

MANAGEMENT OF UNCERTAINTY IN THE SAFETY EVALUATION OF INDIVIDUAL RISK

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ABSTRACT

Risk models used to evaluate whether a transport activity operates at a tolerable safety level or not often contain uncertain elements. An uncertain element could be the values of model parameters which are estimated on the basis of limited accident data. In order to make a more balanced safety evaluation, uncertain elements in the used risk model ought to be taken into account. This paper address how uncertainty in the accident frequency and accident outcome can be accounted for in the evaluation of individual risk posed by a public transport activity. The accident outcome is in the paper represented by the mean number of fatalities per fatal accident. The uncertainty in the parameters are represented by Bayesian probability distributions. It is analysed how uncertainty in the fatal accident frequency and the mean number of fatalities may affect the decision whether the individual risk posed by the transport activity is tolerable or not.

1. INTRODUCTION

Transport of people and goods is liable to disruption in the form of accidents with consequences of varying severity. Such consequences could be fatalities, injuries, pollution, large delays and other socio-economic costs. The frequencies and consequences of the transport accidents can be interpreted as the total *risk* which the transport activity imposes on the society. It is in society's interest that this total risk is kept at a tolerable level. There is therefore a need for methods and tools with which to analyse the risk posed on society by the transport activity.

Decisions on risk tolerability and how many safety measures to implement may in some cases be affected by the uncertainties inherent in the tools and methods used for the risk analysis. It could therefore be argued that the effects of uncertainty in any risk analysis should be elucidated to the extent possible before any final decisions are made. This would enable the safety managers (including relevant safety regulatory authorities) to make more informed and balanced decisions. Uncertainty in risk analysis can be divided into three categories (Baybutt(1989): 1) uncertainties in parameter values used (parameter uncertainty stems usually from a lack of a sufficient amount of relevant accident data); 2) uncertainties in modelling (for example deficiencies of the models in representing reality); and 3) uncertainties in the degree of completeness, that is, the analyst's inability to identify exhaustively all contributions to risk.

Compared to road safety relatively little research has been done into risk and uncertainty in public transport activities in which accidents with a large number of fatalities may occur. This could be due to the relative small number of accidents that happen in public transport.

The purpose of this paper is to show how parameter uncertainty can be taken into account in decisions on risk tolerability in public transport with multi-fatality accidents. Such a public transport activity could be transport of passengers by rail. It is also analysed how parameter uncertainty may affect which of the risk model parameters to improve, in order to enhance the tolerability decision, when constraints on resources makes it impossible to improve all the model parameters. Uncertainty in modelling and uncertainty in completeness are also important issues in safety management of public transport, but these two topics are not dealt with in this paper.

HSE(1988) recommends two criteria to decide whether transport risk is tolerable or not. One is concerned with risks to individuals and is labelled *individual risk*. Individual risk usually expresses the probability for an individual of being killed in an accident per unit time. The individual represents a specific group of people for example passengers. The other risk criteria is *societal risk* and is concerned with the size distribution of the accidents (where size is typically measured in number of fatalities). Societal risk is often used to express the safety managers' averseness towards multi-fatality accidents. Only individual risk is considered in the analysis in this paper. Furthermore, the accidents consequences are in this paper only concerned with fatalities. Fatalities are usually considered one of the worst outcomes of accidents, so restricting the consequences to fatalities only is not unreasonable.

After this introduction the paper proceeds with section 2 where it is discussed how uncertainty can be

taken into account when deciding whether the transport activity is tolerable or not. On the basis of this discussion an individual risk model is set up in section 3. This model is used for the analysis in section 4 and 5. Section 4 describes how uncertainty affects the tolerability decision. Section 5 describes how the safety managers can assess which model parameter to improve in an attempt to enhance the tolerability decision.

2. DECISION RULE ON RISK TOLERABILITY UNDER UNCERTAINTY

The ideas of tolerable and intolerable risk levels are based on the premise that, although some levels of risk are inevitable, there should be an upper limit to tolerable risk levels - here called the just tolerable risk r_t . If the individual risk, R , posed by the transport activity is above r_t , the risk must be reduced by implementing safety measures (in HSE(1988) the just tolerable individual risk for employers is 0,001 per year, and 0,0001 per year for third parties).

