ToD in elasticities is about 28% in the Tour elasticities while 33% in the Kms elasticities. That is to say that the effect of choice of ToD is larger on longer distances, as expected.

Table 7 – Km-cost Tour and Kms elasticities, HW ToD model vs. HW MD model

	HW ToD model	HW MD model
Tour	-0. 1253	-0.1749
Kms	-0.3385	-0.5032

Car travel time

The Car LOS is the total OD travel times. To calculate car travel time elasticities, the total travel time for SOVshort, HOVshort, SOVlong, HOVlong and Car Passenger modes are increased by 10%.

The model predicted Tour elasticity is -0.154, a value in the lower part of the recommended interval of -0.15 to -0.4. As expected, the Car travel time Tour elasticity is higher than the Car km-cost Tour elasticity, i.e. increase in congestion time (which is the part of the total OD travel time) has a large impact on predicted changes.

The model predicted Km elasticity is -0.280. This value is on a low side of the recommended elasticity range.

The PT tour elasticity is +0.406 while the PT km elasticity is +0.512, i.e. car travel time increase ends in a higher shift to PT than the increase in car km-costs does.

[14] 2023 demand elasticities, VD report done by Significance (The Netherlands) and Andrew Daly (independent consultant)

Table 8 – Travel time Tour and Kms elasticities

	HW ToD model	Recommended values all purposes
Tour	-0.1540	-0.15 to -0.4
Kms	-0.2801	-0.3 to -1.1

Distance response

Table 9 shows Distance response in number of tours when Km-cost change for six distance bands, i.e. up to 20km, 20 - 50km, 50-100km, 100-200km, 200-300km and above 300km. Two things appear in the table:

1. Elasticities increase with distances, i.e. when increasing km-costs or travel time then the car tours over longer distances move to tours with shorter distance.
2. Modal shift away from car (except for the 0-20km band in the km-cost test.

Table 9 – HW Tour Km-cost Distance elasticities

	0-20 km	20-50 km	50-100 km	100-200 km	200-300 km	300+ km
Km-cost	+0.06	-0.05	-0.23	-0.48	-0.61	-1.00

5. Some conclusions

The estimation work on the GMM4 WD model is most detailed at two aspects, i.e. Cost Damping and choice of Time-of-Day (ToD) modelling. The sampling procedure applied in the ToD modelling makes this model a state-of-art model withing the tour-based person demand modelling.

Two other novelties are introduction of the COVID19 variable in the PT mode (actually, the COVID19 variable is also included in the GMM4 Frequency models[15]), and introduction of the PFPT variable within the ToD modelling for families with small children.

Finally, the model is fully ready for use when testing Road Pricing – a policy planning tool that is presently very actual in Denmark.

Documents

GMM4 Weekday Person Demand Model; Technical Report on Phase 1 Estimations. Vejdirektoratet, August 15th, 2024.

GMM4 Weekday Person Demand Model; Technical Report on Phase 2 Estimations. Vejdirektoratet, June 30th, 2025.

GMM4 Weekday Person Demand Model; Technical Report on Phase 3 Estimations. Vejdirektoratet, February 25th, 2026.

[15] For instance, under COVID less HW tours are generated per weekday per person.