If the expected individual risk for the transport activity is known for certain it would be straight forward to decide whether the activity is tolerable or not. It would only require a comparison of the known individual risk with the just tolerable risk. Under uncertainty there is no guarantee that the estimated individual risk equals the true risk level posed by the transport activity, thus complicating the evaluation. Under uncertainty the safety managers may make one of two wrong decisions with costs (in a general sense) to society as an end result. The two wrong decisions are:

- 1) The activity is assessed intolerable, but in reality it is tolerable and;
- 2) The activity is assessed tolerable, but in reality it is intolerable.

If the safety managers have reasons to be concerned for the possibility of making a wrong decision the question arises how this concern can be taken into account in the tolerability assessment. If the distribution, $p(r)$, of the individual risk is known then one way of addressing the concern would be to use the decision rule: judge the system tolerable if a given percentile of the risk distribution is less than or equal to the just tolerable individual risk, r_t , and intolerable otherwise. Expressed mathematically:

$$\text{Tolerabel if: } \int_0^{r_t} p(r)dr \geq \mathbf{a} \quad \text{and} \quad \text{Intolerable if: } \int_0^{r_t} p(r)dr < \mathbf{a} \quad (1)$$

The size of \mathbf{a} depends on the safety managers concern for making the wrong decision 1 relative to the concern for making the wrong decision 2. A high \mathbf{a} -value, like 0.95, indicates that making a wrong decision 2 is of more concern to the safety managers than making a wrong decision 1. One of many reasons for this attitude could be that the risk distribution, $p(r)$, is based on a small amount of data which, to the safety managers judgement, does not cover the full range of possible accident sizes in the system. The use of a high \mathbf{a} -value is in this case an attempt to decrease the possibility of having to change the judgement from tolerable to intolerable, when including new data which provides a more complete picture of the range of accident sizes. The use of high \mathbf{a} -values could also be due to the safety managers desire to comply with public demand for high safety standards. In fact high \mathbf{a} -values

are in accordance with HSE's recommendation to "err on the side of safety" when making risk tolerability evaluations under uncertainty. Or put differently, HSE recommends to judge the activity intolerable if the estimated individual risk posed by the activity is less than but close to the just tolerable risk.

For the purpose of the analysis in this paper α is set to 0.95. Apart from settling on an α -value knowledge about the probability distribution $p(r)$ is necessary in order to apply the decision rule. The next section is concerned with how such a probability distribution can be derived.

3. MODELLING THE INDIVIDUAL RISK DISTRIBUTION

The individual risk for a person representing a specific group of people can be calculated as the expected number of fatalities per unit time divided by the group population. From the modelling point of view the stochastic process generating the fatalities may be conceptualised in two equivalent ways: first, as a set of independent superimposed random processes, each of which having its own mean frequency and generates accidents of a determinate size; or, secondly, as a single random process generating the fatal accidents followed by a second independent random process which generates the number of fatalities once an accident has occurred. With the latter conceptualisation the individual risk per unit time is calculated as:

$$R = \frac{AM}{G} \quad (2)$$

G is the group population of the individuals for which the individual risk is calculated. A is the fatal accident frequency, that is, the mean number of **fatal** accidents which the transport activity's accident process generates per unit time. M is the mean number of fatalities per fatal accident. It is assumed that the group population is known for certain. The uncertainty aspect is thus confined to the fatal accident frequency and mean number of fatalities per fatal accident.

Bayesian statistics could be used to derive $p(r)$. With Bayesian statistics (see Maritz & Lwin (1989)) it is possible to derive distributions of the uncertain parameters on the basis of the available data and justified assumptions on the parameter characteristics. The initial assumptions about the parameter characteristics are represented by the so called prior distribution. The prior distribution of, say, the accident frequency describes the degree of belief in the possible values of A before looking at any accident data. On the basis of accident data giving evidence on the parameter the prior distribution is updated. The updated prior distribution is called the posterior distribution. The posterior distribution of A describes the beliefs about the possible values of A in light of new accident data. Once the posterior distributions of A and M have been derived it is possible to derive the posterior distribution of R through the relationship in (2). Section 3.1 and 3.2 describes the derivation of the posterior distributions for A and M . The posterior distribution of the individual risk, R , is derived in section 3.3

3.1 The posterior distribution of mean number of fatalities

Let $p_1(m)$ be the prior distribution of M (m is the value of M) and $p_2(m | (J,F))$ be the posterior distribution of M , where (J,F) is the new data giving evidence on M . J is the number of fatal

accidents and F is the total number of fatalities in the J accidents. By Bayes theorem the relationship between $p_1(m)$ and $p_2(m | (J,F))$ is:

$$p_2(m|(J, F)) \propto l((J, F)/m) p_1(m) \quad (3)$$

The likelihood $l((J,F) | m)$ is the probability of having observed the new data given $M=m$, that is:

$$l((J, F)/m) = \prod_{i=1}^J p(K = k_i/m) \quad (4)$$

$p(K=k_i | m)$ is the probability of observing k fatalities in a fatal accident given the mean number of fatalities per accident is m . From (4) it is seen that an assumption about the accident size distribution is necessary in order to derive $l((J,F) | m)$. By accident size distribution is meant the distribution of the number of fatalities in a fatal accident.

Observed accident size distributions in rail transport often tends to be skewed to the right. That is, the frequency of fatal accidents with many fatalities is very small relative to the frequency of single fatality accidents. In an analysis of the size distribution of observed fatal collisions, derailments and overruns on British Railways Evans & Verlander(1996) found that the *logarithmic series distribution* gave a good fit to the data. Unfortunately, the logarithmic series distribution is a very cumbersome distribution for the intended analysis in this paper (for example given the distribution's parameter the mean number of fatalities, F , can only be found through iteration). Therefore, one could instead use another skew and more tractable distribution like the geometric distribution to represent the variation in the accident size. However, when fitted to the data in Evans and Verlander(1996) the geometric distribution turns out to perform not as well as the logarithmic series distribution. Despite of this result the geometric distribution is used for the analysis in this paper.

The main difference between the two distributions is that the tail of the logarithmic series distribution is longer than the tail of the geometric distribution. In other words if the geometric distribution were used to represent the accident size distribution for the rail activity analysed in Evans & Verlander(1996) the distribution would underestimate the occurrence of large accidents. This result does not imply that there does not exist public transport activities for which the geometric distribution may be an appropriate accident size distribution. One should just be aware of the fact that choice of accident size distribution may influence the conclusions drawn from the risk analysis. Therefore, for the purpose of a more complete picture of the effects of uncertainty on risk tolerability, the issue of choice of accident size distribution for the tolerability analysis ought to be addressed. However, it is out of the scope of this paper to pursue the issue to any great length.

If the accident size is geometric distributed the probability of observing k fatalities in a fatal accident is:

$$p(K = k) = (1 - \mathbf{q})\mathbf{q}^{k-1} \quad \text{for } 0 < \mathbf{q} < 1 \text{ and } k = 1, 2, \dots \quad (5)$$

Since M is the mean number of fatalities per fatal accident the relationship between the value of M and the parameter \mathbf{q} is:

$$m = \sum_{k=1}^{\infty} k(1-q)q^{k-1} = \frac{1}{1-q} \quad (6)$$

With the relationship in (6) it is possible to derive the posterior of M with the help of the posterior distribution of q . Suppose that for the transport activity under scrutiny there is no initial knowledge of the value of q . One way of representing this initial state of ignorance of q would be to chose a prior distribution, $p_1(q)$, of the form:

$$p_1(q) = \frac{1}{q} \quad (7)$$

Other prior distributions than the one given above could be used but the reason why (7) is preferred will be commented on later in the text. After having observed J fatal accidents with a total number of F fatalities the likelihood $l((J,F) | q)$ is:

$$l((J,F)|q) = (1-q)^J q^{F-J} \quad (8)$$

By Bayes theorem in (3) the posterior of q is consequently proportional to:

$$(1-q)^J q^{F-J-1} \quad (9)$$

Apart from a constant, which is the inverse of the Beta function $B(J+1, F-J)$, (9) is identical to the Beta distribution with the parameters $J+1$ and $F-J$ (Maritz & Lwin p. 30). Thus the posterior of q is the Beta distribution :

$$p_2(q|(J,F)) = \frac{(1-q)^J q^{F-J-1}}{B(J+1, F-J)} \quad (10)$$

From (7) it can be deduced that $q = 1-1/m$ and $dq/dm = -1/m^2$. The posterior distribution, $p_2(m|(J,F))$, of M can be derived by multiplying the frequency distribution in (10) with the absolute value of $-1/m^2$ and finally substituting q with $1-1/m$, that is:

$$p_2(m|(J,F)) = \frac{1}{B(J+1, F-J)} \left(\frac{1}{m}\right)^{J+2} \left(1 - \frac{1}{m}\right)^{F-J-1} \quad (11)$$

The mean of $p_2(m | (J,F))$ is equal to F/J (see appendix 1), which is the mean number of fatalities per fatal accident in the observed accident data. This property of the distribution in (11) is due to, and the reason why, (7) is used as the prior distribution of q . Furthermore the prior distribution in (7) requires at least one observed fatal accident before it is possible to make any inferences about q (and therefore also M). This property is a suitable way of representing total ignorance of q . Without any data (7) is an improper probability distribution, that is, it does not equal unity when integrated over the possible values of q , as a proper probability distribution would.

The variance of (11) is in appendix 1 given as $F(F-J)/J^2(J-1)$. From this expression it is seen that if

the observed total number of fatalities, F , in the data equals the observed number of accidents, J , the variance of M is 0. M is for $F=J$ treated as being known for certain with a value of 1. This result is unrealistic. Contrary to the model's suggestion one would in reality not be as certain about the true value of M if the accident data consists of 2 single fatality accidents as opposed to a case where the data consists of 100 single fatality accidents. The unrealistic property of (11) results from the prior distribution in (7) and the observed data. When the observed number of fatal accidents equals the observed number of fatalities the data does not provide any information on the variation in the accident size. The prior distribution in (7) is in itself not able to provide any information on the variation of q (and therefore M) because it is improper. So, if the observed data does not provide this information the model assumes that q (and M) is certain. If there, before any observed data, were initial beliefs about the variation of q the prior distribution of q would reflect these beliefs. q (and M) would consequently not be regarded as certain if J happened to equal F in the empirical data. The comments just made are independent of the assumed accident size distribution. Therefore, given an improper prior distribution, and observed data where J equals F , M would be regarded as being certain whatever the assumptions made about the accident size distribution.

From (5) it is seen that the geometric distribution is only defined for q -values in the interval between 0 and 1. When M is 1, q is 0, which represents the situation just commented on above. A q equal to 1 corresponds to an infinitely large value of M . An infinitely large M is unrealistic in practice for the type of transport activity under scrutiny. Therefore the restriction that q cannot be 1 does not add more unrealistic properties to the model.

3.2 The posterior distribution of the fatal accident frequency

In public transport it is reasonable to assume that the number of accidents over T units of time is Poisson distributed. With this assumption the derivation of the posterior distribution of the fatal accident frequency follows the same steps as in the previous section.

The improper prior distribution below could be used to represent the initial stage with no knowledge of the accident frequency's size distribution.

$$p_1(a) = \frac{1}{a} \quad (12)$$

With J observed fatal accidents during a period of T units of time, the Poisson assumption leads to the likelihood $l((T, J) | a)$:

$$l((T, J) | a) = \frac{(aT)^J}{J!} \exp(-aT) \quad (13)$$

The likelihood above is the probability of observing J accidents when the mean is aT . By Bayes theorem we have that the posterior distribution of A is proportional to:

$$a^{J-1} T^J \exp(-aT) \quad (14)$$

Thus the posterior of A equals the gamma distribution, $\Gamma(J, 1/T)$, given below:

$$p_2(a|(T, J)) = \frac{1}{\Gamma(J)} a^{J-1} T^J \exp(-aT) \quad (15)$$

The mean of (15) is J/T , which is the observed mean number of fatal accidents per unit time. The reason for using (12) to represent the initial state of ignorance of A is that it is equivalent to starting with $J=0$ and $T=0$. So, similar to the prior distribution in section 3.1, the chosen prior distribution of A requires at least one observed fatal accident before any conclusions on A can be drawn. The variance of the fatal accident frequency, A , is J/T^2 .

3.3 The posterior distribution of the individual risk

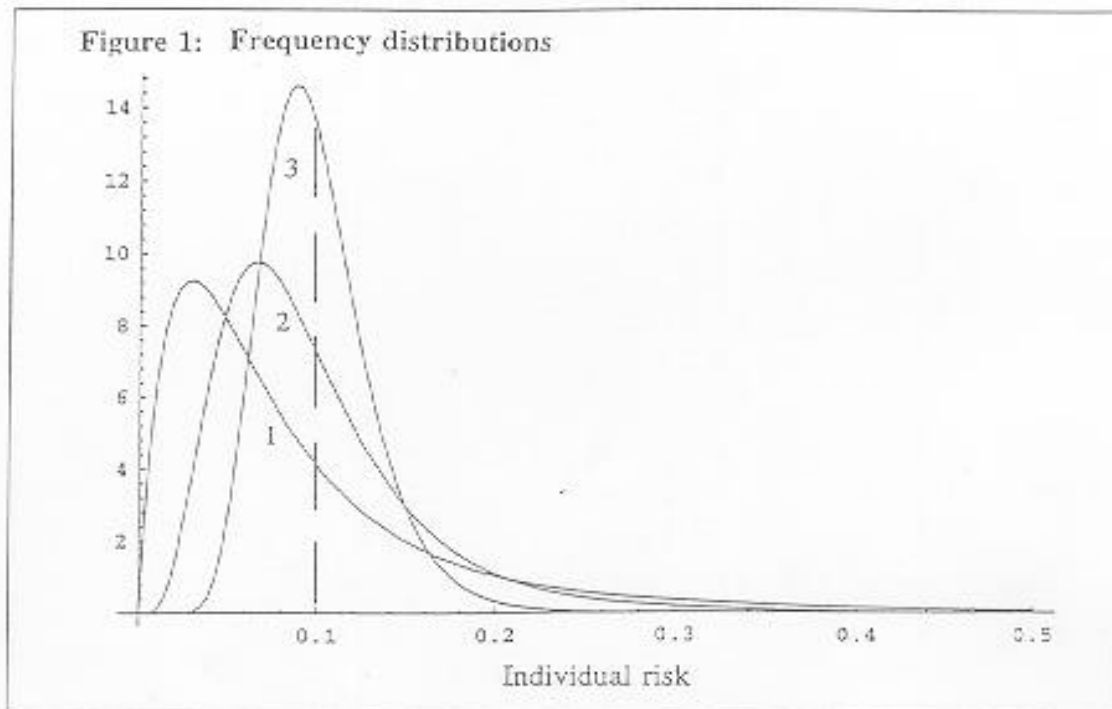
With the posterior distributions from section 3.1 and 3.2 it is now possible to derive the posterior distribution of the individual risk, R , by using the relationship in (2). In appendix 2 it is shown that the posterior distribution of the individual risk is:

$$p_2(r|(G,T,J,F,)) = \frac{(GT)^J r^{J-1}}{B(J+1, F-J)\Gamma(J)} \int_0^1 q^{2J} (1-q)^{F-J-1} \exp(-GTqr) dq \quad (16)$$

G is the group population. J is the observed number of fatal accidents happened during the T units of time. F is the observed number of fatalities in the J fatal accidents. T is the period for which the transport activity is analysed. The mean, $E(R)$, and variance, $V(R)$, of R is (Aagaard(1995)):

$$E(R) = \frac{F}{GT} \quad \text{and} \quad V(R) = \frac{1}{(GT)^2} \left(\frac{2F^2 - FJ - F}{J-1} \right) \quad (17)$$

It is seen that $E(R)$ is the individual risk one would be able to calculate on the basis of the historical data. The certainty of this mean will increase as the data sample giving evidence on the mean increases. This relationship is illustrated in figure 1 next page. The figure shows three posterior distributions of R . Distribution 1 is based on 2 fatal accidents with 10 fatalities observed during 1 year of activity. Distribution 2 is based on 6 fatal accidents with 30 fatalities observed during 3 years of activity. Finally distribution 3 is based on 20 fatal accidents with 100 fatalities observed during 10 years of activity. G is set to 100 individuals for all three distributions. The distributions have the same mean individual risk of 0.1 per year. It is seen that the posterior distribution becomes more concentrated around the mean as the historical data increases. It is also seen that for small amounts of historical data the individual risk distribution is skew to the right. However, as the evidence on R increases the more symmetric is the posterior distribution around its mean.



4. THE EFFECT OF UNCERTAINTY ON THE TOLERABILITY DECISION

With the individual risk distribution in (16) it is now possible to illustrate the use of the tolerability decision rule in section 2. The line labelled A1 in figure 2 represents the relationship between the mean of the individual risk distribution and its corresponding 95-percentile as a function of the number of observed fatalities in 5 fatal accidents during 1 year. The group population has been set to 1000 individuals. An observed mean individual risk of 0.1 per year is equivalent to having observed 100

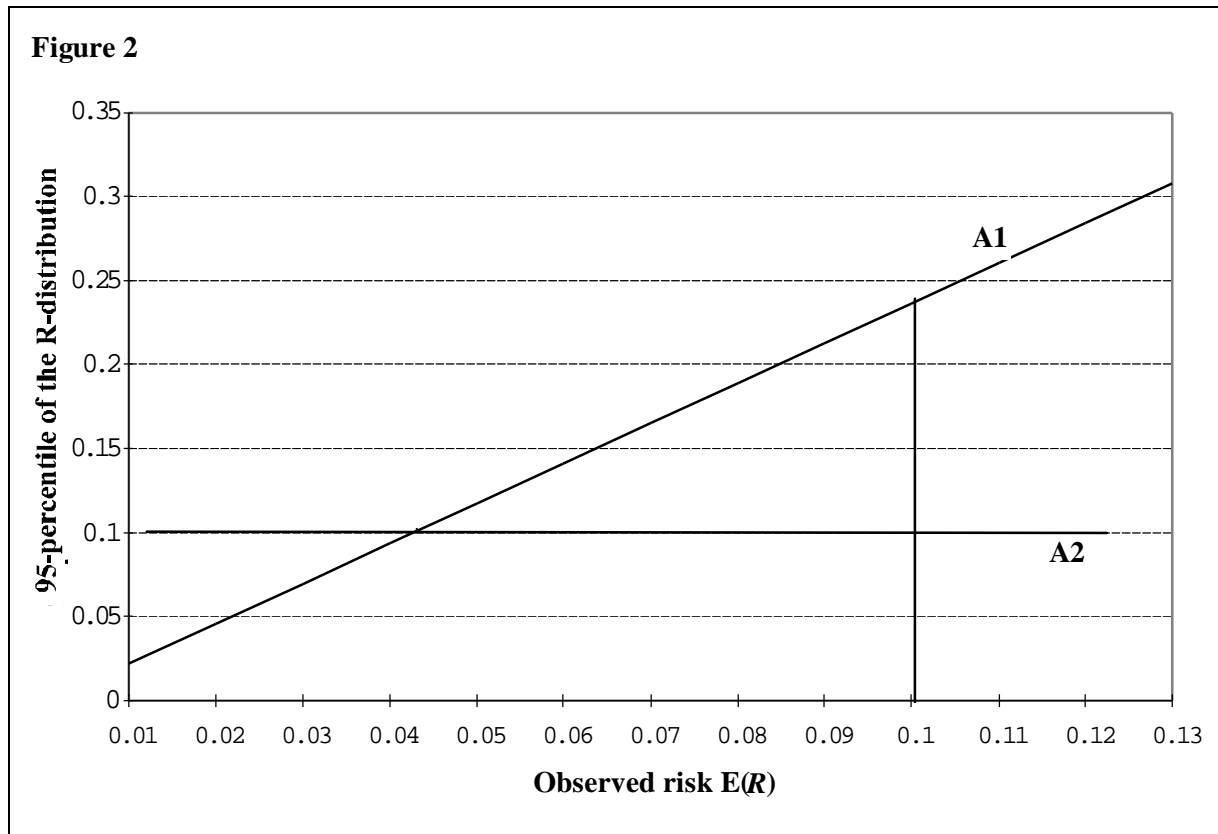
(= $E(R) \times G \times T = 0.1 \times 1000 \times 1$) fatalities in the 5 fatal accidents during the 1 year of activity.

The horizontal line labelled A2 represents the just tolerable individual risk which, without loss of generality, is set to 0.1 per year. By using the decision rule in section 2 a public transport activity would only be judged tolerable if the 95-percentile of the individual risk distribution is equal to or less than 0.1 per year. It is seen from figure 2 that an observed mean of 0.1 per year would render the transport activity intolerable. The 95-percentile which corresponds to a mean of 0.1 is 0.23 per year which is above the just tolerable individual risk of 0.1 per year. Had the uncertainty in the individual risk not been taken into account the activity would have been just tolerable because the mean, 0.1 per year, is equal to the just tolerable risk.

The intersection between line A and line B defines at what observed mean individual risk the activity changes from being tolerable to intolerable. It is seen that the change over point ($E(R) \approx 0.042$ per year) is less than the just tolerable individual risk of 0.1 per year. This is not surprising, since using a 95%-percentile for the decision rule in (2) is in accordance with "erring on the side of safety" under

uncertainty.

If safety managers would find, say, a 80-percentile more appropriate for the tolerability decision the effect would be an change over point closer to the just tolerable individual risk of 0.1 per year. Correspondingly the use of a larger percentile than 0.95 would lead to a change over point smaller than $E(R)=0.042$ per year in figure 2.



From figure 1 it is seen that the individual risk distribution becomes more concentrated around its mean as the amount of data increases. The effect of this relationship on the decision rule is that the change over point moves towards the just tolerable individual risk as the amount of data increases. In other words, the more the evidence the less is it necessary to “err on the side of safety” as one intuitively would expect.

The qualitative results of the change over point's behaviour would not change if the individual risk distribution were used instead of the normal approximation. This is also true had another accident size distribution than the Geometric distribution been used in the individual risk model.

Safety managers may sometimes consider using a 50-percentile for the tolerability evaluation. They may have reasons for using a 50-percentile but a 50-percentile is not in accordance with "erring on the side of safety" under uncertainty. Why this is so is due to that the mean of the individual risk will normally be used to represent the activities safety level when uncertainty is disregarded. From figure 1 it is seen that the individual risk distributions are skew to the right. Because of this skewness the

mean of the individual risk distribution is larger than the 50-percentile. Using a 50-percentile, which is smaller than the mean, could therefore result in cases where the activity is judged tolerable, while the activity's mean individual risk is above the just tolerable risk. Judging an activity tolerable under such circumstances is not in accordance with "erring on the side of safety" under uncertainty. "Erring on the side of safety" under uncertainty would imply judging the activity intolerable when the mean is above or smaller than but close to the just tolerable risk.

Line A1 in figure 2 has been derived on the assumption that (16) can be approximated with a normal distribution with the mean and variance given in (17). The reason for using the normal approximation is that it is very cumbersome to derive the cumulative distribution of (16) for even small numbers of observed fatal accidents, J , and fatalities F . However, numerical analysis have shown that the normal approximation differs at maximum by +/- 10 % from the true 95-percentile of the individual risk distribution; which is acceptable for this analysis.

5 CONCLUSION

The purpose of this paper has been to elucidate how uncertainty in the fatal accident frequency and the fatal accident outcome may affect the evaluation of whether a public transport activity operates at a tolerable individual risk level or not. The analysis is done on the basis of an individual risk model based on Bayesian statistics. In the model the individual risk is a function of the frequency of fatal accidents and the mean number of fatalities per fatal accident. By the use of the individual risk model it is shown that previously tolerable transport activities may be judged intolerable when uncertainty is taken account. This result is the consequence of safety managers' concern under uncertainty of judging a transport activity tolerable when in fact it is intolerable.

The individual risk model can be used for the safety evaluation of any transport activity with multi-fatality accidents. However, the accident process in the transport activity under evaluation must meet the assumptions made in the paper, that is, 1) the fatal accidents happen according to a Poisson process, 2) the number of fatalities in a fatal accident is Geometrically distributed and, 3) the outcome of a fatal accident is independent of the accident frequency.

The assumption that the accident size is Geometrically distributed is believed to be more restrictive for practical applications of the risk model than the assumption that the fatal accidents happen according to a Poisson process. To what extent the obtained results presented in this paper would be affected by other accident process assumptions is not clear at present. More research would be necessary to elucidate this question.

Apart from the research presented in this paper it is in Aagaard(1996) furthermore analysed: 1) how safety managers can decide on which of the two risk model parameters to improve when constraints on resources makes it difficult to improve both parameters, and 2) how uncertainty will affect the amount of resources spend on safety measures implemented to reduce individual risk.

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Appendix 1 Mean and variance of the mean number of fatalities per fatal accident

The mean of M can be calculated as:

$$\begin{aligned}
 E(M) &= \int_1^{\infty} m p_2(m|(J,F)) dm = \int_0^1 \frac{1}{1-q} p_2(m|(J,F)) dq \\
 &= \frac{1}{B(J+1,F-J)} \int_0^1 (1-q)^{J-1} q^{F-J} dq = \frac{B(J,F-J)}{B(J+1,F-J)} = \frac{F}{J}
 \end{aligned}
 \tag{A1}$$

The mean of M^2 can similarly be calculated as:

$$\begin{aligned}
 E(M^2) &= \int_1^{\infty} m^2 p_2(m|(J,F)) dm = \int_0^1 \frac{1}{(1-q)^2} p_2(m|(J,F)) dq \\
 &= \frac{1}{B(J+1,F-J)} \int_0^1 (1-q)^{J-2} q^{F-J-1} dq = \frac{B(J-1,F-J)}{B(J+1,F-J)} = \frac{F(F-1)}{J(J-1)}
 \end{aligned}
 \tag{A2}$$

The variance of M is thus:

$$V(M) = E(M^2) - E^2(M) = \left(\frac{F(F-1)}{J(J-1)} - \frac{F^2}{J^2} \right) = \frac{F(F-J)}{J^2(J-1)}
 \tag{A3}$$

Appendix 2 Derivation of the posterior frequency distribution of the individual risk, R

Let X and L be independent stochastic variables with their respective frequency distributions $h1(x)$ and $h2(l)$. The frequency distribution of $S = XL$ is then given by the equation below:

$$p(s) = \int_{-\infty}^{\infty} \frac{1}{|x|} h1(x) h2\left(\frac{s}{x}\right) dx
 \tag{A4}$$

A and M are assumed independent. A is defined for values larger than or equal to 0 and M is only defined for values larger than or equal to 1. The fatality frequency Y (fatalities per unit time) is equal to AM . The frequency distributions of A and M are given in section 3. Substituting a with y/m and applying (A4) for the derivation of the posterior distribution of Y results in:

$$\begin{aligned}
p_2(y|(T,J,F)) &= \int_1^{\infty} \left[\frac{1}{m} \frac{1}{B(J+1,F-J)} \left(\frac{1}{m}\right)^{J+2} \left(1-\frac{1}{m}\right)^{F-J-1} \frac{1}{\Gamma(J)} T^J \left(\frac{y}{m}\right)^{J-1} \exp\left(\frac{-Ty}{m}\right) \right] dm \\
&= \frac{T^J y^{J-1}}{\Gamma(J)B(J+1,F-J)} \int_1^{\infty} \left(\frac{1}{m}\right)^{2J+2} \left(1-\frac{1}{m}\right)^{F-J-1} \exp\left(\frac{-Ty}{m}\right) dm
\end{aligned} \tag{A5}$$

Substituting $1/m$ with q leads to $(dq/dm = -1/m^2)$:

$$p_2(y|(T,J,F)) = \frac{T^J y^{J-1}}{\Gamma(J)B(J+1,F-J)} \int_0^1 q^{2J} (1-q)^{F-J-1} \exp(-Tqy) dq \tag{A6}$$

The individual risk R multiplied with the group population G is equal to Y . So, substituting y with rG ($dy/dr = G$) leads to the following posterior distribution of the individual risk, R :

$$p_2(r|(G,T,J,F)) = \frac{(TG)^J r^{J-1}}{\Gamma(J)B(J+1,F-J)} \int_0^1 q^{2J} (1-q)^{F-J-1} \exp(-TGqr) dr \tag{A7}$$

The mean of R is:

$$E(R) = \frac{E(AM)}{G} = \frac{E(A)E(M)}{G} = \frac{1}{G} \frac{J}{T} \frac{F}{J} = \frac{F}{GT} \tag{A8}$$

and the variance is:

$$\begin{aligned}
V(R) &= \frac{E((AM)^2) - E^2(AM)}{G^2} = \frac{E(A^2) E(M^2) - E^2(A) E^2(M)}{G^2} \\
&= \frac{1}{G^2} \left(\frac{J}{T^2} + \frac{J^2}{T^2} \right) \left(\frac{F(F-1)}{J(J-1)} \right) - \frac{F^2}{G^2 T^2} \\
&= \frac{1}{(GT)^2} \left(\frac{2F^2 - FJ - F}{J-1} \right)
\end{aligned} \tag{A9}$$